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*The Cole Lectures for 1911
delivered before Vanderbilt University*

Some Great Leaders in The World Movement

By
ROBERT E. SPEER



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THE COLE LECTURES

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“The object of this fund is to establish a foundation for a perpetual Lectureship in connection with the Biblical Department of the University, to be restricted in its scope to a defense and advocacy of the Christian religion. The lectures shall be delivered at such intervals, from time to time, as shall be deemed best by the Board of Trust; and the particular theme and lecturer shall be determined by nomination of the Theological Faculty and confirmation of the College of Bishops of the Methodist Episcopal Church, South. Said lecture shall always be reduced to writing in full, and the manuscript of the same shall be the property of the University, to be published or disposed of by the Board of Trust at its discretion, the net proceeds arising therefrom to be added to the foundation fund, or otherwise used for the benefit of the Biblical Department.”

Foreword

I AM very grateful for the honour and privilege of the appointment to this lectureship. Some of life's best friendships have been with men of this University and of this Department of the University, and it is with thankful remembrance of them that I propose to speak of some of the great leaders who preceded them and whose wide thoughts and high spirit this University has sought to perpetuate.

R. E. S.

New York City.

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LECTURE I

RAYMOND LULL, THE CHRISTIAN
CRUSADER AND HIS CONQUEST

LECTURE I

RAYMOND LULL, THE CHRISTIAN CRUSADER AND HIS CONQUEST

THE two most powerful forces in the world are principles and personalities. It is by these that the living movements of the world are advanced, by the truth which they embody and the men who embody them. We who are here to-day do not hold to the philosophy that the truth in such movements is the creation of the men who hold it. We believe that the truth is the truth whether men find it or miss it, declare it or deny it, and that by itself it is ever at work in the world. Right principles, we hold, exist independent of their discovery and are ever operating on life by the Divine Spirit. We believe this because we believe in God Who is the Truth, Who lives in Himself and not alone in man's apprehension of Him, and works by Himself and not alone in the assenting wills of men. But we hold, also, that God is seeking men who will advance the truth by making it a part of themselves, and who will give to principles a home and a leverage within their personalities, and in so doing recover for personality its true glory and power. Indeed, it is the very glory and power of God Himself that in Him principle and personality are perfectly one. He is what we can only say that we have. Man holds the truth. God is the Truth.

Man does right. God is Right. From the beginning the self-revelation of God was a revelation of principle as personality. "Who shall I say sent me?" asked Moses. "Say 'I am' hath sent you," replied Jehovah,—not He Who has power, but He Who is Power; not He Who declares truth, but He Who is Truth.

The Incarnation is the central fact in history, and the most powerful agency in the life of the world, because it was the supreme and faultless instance of the highest principle and the purest personality perfectly joined and interfused. And human personalities have had lasting beneficent power in proportion as they have resembled the Incarnate Personality and have embodied eternal principles in life and contributed to the ceaseless effort to solve our human problems by ordering life according to the eternal principles which are the character of God. I think, accordingly, that whether we seek guidance or inspiration, needing courage for our own struggle or light on the central issues of life, no study can be more profitable to us than the study of such personalities as these in their wrestle with great problems and their embodiment of the great principles of the kingdom of Christ.

We are to consider six leaders whose lives will illustrate some of these fundamental principles. But they will do more than this. They will recall us to the highest standards of duty and devotion. They will bring us into contact with the deepest problems of human progress. From some of them we shall learn how profound and lasting an impression humble men, without resources or any adventitious aids to

influence, can make upon the life of the world, and others, alas, will show how, in the age-long work of the Church, applying the redemption of Christ to the world, the richest personal influence may appear to spend itself fruitlessly, leaving no enduring impression upon the elements which it sought to mould, abiding only, as far as man may see, in a fragrance of holy life and passionate love in the Church and in an unanswered but immortal summons to the completion of what it attempted to begin.

It is a good thing for us at this time of proper zeal for immediacy in the work of the kingdom of Christ, to consider carefully the lives of some of these men who represented the unconquerable patience of the Christian spirit. Bushnell has advanced as one of the characteristics of Jesus, forbidding His classification with men, the quality of faith which enabled Him to sit down in quietness before a perpetual project. The spirit of a human institution, or the spirit of a man may enable the institution or the man to meet with unwavering courage and determination the sharpest tests of a definite campaign. But the Christian movement is not a definite campaign. It has lasted for centuries. It is full of disappointments which crush the heart and break the will. If any agency embarks on it in the human spirit alone, it is doomed to shameful disaster. Nothing but the Christian spirit has ever shown itself capable of the patience and the sacrifice demanded of men who are not engaged in a four months' or a four years' or a four centuries' campaign, but in a project as perpetual as the need of man and the love of Christ for the lost. Whoever will do truly in the world the

work of the Church, must do it in the spirit of the great but simple men whom we are to consider, who followed Christ and in consequence led their fellow men.

Of the six whose lives we are to review and the lessons of whose leadership we are to study, two, William Carey and Alexander Duff, were great missionaries who had the statesman's vision and power, and whose names are known to all the Christian Church; two, John Lawrence and "Chinese" Gordon, were great political administrators who had the missionary's spirit and devotion and who appealed as much as any two men of the nineteenth century to all that was heroic and chivalrous in the imagination of the world; and two, Raymond Lull and George Bowen, though they stand together in the foremost rank of Christian men who rose to uniqueness of character and conceived and fulfilled original tasks, have had but a small place in Christian literature and perhaps no place at all in the thought of some of us who are gathered here to-day. There is no biography at all of Bowen (perhaps the greatest present lack in missionary literature), and there has been until recently no English life of Lull. We need to become better acquainted with all of these men, especially with these two of whom probably we know least. Such acquaintance with the greatest man who ever went forth to the Mohammedans and with the most holy and revered missionary in India in his day, and perhaps in any day, will help each of us who, realizing his own imperfectness of character, is ready to learn from another what far greater things are possible and how they may be attained.

For after all, all worthy greatness is greatness of character, and while, as the lives of Lull and Bowen and Gordon, also, sadly prove, greatness of character may lavish itself in vain upon those whom it seeks to win, yet nothing else will succeed where it fails, unless it is with a meretricious or inferior success. The high and real work of the world, the work that is essentially and distinctively Christian, is personal influence grounded in and springing from character.

In the missionary propaganda, especially, the supreme element is personality. It is not linguistic skill, or intellectual superiority, or fertility of method, or masterfulness of organization, or physical endurance. All these are valuable and desirable, but the one vital and indispensable thing is holy personality, the presence in the human agent of the divine life, and the unwearied attempt to impart that life through the exposure of personality, so to speak,—its living contact with other living spirits, lacking as yet the divine indwelling. If this mode of speech seems to be mystical, it is only because Christianity cannot be stated otherwise. Even Harnack, who wants no mystical Christology in his Gospel, admits that "it is not as a mere factor that He (Jesus) is connected with the Gospel; He was its personal realization and its strength, and this He is felt to be still. Fire is kindled by fire; personal life only by personal forces. . . . The Gospel nowhere says that God's mercy is limited to Jesus' mission. But history shows that He is the one Who brings the weary and the heavy-laden to God; and, again, that He it was Who raised mankind to the new level and His teaching is still

the touchstone, in that it brings men to bliss and to judgment. . . . For those who followed Him, Jesus was Himself the strength of the Gospel. What they experienced, however, and came to know in and through Him, they have told the world; and this message is still a living force." It is on Harnack's own showing a living force because it carries and kindles a personal life.

The simple truth is that Christianity is Christ, not metaphorically but really, and true Christians proclaim Christ, not metaphorically, but actually. As they live truly, they live Christ and embodying in themselves the divine life, they go out to communicate it to the world. All philanthropic and educational aspects of Christian work are secondary. If they do not flow from the inner divine life as their source, they are unreliable and precarious. If the inner divine spring is there, all such accessory benefits flow forth continually. St. Paul's interpretation is eternally true. What a man is in Christ and what Christ is in a man—these are the only questions of any significance. It is with men in Christ and Christ in men that we are to meet in these leaders in the moral movement of the world.

Of all the men of his century of whom we know, Raymond Lull was most possessed by this love and life of Christ and most eager, accordingly, to share his possession with the world. The world sadly needed it; the Church scarcely less. It brings out the greatness of Lull's character strikingly to note how sharply he rose above the world and Church of his day, anticipating by many centuries moral standards, intellectual conceptions and missionary ambi-

tions to which we have grown only slowly since the Reformation. What the general state of morality was, in the thirteenth century, may be gathered from the character of the popes. "Gregory VII and Innocent III were great popes and mighty reformers of a corrupt priesthood, but they were exceptions in the long list. One of the popes was deposed on charges of incest, perjury, murder and blasphemy. Many were in power through simony. Concubinage and unnatural vices were rife in Rome among the clergy. Innocent IV, who reigned the very year Lull was born, was an outrageous tyrant. Nicholas III and Martin IV were popes towards the close of the century. The pontificate of the first was so marked with rapacity and nepotism, that he was consigned by Dante to his Inferno. The latter was the infamous cause of the terrible Sicilian Vespers."¹

Intellectually, as Symonds says, "there were by no means lacking elements of native vigour, ready to burst forth. But the courage that is born of knowledge, the calm strength begotten of a positive attitude of mind, face to face with the dominant overshadowing sphinx of theology, were lacking. We may fairly say that natural and untaught people had more of the just intuition that was needed than learned folk trained in the schools. Man and the universe kept on reasserting their rights and claims in one way or another; but they were always being thrust back into Cimmerian regions of abstractions, fictions, visions, spectral hopes and fears, in the midst of which the intellect somnambulistically moved upon an unknown

¹ Zwemer, "Raymond Lull," Chapter I.

way."¹ It was an age of superstition coupled with the fear of investigation of the world, when Roger Bacon was sent to a dungeon for too much inquiry ; of superstition coupled with a passion for the investigation of the inner heart, when the Flagellants scourged themselves over Europe and Catherine of Sienna and St. Francis of Assisi saw visions. And far and wide the tides of life were beginning to move.

The most picturesque feature of the times and the one which bore closest relation to the life and work of Lull was the conflict between Christianity and Islam. For nearly a century and a half, at the time of Lull's birth, the Crusades had shaken Europe and the Holy Land, and tide after tide of life, the best and the worst, had poured out of Europe at flood, only to ebb again in weakness and disaster. Here and there, some earnest soul strove to save and not to slaughter the Moslems. Francis of Assisi was one of these. He attempted in 1219 the evangelization of "the Saracen hosts, then besieged in Damietta by a mass of crusading Franks. Although there was a price upon every Christian's head, the missionary, in his mendicant's gray robe and cord of self-denial, chanting the Twenty-third Psalm, crossed over to the infidels and was hurried into the Sultan's presence, to whom he declared : 'I am sent not of man, but of God, to show thee the way of salvation.' His courage, which to the Oriental seemed the inspiration of madness, was his safeguard ; he was dismissed with honour, and he lived to induce the Sultan Nuleddeen

¹"Encyclopedia Brittanica," Vol. XX, p. 383, "The Renaissance."

to treat the Christian captives kindly and to give the Franciscans the guardianship of the sepulchre of Christ.”¹ The preaching friars also “who started into existence the early part of the thirteenth century, had renounced the idea of solitary life in the monasteries and gone forth among the masses; and to Saracens, Moors and Hebrews, as well as to heathens and professing Christians, they had carried such a Gospel as they knew.”² But all this was exceptional, and the general attitude of Europe towards Moslems was an attitude of hatred and contempt. Christians would go to trouble to kill the Mohammedans and to wrest the sepulchre of Christ from them, but not to save them or win them to faith in the Christ Who had occupied the sepulchre. Raymond Lull was raised up to spend a great life in attempting to teach the Church a new attitude towards Islam.

He was born in 1235, in Palma, the capital of Majorca, an island of the Baleiric group, wrested by James I, King of Arragon, from the Saracens, Lull's father serving in the army of conquest. He grew up in luxury, and as he, himself, says, in sensual living. Fond of pleasure, he and his young wife moved from the island to the court of James II, King of Arragon, and there he became Seneschal of the court. He was a musician, playing the cithern with skill, a poet dealing usually with the pleasures of dishonourable love, and the most popular poet, too, of his age in Spain, but sensual and careless. “I see, O Lord,” he wrote in after life in his book on “The Contemplation of God,” “that the trees every year bring

¹ Smith, “Short History of Christian Missions,” p. 101 f.

² Walsh, “Heroes of the Mission Field,” p. 146.

forth flowers and fruit by which men are refreshed and nourished ; but it is not so with me a sinner. For thirty years I have borne no fruit in the world ; yea, rather I have injured my neighbours and friends. If, therefore, the tree which is destitute of reason brings forth more fruit than I have done, I must be deeply ashamed and acknowledge my great guilt."

Lull was not spending all these years in pure idleness, however. Beside his music, he was interested in all sorts of subjects, in science and discovery and most of all in life. He was exceedingly popular and well liked, and all the qualities and training of these wasted days, he was later to find of use.

At the age of thirty-two, he went back to Palma, and it was there that his life was turned. As he sat one evening on his couch with his cithern on his knee, composing a sensual song in praise of a married woman who had failed to respond to his love, he looked up and saw "on his right hand the Saviour hanging on His cross, the blood trickling from His hands and feet and brow, looking reproachfully at him." Deeply impressed, Lull laid aside his cithern and lay down on his couch. Eight days later, he took up his cithern to complete the song, and again the Saviour appeared to him. It was enough. The vision never again left him, and he rose up that day from his life of sin and uncleanness to follow the Saviour Whom he had seen. The old life did not release him without a struggle, and the issue was in doubt until some months after his visions of the crucified Lord, he went on the 4th of October, 1266, to the Franciscan Church in Palma, at the Festival of St. Francis of Assisi. There, he heard a friar

preacher tell the story of St. Francis who, "son of a rich merchant, lover of pleasure and imprisoned in his youth for a brawl, was suddenly arrested by the Spirit of God. Hearing the voice amid the loneliness of the Umbrian Hills, 'My temple is falling into ruins, restore it,' he took poverty as his bride. He founded the Order of Seraphic Brethren, Minorites or Greyfriars, who from their number and care for the poor have been called the democracy of Christianity."¹ Lull listened to the whole story of Francis, of the stigmata of Christ, of his passion of love, of the fearlessness of his missionary spirit, of his zeal for the conversion of the Saracens. The flame burned anew in his own soul. He decided that he also would renounce the world for Christ. This was his vow of consecration: "To Thee, O Lord God, I offer myself, my wife, my children and all that I possess. May it please Thee, Who didst so humble Thyself to the death of the cross, to condescend to accept all that I give and offer to Thee, that I, my wife and my children may be Thy lowly servants." Accordingly, he sold his property, provided for his family and set out upon the hard and lonely mission of his life.

"Yes, without cheer of sister or of daughter,
Yes, without stay of father or of son;
Lone on the land, and homeless on the water,
Pass I in patience till the work be done."

Some biographers speak of a period of retirement, lasting nine years, which he spent in a cell on Mount Roda, enjoying heavenly illumination and conclud-

¹ Smith, "Short History of Christian Missions," p. 101.

ing at the end of it that he was called to preach the Gospel to the Mohammedans. Whether he had these nine years of solitude or not, he had the heavenly illumination and we could not ask any better credential than the fashion of the man's life and the intensity and purity of devotion with which he gave himself thenceforth to the one great object of his life—the evangelization of Islam. No man is likely to choose that for his life-work without some constraint outside of his own will. And the man who gives himself to it as Lull did, is manifestly guided by God.

At the same time it was most natural that Lull should turn to the Mohammedan missionary problem. Majorca and Minorca had but recently been in Moslem hands. His family history and his racial training kept the idea of the Saracens in his mind. And above all, the Crusades were over and a man as discerning as Lull could see that the results were nothing and the method barren. "I see many knights going to the Holy Land beyond the seas," he said, "and thinking that they can acquire it by force of arms ; but in the end, all are destroyed before they attain that which they think to have. Whence it seems to me that the conquest of the Holy Land ought not to be attempted except in the way in which Thou and Thy apostles acquired it, namely, by love and prayers and the pouring out of tears and blood." In this way, Raymond Lull proposed to attempt to reach the Mohammedan world.

Lull realized, as few did, the nature of his problem. It is evidence of his greatness that he realized that it was a problem. Most of his contemporaries were

either largely or totally ignorant of the character, both of Islam and of Mohammed. They did not know Arabic, were unacquainted with the Koran, hated the Saracen as an enemy and recognized no missionary duty towards him as a man. Lull perceived that a radically different attitude of mind was necessary and at once adopted it for himself.

His first plan was to compose a book which would demonstrate to the Mohammedans the truth and superiority of Christianity, but he realized that he was as yet unprepared to write such a book, and unacquainted with Arabic in which it would have to be written if it was to reach Moslems. He thought of going to Paris to prepare himself for the great controversy, but a Dominican kinsman persuaded him to stay in Majorca and study there. His first work was to master Arabic. As no other teacher was available, he bought a Saracen slave and for nine years was engaged in the study of the language with his slave. "After this long, and we may believe, successful apprenticeship," says Dr. Zwemer, "with the Saracen slave, a tragic incident interrupted his studies. Lull had learned the language of the Moslem, but the Moslem slave had not yet learned the love of Christ; nor had his pupil. In the midst of these studies on one occasion, the Saracen blasphemed Christ. How we are not told; but those who are among Moslems know what cruel, vulgar words can come from Moslem lips against the Son of God. When Lull heard the blasphemy, he struck his slave violently in the face in his strong indignation. The Moslem, stung to the quick, drew a weapon, attempted Lull's life and wounded him

severely. He was seized and imprisoned. Perhaps fearing the death penalty for attempted murder, the Saracen slave committed suicide. It was a sad beginning for Lull in his work of preparation. Patience had not yet had its perfect work. Lull felt more than ever before 'He that loves not, lives not.' The vision of the thorn-crowned head came back to him; he could not forget his covenant. Although he retired for eight days to a mountain to engage in prayer and meditation, he did not falter, but persevered in his resolution."¹

If disciples are to be won in all nations, they must be won by preaching the Gospel and the Gospel can only be preached by men who learn the various languages of the earth. Lull saw this, so obvious to us, so new to men in Lull's days. "O Thou true Light of all lights," he prays, "as Thy grace through the true Faith has enriched Christians before unbelievers, so they are bound to demonstrate the true faith to unbelievers. . . . The holy Church, which consists of the souls of just Catholic men, would be far more beautiful, if there were men acquainted with different languages, who would go through the earth, that unrighteous and unbelieving men might hear the praises of the glorious Trinity and of Thy blessed humanity and of Thy painful passion."

While engaged in studying Arabic, Lull was developing his idea of a great apologetic Christian statement designed to meet the difficulties in men's minds in the way of accepting Christianity; and especially to convince Mohammedans of its divine

¹ Zwemer, "Raymond Lull," Chapter IV.

truth. Either Lull or a relative of his had persuaded Thomas Aquinas to write his book, "On the Catholic Faith, or Summary against the Gentiles." Now Lull himself undertook the task and the result was his "*Ars Major sine Generalis*." "This remarkable treatise," says Dr. Zwemer, "while in one sense intended for the special work of convincing Moslems, was to include 'a universal art of acquisition, demonstration, confutation' and meant 'to cover the whole field of knowledge and to supersede the inadequate methods of previous schoolmen.' " Lull's aim was to use the scholastic method not only in the Church, but as a missionary agency, and to win the Moslem and the heathen by convincing them in fair discussion of the superiority of Christianity. "In his assertion of the place of reason in religion, in his demand that a rational Christianity should be presented to heathendom, Lull goes far beyond the ideas and the aspirations of the century in which he lived. . . . In judging the character of Lull's method, and his long period of preparation, one thing must not be forgotten. The strength of Islam in the age of scholasticism was its philosophy. Having thoroughly entered into the spirit of Arabian philosophical writings and seen its errors, there was nothing left for a man of Lull's intellect than to meet these Saracen philosophers on their own ground. Avicenna, Algazel and Averroes sat on the throne of Moslem thought. Lull's object was to undermine their influence and so reach the Moslem heart with the message of salvation. For such a conflict and in such an age, his weapons were well chosen." ¹

¹ Zwemer, "Raymond Lull," Chapter IV.

The method of Lull became well known and his art, the Lullian art, was very famous in his day. Indeed, it has been said that "for two centuries, the name of Raymond Lull was the best known and perhaps the most influential in Europe."¹ The art of Lull seems very absurd and antiquated to us to-day, however. It "goes beyond logic and metaphysic; as a universal topic, it provides a universal art of discovery and contains the formulas to which every demonstration in every science can be reduced. A sort of cyclopedia of categories and syllogisms . . . a mnemonic, or rather a mechanical contrivance for ascertaining all possible categories that apply to a possible proposition. . . . This Lullian method of a wheel within a wheel seems at first as perplexing as the visions of Ezekiel and as puerile as the automatic book-machine in 'Gulliver's Travels.' But it would be unfair to say that Lull supposed 'thinking could be reduced to a mere rotation of pasteboard circles' or that his art enabled him 'to talk without judgment of that which we do not know.' Lull sought not to give a compendium of knowledge but a method of investigation. He sought a more scientific method for philosophy than the dialectic of his contemporaries. In his conception of a universal method and his application of the vernacular languages to philosophy, he was the herald of Bacon himself. He perceived the possibilities (though not the limitations) of comparative theology and the science of logic as weapons for the missionary."²

These two great ideas of vernacular preaching to

¹ Walsh, "Heroes of the Mission Field," p. 145, quoted.

² Zwemer, "Raymond Lull," Chapter VIII.

heathen and Moslem and of an unanswerable apologetic were supplemented by a third. Lull became possessed with the desire to found missionary colleges which would send out over the world men who would be missionaries after his conception. "I find scarcely any one," he says, "who out of love to Thee, O Lord, is ready to suffer martyrdom, as Thou hast suffered for us. It appears to me agreeable to reason, if an ordinance to that effect could be obtained, that the monks should know various languages, that they might be able to go out and surrender their lives in love to Thee. . . . O Lord of glory, if that blessed day should ever be in which I might see Thy holy monks so influenced by zeal to glorify Thee, as to go into foreign lands in order to testify of Thy holy ministry, of Thy blessed incarnation, and of Thy bitter sufferings, that would be a glorious day, a day in which that glow of devotion would return with which the holy apostles met death for their Lord Jesus Christ." Lull even dreamed of monks of holy lives and great wisdom, "forming institutions in order to learn various languages and to be able to preach to unbelievers." As he had anticipated our modern missionary attitude towards the unevangelized, so he anticipated here as Dr. Smith says, "the great modern conception and agency of societies and churches organized for foreign missions. One step further, but some slight response from his Church or his age, and Raymond Lull would have anticipated William Carey by exactly seven centuries."¹

The only successes with which Lull met in his attempt to establish colleges were two. (1) The

¹ Smith, "Short History of Christian Missions," p. 105.

king of Majorca established there a monastery in which monks were "to be instructed in the Arabic language and trained to become able disputants among the Moslems. Lull had no narrow design for this school. He wanted no dry, theoretical instruction. The language of the people to be reached was taught and the monks were to be made familiar also with the lands in which they lived. "Knowledge of the regions of the world," he wrote, "is strongly necessary for the republic of believers and the conversion of unbelievers and for withstanding infidels and anti-Christ. The man unacquainted with geography is not only ignorant where he walks but whither he leads. Whether he attempts the conversion of infidels or works for other interests of the Church, it is indispensable that he know the religions and the environments of all nations." That is an ideal as natural as it is unrealized in most of our theological schools. Lull insisted that in his schools men should be made familiar with the world in which they were going to live and for which they were to work. If there ever was a day when Lull's ideal was not necessary for us, that day has passed away. The theological school is not faithful to its mission whose department of homiletics, or of theology, or of church history assumes that America is the sole field for the Christian minister, and that its problems are either the only problems or the supreme problems of the Church. (2) Beside getting his school started at Majorca, Lull strove to found colleges elsewhere and several times went to Rome to urge his plans. But popes were busy with the great issues of their day, which are too often the

small issues of all days, and when there were no popes, the cardinals were busy making new ones and had no time to waste on projects for preaching the Gospel, least of all to Mohammedans. All that Lull got as the result of his long agitation was the act of the Council of Vienne in 1311, decreeing that professorships of Oriental languages be established in the Universities of Paris, Oxford and Salamanca.

I have called these the only successes which Lull achieved in his scheme for educating missionaries, but these were not small successes. When one considers the obstacles in his way they were very great successes. They are significant to us, however, chiefly as revealing once more the large-mindedness of this solitary worker labouring for the evangelization of the world and forestalling in his plans men who came centuries later—but who forestalled them even more by living in the thirteenth century the missionary life of the nineteenth, and feeling then the missionary motives to which the Church came only many years afterwards.

Having learned Arabic, written his great book and done his best to establish institutions in Europe which would send out a constant stream of missionaries to Mohammedans, Lull gave himself with all his power to a personal mission to Islam and an unrelenting propaganda at home in support of true missions to Moslems. He believed in rational proclamation of the truth. He did not propose to trust to authority, to education, to political influence, but to the living power of the truth of Christ. He did not fear discussion, or hesitate to compare fairly other religions with Christianity. He believed that

Christianity was the truth and that Christ would triumph as the truth alone, by love. The spirit of Christ which was in his heart enabled him to realize that it was no small thing which he demanded when he asked men to change their religion. His position and spirit are illustrated in what he said about his book, "On the Discovery of Truth," written while waiting at Rome in the vain hope of influencing two successive popes to adopt his mission :

" We have composed this treatise in order that believing and devout Christians might consider that while the doctrine of no other religious sect can be proved to be true by its adherents, and none of the truths of Christianity are really vulnerable on the grounds of reason, the Christian faith cannot only be defended from all its enemies, but can also be demonstrated. And, hence animated by the glowing zeal of faith, may they consider (since nothing can withstand the truth, which is mightier than all) how they may be able by force of argument, through the help and power of God, to lead unbelievers into the way of truth, so that the blessed name of the Lord Jesus, which is still unknown in most parts of the world, and among most nations, may be manifested and obtain universal adoration. This way of converting unbelievers is easier than all others. For it must appear hard to unbelievers to forsake their own faith for a foreign one ; but who is there that will not feel himself compelled to surrender falsehood for truth ; the self-contradictory for the necessary ? . . . Of all methods of converting unbelievers and reconquering the Holy Land, this is the easiest and speediest which is most congenial to love and is so much

mightier than all other kinds and methods, in the proportion that spiritual weapons are more effective than carnal ones. This treatise was finished at Rome in the year 1296, on the holy evening before the Feast of John the Baptist, the forerunner of our Lord Jesus Christ. May he pray our Lord, that as he himself was the herald of light, and pointed with his finger to Him who is the true Light, and as in his time the dispensation of grace began, it may please the Lord Jesus to spread a new light over the world, that unbelievers may walk in the brightness of this light, and be converted to join with us in meeting Him, the Lord Jesus Christ; to Whom be praise and glory forever!"

In this spirit, Lull devoted the last twenty years of his life to the personal proclamation of the Gospel to Mohammedans and Jews. The thirteenth century witnessed a great growth of hatred against the Jews. At the middle of the century, they were expelled from France and at the close, from England. They were insulted and oppressed in countless humiliating ways. But Lull included them in his missionary purpose. He wrote books to convince them of the truth of Christianity and in the interval of his visits to Africa he laboured personally to win the Jews of Majorca. But it was the Moslems who were most on his heart.

In 1291 he left Paris, where he had been lecturing, supporting Christianity and attacking Mohammedan philosophy, and went to Genoa to sail for Africa, to "experiment whether he, himself," as he said, "could not persuade some of them by conference with their wise men and by manifesting to them, according to

the divinely given method, the incarnation of the Son of God and the Three Persons of the Blessed Trinity in the Divine Unity of the Essence." There is something very heroic in the lonely man who was proposing to go out without friend or companion, without support or society to meet the most implacable antagonism Christianity has ever encountered. A ship was in the harbour ready to sail and passage was engaged and Lull's books were put on board, when as Lull says, he was "overwhelmed with terror at the thought of what might befall him in the country whither he was going. The idea of enduring torture or lifelong imprisonment presented itself with such force that he could not control his emotions." His books were brought back and the ship sailed without him. But who can wonder at his hesitancy? As Dr. Smith says: "No such enterprise had been attempted in the history of the Church. This was no careless Crusader, cheered by martial glory or worldly pleasure. His was not even such a task as that which had called forth all the courage of the men who first won over Goth and Frank, Saxon and Slav. Raymond Lull, refused aid and sympathy by Europe, was going forth alone to preach Christ to a people with whom apostasy is death, who had made Christendom feel their prowess for centuries, who had steadily advanced and rarely retreated, who up to this hour have yielded the fewest converts to the Gospel and have attracted the fewest missionaries to attempt their evangelization, even in British India where their toleration is assured. In the light of all subsequent missions to the Mohammedans, Raymond Lull's first hesitation at Genoa is a small thing,

and it was soon purged away by the martyr's crown." ¹

Lull felt keenly the humiliation of his retreat. He felt that he had opened the Gospel and his missionary enterprise to reproach. So bitter was his regret that he fell into a fever. In this fever, he learned of another ship about to sail and insisted upon his friends putting him on board. Convinced that he could not live, they brought him back to shore. But Raymond Lull was not a man whom either death or danger could long daunt, and when a third vessel was ready to sail, he insisted again upon being carried on board and sailed away for Tunis. Once on board, his sickness left him and he rose up with full strength for his new undertaking. Upon landing at Tunis he immediately acted upon his principle of free and fearless discussion and invited the Mohammedan doctors to a conference, in which he proposed that they should compare the evidences on each side and that then all should commit themselves to whatever should be demonstrated as the truth. Lull was no novice in argument upon the character of Islam and he went straight to the core of the question of comparative worth between Christianity and Islam in presenting as the two weak points of the Mohammedan conception of God—a lack of love in His being and a lack of harmony in His attributes. "Every wise man,"—so Maclear summarizes his argument, which Lull gives in his own writings,—“must acknowledge that to be the true religion, which ascribed the greatest perfection to the Supreme Being, and not only conveyed the worthiest conception of all His attributes,

¹ Smith, "Short History of Christian Missions," p. 106.

His goodness, power, wisdom and glory, but demonstrated the harmony and equality existing between them. Now their religion was defective in acknowledging only two active principles in the Deity, His will and His wisdom, while it left His goodness and greatness inoperative as though they were indolent qualities and not called forth into active exercise. But the Christian faith could not be charged with this defect. In its doctrine of the Trinity, it conveys the highest conception of the Deity, as the Father, the Son and the Holy Spirit in one simple essence and nature. In the Incarnation of the Son it evinces the harmony that exists between God's goodness and His greatness ; and in the person of Christ displays the true union of the Creator and the creature ; while in His Passion which He underwent out of His great love for man, it sets forth the divine harmony of infinite goodness and condescension, even the condescension of Him Who for us men and our salvation and restitution to our primeval state of perfection, underwent those sufferings and lived and died for man." ¹

Such preaching could only secure the condemnation of the preacher, not the acceptance of his message. There was no openness of mind to conviction in the Mohammedan doctors. They had hoped to silence Lull easily. They were not willing to be persuaded or outreasoned, and Lull was cast into prison to await deportation, one of the Moslems commending the devotion of the missionary and securing a sentence of deportation rather than death. He escaped from the ship, however, and lay in hiding for three months,

¹ Maclear, "History of Christian Missions in the Middle Ages," p. 362 f.

preaching quietly and composing a new scientific work. But finding any further public discussion impossible, he returned to Naples where he lectured and taught for several years and thence went to Rome and back to Majorca. But the regions beyond were still on his heart,—the people to whom Christ had not been preached, and he soon pressed out upon another itineration, going to Cyprus and on to the Armenians in Cilicia.

Cast out of one city in Africa, however, he refused to believe that Africa was closed and in 1307, at the age of seventy-one, he crossed over to Bugia. There were Christians in the city, but there was no Christian propagandism and the spirit of Islam was orthodox and intolerant. The old code of Omar was as far as possible in force. But all this could not stifle the zeal of the old missionary and he went straight to the market-place and preached Christ. The mob would have killed him but the Mohammedan doctors rescued him and put him in prison. When the chief Mullah expostulated with him on the folly of his conduct, he replied, "Death has no terrors whatever for a sincere servant of Christ who is labouring to bring souls to a knowledge of the truth." In prison he was tempted with sensual offers which the old man, who had forty years before left sin for Christ, treated with scorn, and he spent his time writing another book in defense of Christianity against Islam. He was soon deported, this time without opportunity to escape. On his way back to Europe, the ship was wrecked off Pisa, but Lull was rescued and came on to Genoa and Paris.

Now at last surely the aged warrior had earned rest

and might justly have settled down into some quiet professorship or in some peaceful monastery for his remaining days. But this was not the spirit of Lull. "As a hungry man makes despatch and takes large morsels on account of his great hunger, so Thy servant feels a great desire to die that he may glorify Thee." This was his word to God. "He hurries day and night to complete his work in order that he may give up his blood and his tears to be shed for Thee." And in "Contemplation," he wrote: "As the needle naturally turns to the north when it is touched by the magnet, so it is fitting, O Lord, that Thy servant should turn to love and praise and serve Thee: seeing that out of love to him Thou wast willing to endure such previous pangs and sufferings." And again, "Men are wont to die, O Lord, from old age, the failure of natural warmth and excess of cold: but thus if it be Thy will, Thy servant would not wish to die; he would prefer to die in the glow of love, even as Thou wast willing to die for him." The hunger for martyrdom grew keen in Lull's heart. He could have but a few years for work; why not spend them richly in one more effort to reach the Moslem and then die gloriously in the noble army of the martyrs?

Accordingly, Lull sailed once more for Africa in 1314, in the seventy-eighth year of his age and spent a year in Bugia in seclusion, working quietly, strengthening the little band of converts, gathered as a result of his previous visit to the city. To these and to all who would come, he preached love in love: the love of his own heart pleading with men the love of the heart of God. This was the supreme power of

his own faith, a noble realization of the love of God in Christ. "I have sought Thee on the Crucifix," he writes, "and my bodily eyes could not find Thee there. I have sought Thee with the eyes of my soul, and as soon as I found Thee, my heart grew warmer with the glow of Thy love, and my eyes began to shed tears, and my mouth to praise Thee." And this was his supreme argument with Moslems. "If Moslems," he urged, "according to their law, affirm that God loved man because He created him, endowed him with noble faculties, and pours His benefits upon him, then the Christians, according to their law, affirm the same. But inasmuch as the Christians believe more than this and affirm that God so loved man that He was willing to become man, to endure poverty, ignominy, torture and death for his sake, which the Jews and Saracens do not teach concerning Him; therefore is the religion of the Christian which thus reveals a Love beyond all other love superior to that of those which reveals it only in an inferior degree."

For a time, Lull was able to content himself with the private preaching of Christ as the divine love and the evidence of the love of God; but after a while he wearied of the seclusion and the woe of St. Paul, which had filled his soul for half a century, grew too burdensome. So he came forth from his retirement and openly proclaimed Christ to the people in the public market. As he called upon the people to renounce Mohammed and to follow Christ, he received the crown for which he had longed. The infuriated crowd stoned him to death on June 30, 1315, and he fell asleep. The spirit of the good soldier of Thomas

Fuller had always been in him. "The good soldier grudgeth not to get a probability of victory by the certainty of his own death and fleeth from nothing so much as from the mention of fleeing ; and though the world call him a madman, our soldier knoweth that he shall possess the reward of his valour with God in heaven, and making the world his executor leave to it the rich inheritance of his memory." So the old warrior and saint died on the threshold of his eightieth year, and the world has waited in vain for a missionary to the Mohammedans who could approach him in ability, in energy, in fearlessness, in clean discernment of the issues involved, in power of argument, in passion of love.

Raymond Lull was a Protestant before the Reformation. He was not a Protestant in the sense of separation from or opposition to the Church of Rome, or to the popes of Rome. That Church was his Church and though he knew well the character of some of the popes of his day, and must have been made sick by their irreligion and their blindness to the essential character of Christianity as a missionary power, he had other things to do than to reform the Church or failing, to separate from it. The time had not come for the Reformation and probably the thought of it did not occupy Lull's mind. But he was a Protestant in the character of his religious doctrine, retaining still the personal passionate devotion to Christ which has been too often sacrificed in the interest of purity of theological opinion, and which was the noble adornment of the best mediæval piety. Speaking of Lull's conversion, Mr. Noble says : "This new birth, be it noted, sprang from a passion

for Jesus. Lull's faith was not sacramental, but personal and vital, more Catholic than Roman." Perhaps that is the better word. He was not Protestant or Roman, but simply a Catholic Christian, but a Catholic Christian rather than a Catholic churchman. It was Christ not Church with him. He would have thrown away his time and life if he had gone to Islam with a church religion. Those rigid Monotheists and individualists would have been totally untouched by a doctrine of sacrament and organization, which submerged the sense of individual duty and responsibility and obscured the solemn, central truth of religion—the soul and God ; God and the soul. The right religious attitude is the product of psychology, as much as of theology and ecclesiology and it is hard to set right limits to either one. But it may be believed that no character can better serve than Raymond Lull's to indicate to us the right balance of views and sympathies. Churchmen and Christ-men, historical and dogmatic theologians, and practical Christian missionaries, mystics and devotionalists—all those contradictory and discordant types of Christian character and opinion which fill our world to-day find a common meeting place in the great missionary of the thirteenth century.

The attitude of the Roman Catholic Church to the memory of Lull is confirmatory of the view that he was more Catholic than the Church. His works were condemned by the Inquisition, which the Council of Toulouse had created in 1229 to punish a namesake of his, and which was introduced into Spain 163 years after Lull's death to deal after the fashion of the Inquisition with the Mohammedans and Jews

whom Lull had lived and died to win by love. The Jesuits have always been hostile to his memory, and the Church has hesitated whether to regard him as a heretic or a saint. He has never been canonized and others who have been unworthy to unloose his shoes have usurped the devotion of the Church. In his old home, however, his memory lives, and Dr. Zwemer quotes a letter from the present bishop of Majorca in which he speaks of him as "an extraordinary man with apostolic virtues and worthy of all admiration."

If Lull had not been so universal a character, he might have stood better chance of canonization. But his interests were too wide and were sure to conflict at some point with established prejudices. His attitude towards the rudimentary science of his day was enough in itself to lay him open to suspicion. Arnold de Villeneuve, "the alchemist and pious nobleman," was one of his friends and Arnold incurred the censure of the Church for holding that "medicine and charity were more pleasing to God than religious services," and teaching that "the monks had corrupted the doctrine of Christ and that saying masses was useless; and that the papacy is a work of man." Lull, himself, was a prolific writer and an advanced thinker upon natural philosophy. "One of Lull's biographers states that the books written by Lull numbered 4,000. In the first published edition of his works (1721) 282 titles are given; yet only forty-five of these when printed took up ten large folio volumes. To understand something of the scope and ambition of this prodigious intellect," as Dr. Zwemer says, "we must read the partial list of his books given in the bibliography. . . . Lull was a phi-

losopher, a poet, a novelist, a writer of proverbs, a keen logician, a deep theologian and a fiery controversialist. There was not a science cultivated in his age but he added to it. The critical historian, Winsor, states that in 1295 Lull wrote a handbook on navigation which was not superseded by a better until after Columbus. Dr. George Smith credits Lull with the independent invention of the mariner's compass, and not without reason, for we find frequent reference to the magnetic needle in his devotional books. He wrote a treatise on 'the weight of the elements,' and their shape; on the sense of smell; on astronomy, astrology, arithmetic and geometry. One of his books is entitled, 'On the squaring and triangulation of the circle.' In mediæval medicine, jurisprudence and metaphysics, he was equally at home. His seven volumes on medicine include one book on the use of the mind in curing the sick and another on the effect of climate on diseases. He was a dogmatic theologian and wrote sixty-three volumes of theological discussion."

And his writings did not dry out his spirit. His controversial books, even where he is arguing against Mohammedanism, have the irenic tenderness. He would win by love. Thus he concluded one book: "O Lord, my help! till this work is completed Thy servant cannot go to the land of the Saracens to glorify Thy glorious Name, for I am so occupied with this book which I undertake for Thine honour that I can think of nothing else. For this reason, I beseech Thee for that grace, that Thou wouldst stand by me that I may soon finish it and speedily depart to die the death of a martyr out of love to Thee, if it

shall please Thee to count me worthy of it." Another book on the logic of Christianity he closed thus: "Let Christians consumed with burning love for the cause of faith only consider that since nothing has power to withstand truth, they can by God's help and His might bring infidels back to the faith; so that the precious name of Jesus, which in most regions is still unknown to most men, may be proclaimed and adored."

It is possible that, apart from the evident love of Christ which possessed Lull and the admiration of men for his zeal in the hopeless cause of converting the Saracens, as the world regarded it, Lull may have been left free for his work because he was a layman and a little less subject accordingly to sharp ecclesiastical surveillance. His call to follow Christ and to preach to the heathen, the Jew and the Mohammedan did not appear to Lull to be a call to the priesthood. He sought no honour and he felt the need of no ecclesiastical consecration, of no historic credentials. Had he not seen Christ? Did he not have a message? Were there not millions of unreached souls, whom the attested clergy of his time were leaving to their darkness and their doom? We may think of Lull in this regard as reviving the ideals of the first Christian century, or as anticipating the ideals of the twentieth. In other words, though set in a great historic institution, all that Lull felt was essential was the spiritual. Church order is right and necessary, but what is spiritually real, accredited by the immediate anointing of the living God, does not need ecclesiastical sanction. It bears on its face the irrefutable evidence of life and

reality. At the same time, Lull was not a heedless independent. For one thing, he was born in the only Christian institution there was, and to go apart from the Church would have been, unless he had gone to the Waldensians or the Albigenses, to go out of the company of Christian people. Lull felt also the great desirability of organized missions under proper Church responsibility. He strove again and again to have his mission to the Moslems taken up by the Church, but in vain. What his life stands for is the desirability of the union between free individual movement,—men seeing their own visions and following their own gleams,—and such corporate Church responsibility as shall leave no man to fight the battle single-handed, and shall provide a larger generalship than one mind can furnish, and shall have another ready to catch the shield and sword as they drop from the hand of the falling warrior and renew the battle as an eternal strife till the victory is won.

It was probably advantageous that Lull lacked the technical theological education of his day. It might have stifled the freedom of his mind. It certainly would have tended to chill the zeal of his missionary spirit, which the conventional attitude of his time deemed madness. After all has been said that can be said of the general liberalizing influence of education and of the universities as the centres of reforming tendencies in all ages, it yet remains true that the school has its peril of narrowness, of artificiality, of a sort of procrastination, that plays with the forms of the last generation's thought, when a new generation is already on the stage;

or that invents new forms so lightly that one wonders whether the inventors realize that all such changes cost blood and that that is why they are neither to be resisted when they come nor forced with light-heartedness to come prematurely. The theological discipline of his day would have been better than the school of sensuality in which he grew up, but the latter left him with a mind open to new sympathies, to large services, to fresh conceptions. Mediæval scholasticism would have operated severely against all such. It is enough to urge that the supreme thing in missionary work in all ages is the possession of a life, of the love divine that is life divine and the full preparation of such life for contact in the most intricate and unprejudiced way with other life. No school can give the life and while the presumption always is that the school is the best place for preparation, Lull shows that it is not always so and that sometimes it is better to "buy a Saracen" and get ready in exceptional ways. At any rate, when a man appears who has been so made ready, instead of fearing him because of his singularity, our plans should be so flexible as to open a place at once for him,—for any one who has the life and would give it.

And Raymond Lull had it. Somerville, in "St. Paul's Conception of Christ," points out that it was "in the consciousness of what the glorified Christ was to Paul in his personal life, that we are to look for the genesis of his theology." It was in his inner experience of the glorified Christ that we are to look for the secret and source of Raymond Lull's doctrine and life, what he thought, what he was, what he suf-

ferred. And this must be true of all true missionaries. They do not go out to Asia and to Africa to say, "This is the doctrine of the Christian Church" ; or, "Your science is bad. Look through this microscope and see for yourselves and abandon such error" ; or, "Compare your condition with that of America and see how much more socially beneficial Christianity is than Hinduism or Confucianism or Islam." Doubtless, all this has its place : the argument from the historic evidences of Christianity ; the argument from the coherence of Christianity with the facts of the universe ; the argument from fruits. But it is also all secondary. / The primary thing is personal testimony : "This I have felt. This He has done for me. I preach Whom I know, that which was from the beginning ; that which I have heard, that which I have seen with my eyes ; that which I have looked upon and my hands have handled, concerning the Word of Life (and the Life was manifested, and I have seen and bear witness, and declare unto you the life, the eternal life, which was with the Father and was manifested unto me), that which I have seen and heard declare I unto you also, that ye also may have fellowship with me ; yea, and my fellowship is with the Father and with His Son Jesus Christ." The man who cannot say this may be able to change the opinions of those to whom he has gone, to improve their social condition, to free them from many foolish errors and enslaving superstitions, but after all this, the one thing which if done would of itself have attended to these things and a thousand others may be still unaccomplished, namely, the gift of life.

The man who would do Paul's work or Lull's must be able to preach a living Christ, tested in experience, saved from all pantheistic error by the Incarnation and the roots thus sunk in history, by the Resurrection and the personality thus preserved in God above, but a Christ here and known, lived and ready to be given by life to death, that death may become life.

It would be easy to draw other parallels than this between Paul and Lull,—their conversions, their subsequent times of separation, their visions, their untiring toil, their passion for Christ, their sufferings and shipwrecks, their intellectual activity and power, their martyrdoms, the rule of Christ supreme even in death, supreme also in life, its thought, its purpose, its tastes, its use, its friends, its sacrifices. But the essence of all such comparisons is this—the real essence of all true missionary character, namely, the possession by the life of Christ as life and the ability thus to give not a new doctrine only, not a new truth to men, but a new life. The work of missions is just this,—the going out of the Church over the world ; of a body of men and women knowing Christ, and therefore having life in themselves, their quiet residence among the dead peoples and the resurrection from among these peoples of first one, then a few, then more and more, who feel the life and receive it and live.

It will be worth while in closing to notice a few characteristics of Lull in which the great life in him expressed itself.

1. He sought in every way to fit himself for contact with men so that he might reach them in the deepest intricacies of their life, and be able there to

plant the seed of the divine life which he bore. Therefore, he learned Arabic, became a master of the Moslem philosophy, studied geography and the heart of man. And therefore, also, he became a student of comparative religion, as we would call him to-day. There was a great difference between his view, however, and that of a large school of modern students of comparative religion. He did not study other religions with the purpose of providing from them ideals which Christianity was supposed to lack. Nor did he propose to reduce out of all religions a common fund of general principles more or less to be found in all and regard this as the ultimate religion. He studied other religions to find out how better to reach the hearts of their followers with the Gospel, itself perfect and complete, lacking nothing, needing nothing from any other doctrine. With him, there was a difference between Christianity and other religions not in degree only, but in kind. It possesses all that they lack which is desirable. It lacks all that they possess which is unworthy. It alone satisfies. It alone is life. They are systems of society or politics; religions of books, methods, organizations. It and it alone is life, personal life. Lull studied other religions not to discover what they have to give to Christianity, for they have nothing, but to find how best he might give to those who follow them the true life which is life, and which no man shall ever find until he finds it in Christ.

2. Believing in life, he believed in the movement of life. The great dream of his last years was a mighty missionary crusade. "The Saracens write books for the destruction of Christianity," he writes.

"I, myself, have seen such when I was in prison. . . . For one Saracen who becomes a Christian, ten Christians and more become Mohammedans. It becomes those in power to consider what will be the end of such a state of things. God will not be mocked." And one of his proposals was a union of the many religious orders of the knighthood for a new, a spiritual crusade. He tried to organize a Volunteer Movement, a sort of laymen's missionary crusade, seven centuries before at last anything like it came. But this is the way with living men. Life leaps out in unlimited purposes. What we call conservatism is often only life at low ebb, or life overlaid with death. When the Christ, Who laid out for Himself a universal project and calmly prepared for a campaign to last ages, and confidently committed His project at the outset to a dozen fishermen and countryfolk; Who now is planning with a patience that is never ruffled and a wisdom that is never at fault for the victory which is more inevitable than time,—when He lives in the soul of a man, all great visions will come to birth and with them the resistless impulse of mighty undertakings. There is life in the loom at which He weaves. "Room for it, room!"

3. Because the true life was in him, Raymond Lull felt and displayed the true missionary spirit. He would win men personally. He rose right up over the dominant notion of his day, of conversion by authority. The fires of the Inquisition were just beginning to burn and the dungeons of that thought of hell to echo with the wail of old man and little child. The Crusader had rejoiced over the carnage of battle with Islam. But Lull taught the method of the

Saviour. He closes one of his books with this prayer : " Lord of heaven, Father of all times, when Thou didst send Thy Son to take upon Him human nature, He and His disciples lived in outward peace with Jews, Pharisees and other men ; for never by outward violence did they capture or slay any of the unbelievers or of those who persecuted them. Of this outward peace, they availed themselves to bring the erring to the knowledge of the truth and to a communion of spirit with themselves. And so after Thy example should Christians conduct themselves towards Moslems ; but since that ardour of devotion, which glowed in Apostles and holy men of old, no longer inspires us, love and devotion through almost all the world have grown cold, and therefore do Christians expend their efforts far more in the outward than in the spiritual conflict." No sword was in Lull's hand, nor any hate in his heart. The life and love of Christ were there.

4. The love of Christ, because the life of Christ. And what a passionate love it was ! It stopped at nothing. It counted no cost. It dreamed of Christ at night. It saw Him in the joys and not less in the griefs of life. It filled him in all the channels of his being. He wanted nothing but to love Christ more. " O divine love," wrote Livingstone in his journal, " I have not loved Thee deeply, tenderly enough." As Francis had adored his dear Lord and bore in his body His marks, Raymond Lull loved Him and bore in his life evidences as real of the indwelling presence of the Saviour. It was that indwelling life that led him into the loving purpose of bringing, as he said, " by love, by proclaiming the word of truth rather

than by force of arms," the Saracens into "that kingdom which is love and peace and joy in the Holy Ghost." "Crusade had succeeded crusade, leaving the Saracens stronger, calling out in the Turk a more terrible foe, before whom Constantinople was soon to fall, and distant Vienna to tremble, and plunging Europe and the Church into deeper corruption." That was the result of the method of hate and conflict. "Raymond Lull was raised up as if to prove, in one startling case to which the eyes of Christians were turned for many a day, what the crusades might have become and might have done for the world, had they fought for the Cross with the weapons of Him Whose last words from it were forgiveness and peace."¹

5. And lastly, can we hear no call from the old martyr sleeping beneath the stones at Bugia? Shall that great Mohammedan world, which he was so eager to save and which we know is as dear to Christ as our world is to us or to Him, awaken no sympathy in our hearts? The obstacles which Lull encountered were tenfold greater than ours. He went on hard journeys where we can travel with ease. He preached Christ to Moslem people in Moslem states. Now one-half the Moslem people, 125,000,000, are under Christian rulers, where apostasy is not treason and where Christ is not crime. He was alone. Now the Moslem world is girt with missions, and here and there some fearless band is at work in its very citadels. Then the great system was intact, save for some minor schism. Now it is seamed with dissensions and sapped with doubt. That the enterprise is still

¹ Smith, "Short History of Christian Missions," p. 103.

perilous cannot be denied. So was the Incarnation. Christ was crucified and He knew He would be crucified before He came. But alike its peril and its necessity constitute our duty. For thirteen centuries, the world has waited. Even Lull failed to awaken the Church. "In Persia, one thousand years after Islam, the first missionary came. Arabia waited twelve centuries." Every other Christian enterprise has won its friends and moved on, but this one waits, its heralds calling still, voices in the wilderness, sentinels before the dawn. Even the Roman Church, which fears nothing,—disease, leprosy, martyrdom,—since the days of Lull has passed Islam by. Will the morning never break, and the spirit of Lull never come to life again in men who will take up the work which Lull laid down, fearless and sure and steadfast in the faith of Lull's great word : "He who loves not, lives not : he who lives by the Life cannot die" ?

LECTURE II

WILLIAM CAREY, THE CHRISTIAN
PIONEER AND HIS PROBLEMS

LECTURE II

WILLIAM CAREY, THE CHRISTIAN PIONEER AND HIS PROBLEMS

THE names of William Carey and Alexander Duff are bound together by varied associations. Each was the first missionary representative of his country and his Church. They laid together the foundations of missions and missionary influence in India and in the same part of India. They established certain missionary principles at the beginning of the modern missionary enterprise. Each left his influence on the political destiny and development of the Indian government, and from the life of each flowed streams of originating influence over the countries from which they came and over other lands. Even before they died, men discerned the true greatness of their lives and looking back now, we see them overtopping scores of governors and viceroys and taking their places, not only beside them,—Carey beside Clive as the spiritual and secular founders, and Duff beside Hastings as the spiritual and secular consolidators of the Indian Empire, as Dr. George Smith says,¹—but above them, by reason of their moral superiority of character and the daring and originality of their schemes. Work is great or small in proportion to the greatness of the idea it embodies and the faithfulness of the spirit in which it is done. In this

¹ "Life of William Carey," p. 59.

view, no viceroy ranks above these pioneers of missions.

Shortly after his arrival in India, Duff went to consult Carey regarding his educational plans. It was only three years before Carey's death and he was then "a little yellow old man in a white jacket who tottered up to the visitor of whom he had already often heard, and with outstretched hands solemnly blessed him."¹ Only one other incident of their intercourse is preserved in their biographies. It was near Carey's end.

"Among those who visited him in his last illness was Alexander Duff, the Scotch missionary. On one of the last occasions on which he saw him—if not the very last—he spent some time talking chiefly about Carey's missionary life, till at length the dying man whispered, 'Pray.' Duff knelt down and prayed, and then said good-bye. As he passed from the room, he thought he heard a feeble voice pronouncing his name, and turning, he found that he was recalled. He stepped back, accordingly, and this is what he heard, spoken with a gracious solemnity, 'Mr. Duff, you have been speaking about Dr. Carey; when I am gone, say nothing about Dr. Carey—speak about Dr. Carey's Saviour.' Duff went away rebuked and awed, with a lesson in his heart that he never forgot."²

This was the spirit in which Carey had always lived and worked. Probably he did not comprehend in any large measure the magnitude of the influence he was launching in India. He always

¹ Smith, "Life of Duff," Vol. I, p. 105 f.

² Culross, "William Carey," quoted in Smith, "Life of William Carey," p. 365.

disavowed any greatness and said once to his nephew Eustace, "Eustace, if, after my removal, any one should think it worth his while to write my life, I will give you a criterion by which you may judge of its correctness. If he gives me credit for being a plodder he will describe me justly. Anything beyond this will be too much. *I can plod*. I can persevere in any definite pursuit. To this I owe everything." Carey's American biographer in his quaint life of the missionary justly develops Carey's own estimate :

"In Dr. Carey's mind and habits of life, there was nothing of the marvellous; no great and original transcendency of intellect; no enthusiasm and impetuosity of feeling; nothing to dazzle or surprise. Whatever of usefulness and of consequent reputation he attained to was the result of an entire and patient devotion of a single heart and clear intelligence to a well-defined, great, and practicable object;—an object which demanded great labour, but which presented great attraction, and ultimate success. He had nothing of the sentimental, the tasteful, the speculative, or the curious in his mental constitution. He had, therefore, no help from the warmth of feeling, or the glow of spirits, from the fervour or the fire which actuate painters and poets, and by which even some zealots in religion and morals show themselves to have an existence. To this want of excitement may be traced many of those upbraidings of himself for his imagined inactivity and want of zeal. He was often heard to say, 'I think no man living ever felt inertia to so great a degree as I do.' He was a man of principle, not of impulse."¹

¹ Belcher, "William Carey," p. 237f.

Though it may be that Carey was only a plodder and no genius, yet at least he had a genius for plodding, and the history of the missionary enterprise in many lands has shown the power of a long tenacious service. If a good man, whatever his intellectual gifts, will stay for a generation in one place anywhere in the world and work, he will have an abiding result. Human influence is cumulative and forty years of steady honest toil will yield far more of the kind of fruitage which can be built into life and institution than ten years of erratic brilliance. One great problem of the missionary work is how to secure such continuous, indefatigable plodding as Carey gave to it.

Of Carey's early life, it is difficult to say whether it chiefly offered a field for the triumphant exercise of inherited qualities of industry and persistence, or whether it was the difficulties and discouragements of these years which developed the qualities required to overcome them. On the one hand, there assuredly was the latent power in the weaver's son. The family line had known loftier days. "For two and a half centuries, from the second Richard to the second Charles, they gave statesmen and soldiers, scholars and bishops, to the service of their country." The father and grandfather of William Carey had been the schoolmasters and parish clerks of the village, and the boy was given the ordinary education of the boys of his time and condition. He grew up in poverty and for years knew nothing else. While engaged in the trade of shoemaker, which provided his support as a preacher, his employer died. Mr. Carey took over his stock and business and mar-

ried his sister before he had reached the age of twenty.

The condition of Carey's early life and training comprised three elements of missionary equipment, two advantageous and one unsatisfactory. (1) He had good stuff in him. No training will create what does not exist in capacity. The missionary work is seeking for men and women who have in them, thanks to the past, the latent power to do things. (2) The discipline of poverty is a good discipline to the man on whom its blessings are not lost. There are those whom poverty simply prepares for self-indulgence and presumption when their circumstances change and who are made exacting and not adaptive by it. A wise and experienced missionary woman in one of our oldest missions remarked that she had observed that it was often the missionaries who had had least in their home lives who demanded most on the mission field, while those who had known the greatest comfort and luxury were the most cheerful in deprivation and sacrifice. Perhaps only those of the rich or well-to-do who had such a spirit of self-denial would venture to the mission field and some of other classes might come in a different spirit. But as a rule the education of frugality is the best education for all services. (3) Perhaps the folly of his first marriage gave to Carey's capacity for folly a pretty exhaustive trial. He assuredly led a wise life afterwards and he assuredly did a foolish thing when he married the melancholic sister of his former employer. Whether it is wise for missionaries to go out married or not, as some debate, it is surely well for them to go wisely married if they are to be married at all. Carey's

wife was never anything but a clog to him. She hindered his going and in India she merely cumbered his life. No word of complaint escaped him and he behaved towards her as a gentleman and a lover until the end. That is more than can be said of some richer and better born men in like circumstances. He was married twice afterwards. When he died he prescribed that his name should be cut on the same gravestone with the second wife.

Poor and unadvantaged as he was, there was no contraction or timidity in Carey's moral and intellectual outlook. He opened a little school, and as he taught he dreamed and his dream was great. Mr. Fuller has related that, on going to his little workshop, he saw a large map suspended on the wall, composed of several sheets pasted together, in which he had entered every particular he had been able to glean relative to the natural characteristics, the population, and the religion of every country, as then known. Mr. Fuller, himself, who in after years built up the mission at home, was startled by the boldness and novelty of Carey's project, and described his feelings as resembling those of the unbelieving courtier in Israel: "If the Lord should make windows in heaven, might such a thing be!"

Novel and bold as the project seemed, it was no sudden creation of Carey's brain. Streams which had been running through the conscience of the Church simply came to practical vital utterance in the young Baptist preacher's life. In 1784, two years before the meeting at Northampton, the Northamptonshire association had issued a resolution regarding united prayer for the churches of the

association, but also for wider objects : " We trust you will not confine your requests to your own societies, or to our own immediate connection ; let the whole interest of the Redeemer be affectionately remembered and the spread of the Gospel to the most distant parts of the habitable globe be the object of your most fervent requests. We shall rejoice if any other Christian societies of our own or other denominations will unite with us, and we do now invite them most cordially to join heart and hand in the attempt. Who can tell what the consequence of such a united effort in prayer may be ? "

None of those concerned in the call dreamed of what its issues were to be. This spirit of prayer was intensified by the publication in 1789 by Mr. Sutcliff of Jonathan Edwards' " Humble Attempt to Promote Explicit Agreement and Visible Union of God's People in Extraordinary Prayer for the Revival of Religion. " Three years later, Carey published his pamphlet, " An Enquiry into the Obligations of Christians to Use Means for the Conversion of the Heathens, in which the Religious State of the Different Nations of the World, the Success of Former Undertakings, and the Practicability of Further Undertakings, are considered by William Carey. "

The tract is divided into five sections. The first consists of an inquiry, " Whether the commission given by our Lord to His disciples be not binding on us ? " The second contains " a short review of former undertakings for the conversion of the heathen. " The third furnishes a " survey of the present state of the world. " The fourth consists of " considerations on the practicability of something being done more

than what is done, for the conversion of the heathen." The fifth embraces "an enquiry into the duty of Christians in general, and what means ought to be used, in order to promote this work." These are fervent and united prayer, and exerting ourselves in the use of means. Here he suggests the formation of a society such as was afterwards organized. "Christians," he says, "are a body whose truest interest lies in the exaltation of the Messiah's kingdom. Their charter is very extensive, their encouragement exceedingly great, and the returns promised infinitely superior to all the gains of the most lucrative fellowship. Let, then, every one, in his station, consider himself as bound to act with all his might, and in every possible way, for God." He concludes his pamphlet in these earnest words: "Surely it is worth while to lay ourselves out, with all our might, in promoting the cause and the kingdom of Christ."

This "Enquiry" had been circulated before the meeting of the ministers' association at Nottingham in May 30, 1792, where Carey preached his great sermon on Isaiah liv. 2, 3. Even after that, there was hesitation. Mr. Fuller said, "Some of the greatest difficulties we had to encounter were the following:—we were inexperienced in the work; we knew of no opening for a mission in any one part of the world more than another; we had no funds to meet the expense that must attend an undertaking of the kind; our situation in an inland part of the country was inconvenient for foreign correspondence; the persons who would have the management would live at such a distance from each other as to render frequent consultation impracticable; and, finally, in forming such

a society there would be danger of its falling under irreligious influence. From these and other considerations, those who expected to engage in the work entered upon it with much fear and trembling." But Carey persisted and at the next meeting at Kettering on October 2, 1792, the Baptist Missionary Society was formed. And so the great enterprise began, Carey's faith having held to it immovably and his own life being offered as its embodiment.

It is interesting to recall that Carey, like Livingstone, desired originally to go to some other field than that to which he was sent. Tahiti or Western Africa was the field of which he had the first thought. The matter was decided in favour of India through the availability as a companion to Carey of John Thomas. He was a ship surgeon who had been employed in independent missionary work in Bengal. He was eccentric, undependable, irascible, fluent, spiritually ecstatic and generally incapable. He was the means of getting Carey started in the service, but he lost him the sympathy of the best men in Calcutta and perhaps hindered more than he helped. But we cannot say. Perhaps without him the new mission might have begun even worse. In every living enterprise, you must allow a large percentage for waste.

Carey was thirty-three years old when he began in India. Nowadays, men seem to find it hard to learn a new language at that age. But Carey learned half a dozen. Undoubtedly, he did have a genius for languages, or as he would have said—a will to plod at them. "It is well known," as he said in the "Enquiry," "to require no very extraordinary

talents to learn, in the space of a year, or two at most, the language of any people upon earth." He first learned Bengali, then Sanskrit. When he had been in India about three years, he wrote : "I am now learning the Sanskrit language, that I may be able to read their Shasters for myself ; and I have acquired so much of the Hindu or Hindustani as to converse in it and speak for some time intelligibly.

. . . Even the language of Ceylon has so much affinity with that of Bengal that out of twelve words, with the little Sanskrit that I know, I can understand five or six." When his son Jabez was sent as a missionary to Amboyna, one of the Malayan islands, at the cost of the East India Company in 1814, his father wrote to him : "Labour incessantly to become a perfect master of the Malay language. In order to do this, associate with the natives, walk out with them, ask the name of everything you see, and note it down ; visit their houses, especially when any of them are sick. Every night arrange the words you get in alphabetical order. Try to talk as you get a few words, and be as much as possible one of them. A course of kind and attentive conduct will gain their esteem and confidence and give you an opportunity of doing much good." This counsel illustrates his own spirit towards the people as well as his idea of how to acquire a language. He worked over grammars and dictionaries, which, for the most part, he had to produce himself, but he knew that the way to learn a language is to go out and talk with the people and take it in through the pores.

It is one thing for a missionary to go out to India now. It was quite a different thing for Carey. The

East India Company was opposed to the presence of missionaries in the country, and Carey lived in their territories for the first years only as an indigo planter. Indeed that was the only way he lived at all. He and Thomas were employed as assistants in Mr. Udney's indigo factories at Malda. Each received a salary equivalent to £250 a year, with the prospect of a commission, and even a proprietary share. Carey's remark in his journal on the day he received the offer was: "This appearing to be a remarkable opening in divine providence for our comfortable support, I accepted it. . . ." On receiving the rejoinder to his acceptance of the offer, he set this down: "I am resolved to write to the Society that my circumstances are such that I do not need future help from them, and to devote a sum monthly for the printing of the Bengali Bible." "This he did, adding that it would be his glory and joy to stand in the same relation to the Society as if he needed support from them. He hoped they would be the sooner able to send another mission somewhere—to Sumatra or some of the Indian Islands. From the first, he lived with such simplicity that he gave one-fourth to one-third of his little income to his own mission at Mudnabati."¹

This idea of self-support in one form or another had been in Carey's mind from the beginning. He had embraced it in his "Enquiry," where his proposal was that each mission should be a little community and be self-sustaining. His experience in the indigo factory confirmed his view. When Mr. Udney was forced to give up his business, Carey wrote to Mr. Fuller:

¹ Smith, "Life of William Carey," p. 77.

“The experience obtained here, I look upon as the very thing which will tend to support the mission. I know now all the methods of agriculture that are in use. I know the tricks of the natives and the nature of the lowest rate of housekeeping in this country. Having had a monthly allowance, I have made all experiments on these heads, which could not have been made without ruin had I not had these resources, and I will now propose to you what I would recommend to the Society; you will find it similar to what the Moravians do. Seven or eight families can be maintained for nearly the same expense as one, if this method be pursued. I then earnestly entreat the Society to set their faces this way and send out more missionaries. We ought to be given seven or eight families together; and it is absolutely necessary for the wives of missionaries to be as hearty in their work as their husbands. Our families should be considered nurseries for the mission; and among us should be a person capable of teaching school, so as to educate our children. I recommend all living together, in a number of little straw houses, forming a line or square, and of having nothing of our own, but all general stock. One or two should be elected stewards to preside over all the management, which should, with respect to eating, drinking, worship, learning, preaching, excursions, etc., be reduced to fixed rules.”

In anticipation of the coming of reinforcements Carey purchased the indigo factory at Kidderpore and made ready to realize his communistic plan, but on their coming it seemed wiser to move to the Danish settlement of Serampore to escape the difficulties which were certain in the territory of the East India Company. So Carey abandoned his investment

at Kidderpore. That is what missionaries always find it hard to do. Many a piece of unprofitable missionary work is maintained and many an unsatisfactory station, because sometimes the courage is lacking to sacrifice what has been established for something better, or because at other times the Christian spirit simply will not relinquish what has once been begun. The new missionaries arrived in 1799. Early the following year, Carey and they agreed upon a set of rules to govern their little community.

It was determined to form a common stock, to dine at a common table, and to give each family a trifling allowance—Mr. Fuller's "pocket-money"—for personal expenses. All the missionaries were to be considered on a footing of equality, and to preach and conduct social devotions in turn. The superintendence of domestic arrangements and expenditure was to be entrusted to each missionary in rotation for a month. Mr. Carey had charge of the public chest as treasurer, and also of the medicine chest, for India was then considered so unhealthy that a constant resort to medicine was deemed essential to existence. Mr. Fountain was appointed librarian. One evening in the week was to be devoted to the adjustment of differences and the renewal of their pledge of mutual love; and it was resolved that no one should engage in any private trade, and that whatever might be earned should be credited to the common stock.

The first year the brotherhood was more than self-supporting. The next year it had a surplus for the extension of the work, and this success continued.

There was nothing new in this idea of Christian communism. As Marshman's son says in the story

of the Serampore Mission, "The Serampore missionaries, therefore, when they resolved to support themselves while labouring to spread the Gospel, simply adopted the principle on which missions had been conducted before the modern missionary system was organized on its present footing, and the relation between the missionary and the Society assumed its present more secular type. The only difference in their case was the adoption of the novel principle of divesting themselves of all right of property in their own earnings, and consecrating it exclusively to the cause in which they had embarked, by the formation of a common stock."¹

But this also was an old idea. The Roman Catholic orders and the monasteries and convents rested on it, and it was the Moravian idea. It was a unique experiment, however, and it has had no worthy successor. It is not necessary to tell its whole story in detail. It broke down at last not because of any invalidity in its principles, but because of personalities. "The difficulty of management here is that our union is of that nature that it cannot exist until all selfish and turbulent passions are subdued," said Carey in 1808 at one time of trial. Some of the missionaries were accused of extravagance and self-indulgence. Young men who joined the mission were charged with wilfulness and insubordination. The numbers grew too large for as close acquaintance and constant conference as marked the little group at Serampore. The old administrators at home died and new men came in. It was in part the story of the early Jerusalem Church

¹ Marshman, "Life and Times of Carey, Marshman and Ward," Vol. I, p. 209.

reproduced. But there will always be those who will long for the return of the day when in the Church and in missions, this experiment may be repeated and fulfilled, when, to the destruction of neither but to the enrichment of both, individual energy and unselfish love may be joined and merged.

Where the missionaries worked for their own support, there was less danger than in modern missions where all the funds are from home, of the neglect of self-support among the native Christians. One of the first converts received at baptism "a new white dress with six shillings; but such a gift, beautiful in itself, was soon discontinued."

In 1806 the missionaries prepared a statement, setting forth their principles, both as to the self-support and the self-propagation of the Church.

Happy is the mission which perceives its duty in such matters at the outset and forms and steadfastly carries out a true and consistent policy. But it is a rare thing to find such clearness of perception and such persistence of policy. New men come, unaware of old arrangements, and as the years pass the heavy inertia of every society, even a new Christian Society, drags down fine ideals and constrains men to acquiesce in what is inferior and unsatisfactory.

Carey and his associates dealt with this as with most problems, with good sense. The missionaries rigidly refused to compromise with caste. They arranged for a burial in which a converted Brahman and a converted Mohammedan carried the dead body of another believer to his grave on their shoulders. This procedure completed the destruction of caste

in that Christian community. In the matter of the conditions prerequisite to baptism, the Serampore group took the view that "We think it right to make many allowances for ignorance, and for a state of mind produced by a corrupt superstition. We therefore cannot think of demanding from them, previous to baptism, more than a profession of dependence on Christ, from a knowledge of their need of Him, and submission to Him in all things."

Their high standards broke down, however, in the matter of polygamous applicants for baptism and they admitted such, without requiring them to cease their polygamous relationship, but kept them out of church office, while "pressing on the conscience of all the teaching of our Lord in Matthew xix. and of Paul in 1 Corinthians vii."

Apart from the direct problems of missions arising in the fulfillment of its immediate aims of the conversion of souls and the creation of a native Church, there are complicated questions springing from its relationships. Carey and his associates had to deal with these. In some cases, they had no precedents and in many, the problems were very intricate and perplexing. There was, first of all, the problem of their spiritual relationship and responsibility in the case of other foreigners. As early as 1793, the year he landed in India, Carey wrote :

"A missionary must be one of the companions and equals of the people to whom he is sent, and many dangers and temptations will be in his way. In a country like this, settled by Europeans, the grandeur, the customs, and prejudices of the Europeans are exceedingly dangerous.

They are very kind and hospitable, but even to visit them, if a man keeps no table of his own, would more than ten times exceed the allowance of a mission ; and all their discourse is about the vices of the natives, so that a missionary must see thousands of people treating him with the greatest kindness, but whom he must be entirely different from in his life, his appearance, in everything, or it is impossible for him to stand their profuse way of living, being so contrary to his character and so much above his ability."

But Carey realized the impossibility of a missionary's separating himself from all contact with other foreigners. And he saw the necessity of effort to make of foreign communities in the heathen world centres of pure influence rather than fountains of corruption. A great change was wrought during his lifetime in this regard in Calcutta and in India as a whole.

When Carey went to India the conditions were atrocious. Here and there, an individual stood out as a rare exception of conscience and purity. Charles Grant was one of these. "Amidst the universal scepticism of the day, he exhibited in his principles and his practice a noble specimen of the Christian character. While all around him, with rare exceptions, were absorbed in the pursuit of wealth, he devoted his attention to the moral and religious improvement of the natives. At that early period, he appears to have adopted the opinion subsequently enunciated by Sir Charles Metcalfe, that divine providence had assuredly some higher object in bestowing the empire of India on England, than to facilitate the export or

import of cotton-piece goods.”¹ The general tone, however, was dismally abased. The moral condition will be sufficiently illustrated by recalling the agitation of 1804 in Bengal over the Civil Fund question. “The members of the Civil Service became anxious to establish a fund for the support of their widows and orphans, and the plan of a Civil Fund was drawn up and circulated among them. The old gentlemen of the service, who had grown gray in Indian association, were desirous to extend the benefit of it to their dark illegitimate children. But the younger civilians connected with the college, in a number between fifty and sixty, whom it had been the aim of Lord Wellesley to train up in the principles of virtue and religion, united with one voice in deprecating a proposal which, as they had justly observed, involved ‘the total violation of one of the great ordinances of the divine law. . . .’ After the discussion had been carried on for some time, the two parties formally divided. The Civil Service at that time consisted of 350 members, one-half of whom voted for the admission of bastards, the other half against it. The question was then submitted to the arbitration of Lord Wellesley, who did not hesitate for a moment to limit the benefit of the fund to children born in wedlock, leaving upon the 175 civilians the task of providing, from their own funds, for the offspring of the loathsome zenanas. Not disheartened by this decision, they appealed to the Court of Directors, affirming that their masters would never vote with the college, because they would see how ‘unfriendly it

¹ Marshman, “Life and Times of Carey, Marshman and Ward,” Vol. I, p. 28.

was to ancient institutions.' But the Court not only sanctioned the clause which excluded illegitimate children, but endowed the fund with an annual subscription of £2,500."

Many influences coöperated to terminate conditions so disgraceful. Moral sentiment at home was steadily rising. The idea that India must be governed on moral principles and that England was responsible for improving the condition of the people intellectually and morally, indicated in the various revisions of the charter of the East India Company, accomplished a great deal; but on the other hand, this change of view at home was largely due to the missionary enterprise, and in India the presence of the missionaries and their example and influence operated constantly as a purifying power. This was conscious and aggressive as well as passive. Perilous as the entanglement of missionaries with the social life of other Western representatives undoubtedly is, one of the great needs of the day is such a closer relationship as will save to Christianity hosts of men from Christian lands who are going to the heathen world to lose their Christianity, and become paganized. If the missionary enterprise cannot care for the problem without injury to its own first business, some other agency must be developed to look after it. The old Foreign and Christian Union did a good work in this direction a generation or more ago. But now, a thousand times more than then, an effort is needed to save to purity and godliness the increasing streams of young men, who are pouring out from America and losing their souls in the non-Christian world. This effort must be made by

the Church directly and not alone through useful auxiliary agencies.

Carey had to deal with this difficult relationship. He had to deal also with the problem of the duty of a mission in the matter of gross moral and legal evils which might be removed at once by influencing political authority already in existence, without waiting for their slow destruction through generations by the reformation of individuals and the education of society. And the moral conditions of Hindu life were simply indescribable when Carey went to India.¹ He presented memorials against female infanticide, voluntary drowning and widow-burning. He collected testimony and educated opinion. He antagonized slavery and made provision for the care of lepers. He was not of those who hold it to be wrong to interfere with native customs, even though evil. It was on the basis of a report of his that the Governor-General ventured to issue a proclamation, prohibiting the sacrifice of children at the great annual festival of Gunga Saugor.² Missions may easily be diverted from their own business and turned into general educational or reformatory bureaus, and it is an evil thing when this is allowed to happen, but there is no peril, and there is much gain which it would be wrong to sacrifice when missions, which have powerful headway in the direction of their own supreme end, throw out their energies and influences to stop all possible moral evil,—the slave trade, widow-burning, the traffic in opium, the sale of liquor to the dependent races. The

¹Smith, "Life of William Carey," pp. 60, 65 f., 98.

²Myers, "William Carey," p. 123.

destruction of all such evil is a duty in itself and it prepares a better atmosphere in which to spread the truth.

By the methods of work which he used in missions and by his own direct influence in the service of the government, Carey affected greatly the development of the agencies of education and improvement which the new conscience, slowly coming to birth, was creating in India. Long before the government awoke to its educational duty, the Serampore missionaries were working at the problem. They had propounded a great scheme, which had been sent to Fuller. After its transmission, "they gradually augmented the number of their schools and endeavoured to improve their character. Under the new and more favourable aspect which the question of education had assumed, they determined to appeal to the public for the means of enlarging their efforts. Under the modest title of 'Hints,' they proposed a well digested system of national education, susceptible of indefinite expansion, which no subsequent efforts have rendered obsolete. The plan was never carried out to its legitimate extent; but there can be little doubt that if the Serampore missionaries had been enabled to prosecute it with their usual ardour, the lower provinces of Bengal would have presented a different aspect to that which they now exhibit. At the end of forty years of comparative inaction, the Board of Control has sanctioned a system of vernacular education in India on an extended scale; but the germ of it is to be found in the pamphlet of 'Hints' published at Serampore in the year 1816."¹

¹ Marshman, "Life and Times of Cary, Marshman and Ward," Vol. II, p. 119 f.

As we shall come to the problem again in connection with Duff's work, it may be well now to recall just what the issue was at this time. Three different plans were advocated at this period by three parties, that of the Orientalists, the Anglicists, and the Vernacularists.

"The Orientalists, without repudiating in theory the value of instruction through the medium of English, or of the native tongues, contended that the patronage of the state should be given primarily to the encouragement of Oriental literature, Hindu and Mohammedan, that the parliamentary grant was designed to promote this object, and should be exclusively appropriated to it. . . . With the new educational movement another party arose at this time, at the head of which was Sir Edward Hyde East and the Rev. T. Thomason. They considered that the intellectual progress of the country would be most effectually promoted by a liberal education through the exclusive medium of the English language. . . . They were buoyed up with the hope that by strenuous and continued exertion English might come to occupy the same place in India which Latin had formerly occupied in the Roman Empire. Some were even so sanguine as to expect that it might in the lapse of time supersede the mother tongue of India, and become the general medium of communication, and thus bind the people to their conquerors by the bond of a common language. They maintained, moreover, that it was more advisable to give a complete education to a few than an imperfect education to the multitude, and to promote the cultivation of a language already enriched with a noble literature, than of one in which a literature had to be created. The Serampore missionaries stood alone in advocating a

vernacular education as the only means by which the great body of the people, who had no leisure for the acquisition of a foreign language, could be rescued from the evils of ignorance and superstition. It was upon these views that the 'Hints' for native schools was based."¹

Carey did not fail to appreciate the importance of the English language in a land like India. As Dr. Smith says :

"He had not been six months in Serampore when he saw the importance of using the English language as a missionary weapon, and he proposed this to Andrew Fuller. The other pressing duties of a pioneer mission to the people of Bengal led him to postpone immediate action in this direction. . . . But meanwhile, the vernacular schools, which soon numbered a hundred altogether, were most popular, and then as now proved most valuable feeders of the infant Church. 'Without them,' wrote the three missionaries to the Society, 'the whole plan must have been nipped in the bud, since, if the natives had not cheerfully sent their children, everything else would have been useless. But the earnestness with which they have sought these schools exceeds everything we had previously expected. We are still constantly importuned for more schools, although we have long gone beyond the extent of our funds.' It was well that thus early in schools, in books, and tracts, and in providing the literary form and apparatus of the vernacular languages, Carey laid the foundation of the new national or imperial civiliza-

¹ Marshman, "Life and Times of Carey, Marshman and Ward," Vol. II, pp. 120-122.

tion. When the time for English came, the foundations were at least above the ground."¹

In 1801, Lord Wellesley as Governor-General established at Calcutta the college known as Fort William College for the training of civil servants, a new departure and a great progressive step. The college was intended to fit the young men from home for their duties in India. They were to begin their course by a three years' study of the vernaculars which they were to use and the college was to be also: "a centre of Western learning in an Eastern dress for the natives of India and Southern Asia, alike as students and teachers." Carey was called to be professor of Bengali and Sanskrit in this college, and accepted the position. His salary was £700 a year and soon increased to £1800. It was turned in, of course, to the common fund of the Serampore brotherhood. Here Carey had an opportunity to exert an even greater influence in behalf of his ideals of education and administration. He gave utterance to these and also avowed his missionary character in his address in 1804, when the first Sanskrit class was graduated. This address greatly pleased Lord Wellesley, and referring to its words of praise for him, as the founder of such an institution, he said, "I esteem such a testimony from such a man a greater honour than the applause of Courts and Parliaments."

Carey had not accepted the position without misgivings. When the call came to him, he says :

¹ Smith, "Life of William Carey," p. 136.

“I had but just time to call our brethren together, who were of the opinion that for several reasons I ought to accept it, provided that it did not interfere with the work of the mission. I also knew myself to be incapable of filling such a position with reputation and propriety. I, however, went over, and honestly proposed all my fears and objections. Both Mr. Brown and Mr. Buchanan were of the opinion that the cause of the mission would be furthered by it; and I was not able to reply to their arguments. I was convinced that it might. As to my ability, they could not satisfy me; but they insisted on it that they must be the judges of that. I therefore consented with fear and trembling. They proposed me that day, or the next, to the Governor-General, who is patron and visitor of the college. They told him that I had been a missionary in the country for seven years or more; and as a missionary I was appointed to the office. . . . When I was proposed, his lordship asked if I was well affected to the state, and capable of fulfilling the duties of the station; to which Mr. B—— replied that he should never have proposed me if he had had the smallest doubt on those heads. I wonder how people can have such favourable ideas of me. I certainly am not disaffected to the state; but the other is not clear to me.”

His friendly relation to the government, he says, was not doubtful. That was one clear point. He had had occasion to speak of this to others. He warned young men against meddling and political pugnacity. He himself at times adopted a very mild course in the face of political hindrance. But he never betrayed his work. In the timid times after the Vellore Mutiny, the government was extremely

cautious in allowing missionary activity. The Governor-General, Sir George Barlow, even expressed a wish that he (Carey) should not interfere with the prejudices of the natives by preaching to them or distributing books or pamphlets amongst them ; that his colleagues were to observe the same line of conduct ; and further, that the converted natives were not to go into the country to spread Christianity among the people. Carey was no man to surrender. He had gone to India in the face of the East India Company's prohibition, and when at various times the government set itself against his work he would await a change in the government's mood, but not for a moment would he yield his principles. He was well disposed to the government, he said, and he strove to obey it, but he was subject to a higher law and when the two clashed, he knew which to choose.

But being friendly and obedient to the government was quite a different matter from accepting employment under it, and it is not surprising that Carey hesitated. When his son Felix withdrew from his missionary work to become the Burmese ambassador of the Governor-General of India in 1814, Carey wrote, lamenting the step, to Ryland : "Felix is shrivelled from a missionary to an ambassador." There have been cases where such a change has not been a shrivelling, but Carey, himself, felt that nothing was comparable with the missionary service and in his own case he was a missionary all the while he held government office. And in his case this fact, the sheer power of the pure missionary purpose in him which robbed all accessory relationships and activities of peril, and the commu-

nistic principles of the Serampore Brotherhood, which destroyed all danger of secularization through money, saved Carey from perils which are often fatal to other men who turn from their direct religious work to some other service and with whom too often this turning is like that of Demas.

This exaltation of the pure missionary service as the noblest service in the world was characteristic of Carey. In 1809, he wrote to his son William with equal lowliness and spiritual nobility :

“Should you, after many years’ labour, be instrumental in the conversion of only one soul, it would be worth the work of a whole life. . . . I am not sure that I have been of real use to any one person since I have been in this country, yet I dare not give up the work in which I am engaged. Indeed, notwithstanding all the discouragements which I feel from my own unfitness for any part of it, I prefer it to everything else, and consider that in the work of my Redeemer I have a rich reward.”

Again he wrote to the same son : “Now, dear William, what do we live for but to promote the cause of our dear Redeemer in the world? If that be carried on, we need not wish for anything more. . . . Indeed, were you never to be blessed to the conversion of one soul, still the pleasure of labouring in the work of the Lord is greater than that of any other undertaking in the world, and is of itself sufficient to make it the work of our choice.”

“I am not sure that I have been of real use to any one person since I have been in this country.” What a remarkable word to come from Carey after sixteen years in India ! Did he not know of one soul brought

to Christ by his work? Assuredly he knew of many, and he knew of other incalculable influences which he had set in motion. But he was one of those who do not think highly of themselves; who think soberly and humbly and without self-esteem. No one ever picked a quarrel with him over a question of personal honour or precedence. How deep was his self-depreciation is shown by the account he gave of himself to Dr. Ryland in 1804, when he had been a missionary for eleven years:

“I am convinced that some sins have always attended me, as if they made a part of my constitution; among these I reckon pride or rather vanity,—an evil which I have detected frequently, but have never been free from to this day. Indolence in divine things is constitutional; few people can think what necessity I am constantly under of summoning all my resolution to engage in anything which God has commanded. This makes me peculiarly unfit for the ministry, and much more for the office of a missionary. I now doubt seriously whether persons of such a constitution should be engaged in the Christian ministry. This, and what I am going to mention, fill me with continued guilt. A want of character and firmness has always predominated in me. I have not resolution enough to reprove sin, to introduce serious and evangelical conversation in carnal company, especially among the great, to whom I have sometimes access. I sometimes labour with myself long, and at last prevail sufficiently to break silence; or, if I introduce a subject, want resolution to keep it up, if the company do not show a readiness thereto.”

He closes the letter with the words:

"I have now only to desire of you that the above (his autobiographical sketch) may not be published ; though I have no objection to your publishing any parts thereof, provided you so conceal names and other allusions, as that it may never be known that it is an account of me. Every publication of this kind, if the author be known, makes him more public ; and as it is very uncertain whether I shall not dishonour the Gospel before I die, so as to bring a public scandal thereupon, the less is said about me the better."

One of the most familiar stories about him relates to his quiet remark to a young snob from England whom he overheard asking some one at one of the Governor-General's parties in Calcutta, whether Dr. Carey had not been a shoemaker. "No," said Carey, turning about and replying, "only a cobbler." It was not said vindictively. It was the man's natural humility of soul. When complaint was made once of the absence of inscriptions in the mission burying ground, he replied : "Why should we be remembered ? I think when I am dead, the sooner I am forgotten, the better." And on his own gravestone he directed that nothing more should be cut than the words :

*"William Carey, born August 17, 1761,
died June 9, 1834.*

*"A wretched, poor, and helpless worm,
On Thy kind arms I fall."*

There was nothing insincere in all this. Carey was a great man in spite of his own diffidence and self-

distrust. He drove himself to duty finding it hard to do. He appealed to others to attempt great things and he stirred himself to respond to his own appeal. If others had only put forth a fraction of his effort, great things would have been achieved. Undoubtedly, there must have been great latent talents of ability and of character in him, but we must believe his own account of himself. He says he had to fight constantly with indolence.

"Indolence," he wrote Dr. Ryland in 1802, "is my prevailing sin, and to that are now added a number of avocations which I never thought of; I have also so continual a fear that I may at last fall some way or other so as to dishonour the Gospel that I have often desired that my name may be buried in oblivion; and indeed I have reason for those fears, for I am so prone to sin that I wonder every night that I have been preserved from foul crimes through the day, and when I escape a temptation I esteem it to be a miracle of grace which has preserved me. I never was so fully persuaded as I am now that no habit of religion is a security from falling into the foulest crimes, and I need the immediate help of God every moment. The sense of my continual danger has, I confess, operated strongly upon me to induce me to desire that no publication of a religious nature should be published as mine whilst I am alive. Another reason is my sense of incapacity to do justice to any subject, or even to write good sense. I have, it is true, been obliged to publish several things, and I can say that nothing but necessity could have induced me to do it. They are, however, only grammatical works, and certainly the very last things which I

should have written if I could have chosen for myself."

The moral sensitiveness which Carey possessed is a great necessity and safeguard in missionary life and in all highest character. No one who has been in a position to know the inner lives of good men can fail to appreciate the importance of such a sense of moral peril and need of supernatural help to be kept in a blameless life.

How Carey overcame his indolence his life well illustrates. He simply toiled. A disinclination to work is no reason for not working. Any man can work who will whether he has a disposition for it or not. And on the mission field men must work. There is no place there for drones or shirks. As Mr. Ward wrote to Mr. Fuller :

"It was work in which half the dissenting ministers in England, who merely preach twice or thrice a week, when people come to hear the Word, would be of little use. A man who shall do good here must be on his legs, or in the saddle or in his boat. In the hands of a mere domesticated man, who prays at home, but never goes out into the highways and ditches, things die a natural death. Men must go out a-fishing ; the fish will never leave their natural element and walk into their nets, and they must be patient, too, though they toil all night and catch nothing."

Here is a specimen day during Carey's life in Calcutta in 1807 :

"He rose at quarter before six, read a chapter in the Hebrew Bible, and spent the time till seven in private devotions. He then had family prayer in Bengali

with the servants, after which he read Persian with a Moonshee who was in attendance. As soon as breakfast was over, he sat down to the translation of the Ramayana, with a pundit till ten ; when he proceeded to the college, and attended its duties till two. Returning home, he examined a proof-sheet of the Bengali translation of Jeremiah, and dined with his friend, Mr. Rolt. After dinner, with the aid of the chief pundit of the college, he translated a chapter of Matthew into Sanskrit. At six, he sat down with the Telinga pundit, to study that language, and then preached an English sermon to a congregation of about forty. The service being ended at nine, he sat down to the translation of Ezekiel into Bengali,—he had thrown aside his former version, and was now translating the prophets. At eleven, the duties of the day closed, and after reading a chapter in the Greek Testament, and commending himself to God, he retired to rest.”

At the age of seventy, when the financial affairs of the Serampore Brotherhood were very uncertain, one of his colleagues wrote of him : “Though thus reduced in his circumstances the good man, about to enter on his seventieth year, is as cheerful and happy as the day is long. He rides out four or five miles every morning, returning home by sunrise ; goes on with the work of translation day by day, gives two lectures on divinity, and one on natural history every week in the college, and takes his turn of preaching both in Bengali and in English.” All his life, he knew how to work, even though he was indolent. He held tenaciously to duty. He portioned out time so as not to fritter it away. He knew how

to use its fragments. By devoting to some of his greatest literary works mere fractions of time, he succeeded in completing them. "Hence his Sanskrit grammar of a thousand pages. Hence, too, his Bengali dictionary of three quarto volumes, designed and executed on a painfully elaborate plan. And hence, also, his translation of the celebrated Sanskrit poem, the Ramayana; which last work, to the extent of several volumes, he effected by dictating to an amanuensis about two hours only once in seven days. By this means his Scriptural translations advanced by slow, but regular degrees, until, in the course of years, the work arrived at so prodigious an aggregate as to require no ordinary effort to believe it possible that any one man, let his advantages be what they might, could accomplish so vast an achievement. But invincible patience in labour, and unceasing constancy, secured his triumph over every obstruction."¹ The world's output of good work would be quadrupled if men knew this open secret of the plodding cobbler.

But to call him a plodder does not explain Carey. He was a great man, whatever he says of himself, and yet his greatness, barring his linguistic genius, was of the kind accessible to every man. Every man may have his great vision. He saw far and wide. We have seen how great were his dreams as the shoemaker teacher. The first years he was in India, he wrote :

"I hope the Society will go on and increase, and that the multitudes of heathen in the world may hear the glorious words of truth. Africa is but a little

¹ Belcher, "William Carey," p. 243.

way from England ; Madagascar but a little way farther ; South America, and all the numerous and large islands in the Indian and Chinese seas, I hope will not be passed over."

He urged the occupation of Thibet, Afghanistan, where he thinks the lost Ten Tribes are, and his eyes sweep the whole world. In 1809, he writes :

" The state of the world occupies my thoughts more and more ; I mean as it relates to the spread of the Gospel. The harvest truly is great and labourers bear scarcely any proportion thereto. Hindustan requires ten thousand ministers of the Gospel, at the lowest calculation, China as many, and you may easily calculate for the rest of the world. I trust that many will eventually be raised up here, but be that as it may, the demands for missionaries are pressing to a degree seldom realized. England has done much, but not the hundredth part of what she is bound to do. In so great a want of ministers ought not every church to turn its attention chiefly to the raising up and maturing of spiritual gifts with the express design of sending them abroad ? Should not this be a specific matter of prayer, and is there not reason to labour hard to infuse this spirit into the churches ? A mission into Siam would be comparatively easy of introduction and support on account of its vicinity to Prince of Wales Island, from which vessels can often go in a few hours. A mission to Pegu and another to Arakan would not be difficult of introduction, they being both within the Burman dominions. Missions to Assam and Nepal should be speedily tried. Brother Robinson is going to Bhotan. I do not know anything about the facility with which missions could be introduced into Cochin China, Cambodia and Laos, but

were the trial made, I believe difficulties would remove. It is also very desirable that the Burman mission should be strengthened. There is no full liberty of conscience, and several stations might be occupied; even the borders of China might be visited from that country if an easier entrance into the heart of the country could not be found. I have not mentioned Sumatra, Java, the Moluccas, the Philippines, or Japan, but all these countries must be supplied with missionaries. This is a very imperfect sketch of the wants of Asia only, without including the Mohammedan countries; but Africa and South America call as loudly for help and the greatest part of Europe must also be helped by the Protestant Churches, being nearly as destitute of real godliness as any heathen country on the earth. What a pressing call, then, is there for labourers in the spiritual harvest, and what need that the attention of all the Churches in England and America should be drawn to this very object."

Such greatness of dream and desire as his is denied no man. And all men may have his dauntless courage and persistence in overcoming difficulties. He knew little else in his early years. The whole enterprise had to be created. And he had no resource, no experience, no standing, no sympathy and support in his own family. But he understood that obstacles exist to be surmounted. As General S. C. Armstrong said, "What are Christians in the world for but to do the impossible by the help of God?" Carey was like Armstrong in another regard. He was a child all his days, and so was free from envy and distrust, and jealous meanness and all malice. Cannot every man remain or become again as a little

child? Is there any hope of the kingdom of heaven for men who cannot?

But childlikeness was joined in him as in Armstrong with the shrewdest and most careful judgment in business matters. And yet it was not so much shrewdness in either case as high conscience. The greatest cleverness is always righteousness. Carey's views as to the expenditure of mission money will suffice for illustration. He was giving Mardon and Chater instructions with reference to their mission to Burmah. He spoke of the language:

"With respect to the Burman language, let this occupy your most precious time and your most anxious solicitude. Do not be content with acquiring this language superficially, but make it your own, root and branch. To become fluent in it, you must attentively listen with prying curiosity into the forms of speech, the construction and accent of the natives. Here all the imitative powers are wanted; yet these powers and this attention, without continued effort to use all you acquire, and as fast as you acquire it, will be comparatively of little use."

And he went on later to add: "In prosecuting this work, there are two things to which especially we would call your very close attention, viz., the strictest and most rigid economy, and the cultivation of brotherly love. Remember that the money which you will expend is neither ours nor yours, for it has been consecrated to God; and every unnecessary expenditure will be robbing God, and appropriating to unnecessary secular uses what is sacred and consecrated to Christ and His cause. In building, especially, remember that you are poor

men, and have chosen a life of poverty and self-denial, with Christ and His missionary servants. If another person is profuse in expenditure, the consequence is small because his property would perhaps fall into hands where it might be devoted to the purpose of iniquity ; but missionary funds are in their very circumstances the most sacred and important of anything of this nature on earth. We say not this, brethren, because we suspect you, or any of our partners in labour ; but we perceive that when you have done all, the Rangoon mission will lie heavy upon the missionary funds, and the field of exertion is very wide." The old lesson can never be too well learned that all such trust funds should be so spent that the use to which each penny is put will bear the closest scrutiny.

Carey says that in the matter of work, he had to achieve his results in the face of a heavy spirit of indolence. In other directions also he was obliged to succeed against disabilities. He felt his social limitations keenly and often complained of himself for what he called his misanthropy.¹ He was so humble and unassuming that he lacked aggressive tact. He wanted ease and elasticity in society. All this he realized and yet his work called him into society and he knew that the Spirit of God could enable him to meet his duty there as well as in less conspicuous spheres. He followed, himself, the advice he gave to his son Jabez when he went to Amboyna : "Behave affably and genteelly to all, but not cringingly towards any. Feel that you are a man, and always act with dignified sincerity and truth which will command the esteem of all. Seek not the society of worldly men,

¹ Belcher, "William Carey," p. 254.

but when called to be with them, act and converse with propriety and dignity. To do this, labour to gain a good acquaintance with history, geography, men, and things. A gentleman is the next best character after a Christian, and the latter includes the former. Money never makes a gentleman, neither does a fine appearance, but an enlarged understanding joined to engaging manners." Living by these principles, himself, Carey associated without difficulty and with constant usefulness and the unfailing respect and regard of men with governor-generals, merchants, scholars, and soldiers and "was ever the saint and gentleman whom it was a privilege to know."¹

These characteristics and Carey's success illustrate the great truth to which his whole life testifies that nothing is impossible to any man in the line of his duty. Whether the hindrance was an outer obstacle or an inner limitation, Carey conquered it. In this, he was a true pioneer of missions, and young men and women of our day lose a great deal if they rest on great organizations and are content with small routine performances, and excuse themselves from the successful effort to achieve the impossible. They are no worthy followers of the uneducated cobbler, who became one of the most respected scholars and gentlemen in India.

And yet Carey was in India just what he was in England. Men will be what they have been. He became a great scholar in India because he had been an earnest student at home. He had starved himself to buy books. Dutch and French he taught himself.

¹Smith, "Life of William Carey," p. 162.

"He never sat on his stall without his book before him, nor did he painfully toil with his wallet of new made shoes to neighbouring towns or return with leather, without conning over his lately acquired knowledge and making it forever in orderly array his own."¹ This is far superior to the mere form of education in regular institutions. In China today two of the best speakers of Mandarin are men who had no collegiate training. One was a blacksmith and went out late in life. But they were men like Carey with working minds and the power of toil. Carey has been spoken of by one acquaintance at least as easy going. He had said to this friend, "Brother Swan, I am not fitted for discipline. I never could say no,"² and his kindliness of disposition was notable and saved him from attacks made upon his associates. But he did not lack decision and positiveness of character. He revealed enough of it in India and it emerged then because it had always been in him. He believed in effort and effort on the instant. He was no dilatory, hesitant man. When he had preached his great sermon at Nottingham in 1792 and though every one was moved, the meeting was breaking up with no practical action, Carey seized Fuller by the hand "in an agony of distress," and asked imploringly, "Are you after all again going to do nothing?" He was no man to dawdle forever over duty. He believed in rising from sloth and striking when the hour was come.

As at home he was great enough to be kind and unselfish in his judgment, so he remained. He was

¹ Smith, "Life of William Carey," p. 23.

² Belcher, "William Carey," p. 256.

enough a true son of God to know how to forgive. In 1810, he wrote to his son William, who in the absence of the pastor of a church had excluded two members, with the result of coolness on the part of the pastor, who deemed his rights invaded :

“ I advise you to write to Mr. Fernandez immediately, and acknowledge that you did wrong in proceeding to the exclusion of the members without having first consulted with him, and state that you had no intention of hurting his feelings, but acted from what you thought the urgency of the case, and request of him a cordial reconciliation. I should like much to see a copy of the letter you send to him. I have no object in view but the good of the Church, and would therefore rather see you stoop as low as you can to effect a reconciliation, than avoid it through any little punctilio of honour or feeling of pride. You will never repent of having humbled yourself to the dust that peace may be restored ; nothing will be a more instructive example to the heathen around you, nothing will so completely subdue Brother Fernandez’s dissatisfaction and nothing will make you more respected in the Church of God.”

Only true Christian men can write such counsel and no one gives it who does not try to practice it. In the matter of such humiliation and forgiveness there is little hypocrisy in the world. It was the possession of this spirit that carried Carey through all the bitter experiences of his missionary life. Some arose from changes at home, new men succeeding Fuller and the older men who had held the rope from the beginning : some from changes on

the field, difficulties sometimes arising in the mission, but Carey came through all unscathed, because he was a man of love and truth and forgiveness who would not contend.

This would suffice as testimony to his greatness, but there are other evidences more familiar to the world because more of its kind. Even in the world's measurements, he was a great man. He was great in his ability as a scholar. The British and Foreign Bible Society recognized this in the minute adopted at his death :

“ For this arduous undertaking (Bible translation) he was qualified in an extraordinary degree by a singular facility in acquiring languages—a faculty which he had at first shown and cultivated amidst manifest disadvantages in the retirement of humble life. The subsequent extent of his talent as well as of his diligence and zeal may be judged of by the fact that, in conjunction with his colleagues, he has been instrumental in giving to the tribes of Asia the sacred Scriptures, in whole or in part, in between thirty and forty languages.”

The Asiatic Society of Calcutta adopted the following minute :

“ The Asiatic Society cannot note on their proceedings the death of Dr. Carey, so long an active member and ornament of this institution, distinguished alike for his high attainments in the original languages, for his eminent services in opening the stores of Indian literature to the knowledge of Europe, and for his extensive acquaintance with the sciences, the natural history and botany of this

country, and his useful contributions in every branch, towards the promotion of the objects of the society, without placing on record this expression of their high sense of his value and merits as a scholar and a man of science, their esteem for the sterling and surpassing religious and moral excellencies of his character and their sincere grief for irreparable loss."

He was more than a scholar. He was a man of practical action. He accomplished things. He moved men. He altered social conditions. He entered into all the activities of life. From the outset, he was interested in agriculture and industry. He founded the Agricultural Society of India. He was an eminent botanist and first advocated forestry in India. He introduced the first steam engine and the manufacture of paper. It is not too much to say of him that he was the greatest personal force outside of the government—and probably greater than any single individual force in the direction of reform and improvement. No one did more than he towards the uplifting of Indian life—the abolition of its wrongs, the elevation of the character of English influence and the exaltation of Great Britain's ideas of duty towards the millions of India.

Upon the Churches at home his reflex influence was incalculable. When he went out, the home Churches were not only indifferent, they were hostile to the missionary idea. In Scotland the Moderates openly denounced the missionary proposal. In Parliament in 1793, when Wilberforce advocated the authorization of missions in connection with the East India Charter act and proposed clauses to this effect,

they were rejected and in the House of Lords the Bishop of St. David's questioned the right of any people to send their religion to any other nation. The mission of Carey put an end to this attitude on the part of the home Churches, and the modern missionary era began, an era in which each branch of the Christian Church recognized its duty to help to spread Christianity over the whole world.

Carey never returned to England. His idea of missions was that missionaries should go out to take permanent root in the soil, to derive their support thence and to live there until their death. "The Serampore missionaries," says Marshman's son, "had themselves relinquished every idea of ever returning to England, and they resolved to make India not only their own home, but the home of their families." This idea of permanent stay in India was entertained by Duff also. As he told his converts in India when finally leaving them for home: "It was when a student at college, in perusing the article on India in Sir David Brewster's 'Edinburgh Encyclopedia,' that my soul was first drawn out as by a spell-like fascination towards India. And when, at a later period, I was led to respond to the call to proceed to India as the first missionary ever sent forth by the Established National Church of Scotland, my resolution was, if the Lord willed it, never, never to return again."¹ Not only did Carey never return to England, but he took no vacations on the field. Hill stations for vacation purposes had not been opened then. The punkah had not been invented, and of course there was no ice. Yet Carey stayed at his

¹ Smith, "Life of Duff," Vol. I, p. 43 f.

st and worked for over forty years. It is not necessary to open the question as to whether his plan is preferable to the present one of frequent furloughs. The fact that the best missionaries have decided in favour of the latter is sufficient evidence.

It is a vital question as to how the present system, the interruption of work, the disarrangement of plans, the detachment from the people inevitable and compensated for by improved tone and vigour, can be overcome. If men have Carey's spirit, they will overcome them. With that spirit place and travel matter nothing. The will of God and the service of man are all. But how can we win and display the same spirit of devotion to these high ends which William Carey won and displayed, when he went forth alone into Asia and for nearly half a century remained there, supporting himself and planting new missions with his surplus earnings, and living and dying for the evangelization of India? In some way, we must discover and retain the secret of that single and supreme devotion. Or has the mould in which men are cast to-day contracted? Are the resources in God which were open to Carey sealed to us? Will God do through a man to-day in a world of deepened need and richer opportunity what He did through the plodding cobbler who changed the spirit of an empire and retaught the Church the nature of her Gospel and the glory of her mission? Surely He will. Surely He will do it through us if we will give Him room.

LECTURE III

ALEXANDER DUFF, THE
CHRISTIAN STUDENT AND
THE WORLD'S EDUCATION

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I HAVE already associated the names of Carey and Duff as men who wrought together at the establishment of Christianity in India, and who at the close of Carey's life and the beginning of Duff's met personally to talk of what they were doing. Alike in their spirit and aim, they were yet of completely different types and they represented on one great problem of missionary effort and of general educational policy if not divergent views, at least dissimilar proportions of emphasis.

Duff went to his work with the best educational preparation. From Kirkmichael School, whose dominie he ever remembered with love and to whom he sent a copy of every book and pamphlet which he wrote, and then from the grammar school of Perth, he went up to St. Andrews' University, where Chalmers had taken the chair of Moral Philosophy just in time to impress the raw power of this Scotch boy. "Though outrageously thoughtless," writes one of his old fellow students, "I was much impressed by Duff. There was a weight and a downright earnestness about him which everybody felt. He was the boast of the college and was greatly regarded by the townsfolk of St. Andrews. His ap-

pearance as he passed with hurried step is indelibly photographed on my mind, and is thus put in my 'Historical Antiquities' of the city, 'That tall figure crossing the street and looking thoughtfully to the ground, stooped somewhat in the shoulders and his hand awkwardly grasping the lappet of his coat, is Alexander Duff, the pride of the college, whose mind has received the impress of Chalmers' big thoughts and the form of his phraseology.' Under Chalmers, he was, in St. Andrews, the institutor of Sabbath schools and the originator of the Students' Missionary Society."¹ This society was a sort of mission study class. Duff was librarian. The object of its members was "to study foreign missions so as to satisfy themselves of the necessities of the world outside of Christendom"; and it numbered at least three beside Duff who offered themselves for missionary service.

William Carey had himself awakened his Church to the missionary duty and to the willingness to send him. Duff found his Church already awakened and went out in response to its direct call.

In recommending Duff to the assembly, the committee described him as "a person possessed of such talents and acquirements, literary, scientific, and theological, as would do honour to any station in the Church; who also combines with these the prudence and discretion which are so peculiarly requisite in the discharge of the duties which will devolve upon him; and is, at the same time, animated with such zeal in the cause to which he devotes himself, as to make him think lightly of all the advantages which he foregoes in leaving his native land." These words

¹Smith, "Life of Duff," Vol. I, p. 23.

inadequately described the immense force which was about to be projected into India. And even taken on their level, they omitted one consideration on which Duff later laid emphasis in writing home regarding some recent reinforcement :

“He does not appear to be strong, nor capable of undergoing much bodily fatigue, nor exertion in speech, all of which is so essential to the active discharge of a missionary’s duties. I wish the committee would bear in mind that a constitutional vigour of body is just as requisite as a vigorous activity of mind, and piety and learning. Indeed it is not studying men that we want, but hard-working men who have been and still are students.” He met his own requirements. He was a working man for forty-five years, and he wrought with the energy of a torrent that never subsides.

Duff instinctively gave himself before he went to India to the deepening of the missionary interest at home. He was the forerunner in this of the host of students in our own day who realize that they ought to produce, before they go out, sufficient increment of missionary conscience in the Church at home to provide for the increased expense involved in their going. With intense enthusiasm and power Duff pled the cause, and his natural Scotch reserve never resisted the heat of his heart in these appeals. In the old kirk of Leuchars, he exclaimed : “There was a time when I had no care or concern for the heathen ; that was a time when I had no care or concern for my own soul. When by the grace of God I was led to care for my own soul, then it was I began to care for the heathen abroad. In my closet, on my bended

knees, I then said to God, 'O Lord, Thou knowest that silver and gold to give to this cause I have none ; what I have I give unto Thee, —I offer Thee myself, wilt Thou accept the gift?' "

Duff was married before going out to India. An old friend had questioned him on the subject. In reply to his inquiry, Duff had told him that he had been studying too hard to think of such things and had not as yet met any one whom he felt he could ask to go with him. "Well," said the old gentleman, "I do not approve of young men fresh from college taking wives to themselves when newly married to their Church, before they can possibly know the requirements of their work. But your case is wholly different. You go to a distant region of heathenism, where you will find little sympathy among your countrymen, and will need the companionship of one like-minded to whom you may unbosom yourself. My advice to you is this, be quietly on the lookout ; and if, in God's providence, you make the acquaintance of one of the daughters of Zion, traversing, like yourself, the wilderness of this world, her face set thitherward, get into friendly converse with her. If you find that in mind, in heart, in temper and disposition you congenialize, and if God puts it into her heart to be willing to forsake father and mother and cast in her lot with you, regard it as a token from the God of providence that you should use the proper means to secure her Christian society." Duff acted upon this counsel and his wife was very different from the wife who reluctantly accompanied William Carey to the field.

There was a world of difference also in the man-

ner of their going. Things had changed for the better in India. The struggle was not over, but Christianity was not the outlawed thing it had been when Carey went. Duff had letters and recommendations to Lord William Bentinck, the Governor-General, to the Earl of Dalhousie, who was commander-in-chief of the Indian armies, and to other men of influence.

On May 27, 1830, after an eight months' voyage, Duff and his wife reached Calcutta and were warmly welcomed, the Governor-General inviting him to dinner and freely encouraging his plans. So at the age of twenty-four the young Scotchman began his work.

He lost no time in beginning. He had been sent to establish an educational institution. It was not to be in Calcutta but in the country. He visited all the missionaries in or near Calcutta and at once decided that the decision as to the location of the new college was wrong. Calcutta was the place to strike. The next question was as to the character of his institution. There were already a few colleges in Bengal. Fort William College had been established as a training institution for the civil service and was still in existence, though its work was confined to the work of teaching languages to the civilians of the Bengal presidency alone. The three institutions which were in the class with Duff's proposed college were the Hindu College, the Serampore College and the Bishop's College. The first was under Hindu auspices, though aided by the government, which appointed a visitor. "So far as the government was concerned," Duff testified in 1853 to a committee of

the House of Commons, "their views at the outset, with regard to the best mode of communicating European literature and science, were somewhat peculiar and contracted ; in other words, their views seemed to be that whatever of European literature and science might be conveyed to the native mind should be conveyed chiefly through native media, that is to say, the learned languages of India,—for the Mohammedans, Arabic and Persian ; and for the Hindus, Sanskrit. This was the predominant spirit and intent of the British government." Still, English had been introduced into the Hindu College and it was opening up Western learning to the Hindus. The Bishop's College was established by Bishop Middleton in Calcutta in imitation of the Serampore College. The Serampore College was distinctively Christian. In this it differed from the Hindu College which was secularist and obstructive. It taught English and Sanskrit and Arabic, but it did its work in the vernaculars. It rejected the principle of Anglicism which Duff took up and it was also too far from Calcutta to fill the need in the city's life which, as Duff saw, presented him his great opportunity.

He believed thoroughly in the use of education as a missionary agency. That was the axiom with which he had come to India. He believed in it because of its general destructive power in dissolving error and of its general illuminating power in revealing the truth. "While you engage in directly separating as many precious atoms from the mass as the stubborn resistance to ordinary appliances can admit," he said to missionaries engaged in other forms of work, "we shall, with the blessing of God, devote

our time and strength to the preparing of a mine, and the setting of a train which shall one day explode and tear up the whole from its lowest depths." Duff confidently expected his educational method also to produce the trained leaders who would do among the people more than any foreign preacher could ever hope to accomplish. But Duff's policy was not only Christian education but education in English. In the former he parted from the Hindu College, which was secularist, and even worse, teaching Oriental literature and Western atheism. In the latter he pursued a different course from the Serampore College, which taught English but proceeded upon a vernacular basis.

Duff did not ignore the importance of the vernacular. He explained "that while the English language would thus be used as the channel of conveying all higher and improved knowledge, he was determined that the vernacular should be thoroughly taught to the pupils at the same time, as a channel of distribution for the masses." No one was allowed to begin English who could not read with ease his own vernacular. This new study of the vernacular which Duff introduced had two results of vast national importance. "It tended," says Dr. Smith, "to the enriching of the vernacular language with words, and the then barren literature with pure and often spiritual ideas."¹ Writing later Duff said :

"I saw clearly and expressed myself strongly to the effect that ultimately, in a generation or two, the Bengali, by improvement, might become the fitting medium of Eu-

¹ Smith, "Life of Duff," Vol. I, p. 128.

ropean knowledge. But at that time it was but a poor language, like English before Chaucer, and had in it, neither by translation nor original composition, no works embodying any subjects of study beyond the merest elements. As a native of the Highlands, I vividly realized the fact that the Gaelic language, though powerful for lyric and other poetry and also for popular address, contained no works that could possibly meet the objects of a higher and comprehensive education. Hence those who sought that found it in English colleges, and returned as teachers and preachers to distribute the treasures of knowledge acquired through English among the Gaelic people."

Yet there was a real issue between the Serampore plan and his. At Serampore the vernacular and not English was encouraged and used as the medium of education. It was on the desire for English that Duff built his school, and he deliberately threw all his influence to increase the desire, believing that the English language would be an agency of disaster to error and of victory to the truth.

These were the young missionary's plans. At the age of twenty-four he was to attempt single-handed their realization. His school instantly became immensely popular. The young teacher proved to be a genius, a man of fascination and of resistless power. One of his pupils has described his first sight of him in the year 1834 :

"It was about a month after I had been admitted into the institution that I caught a near view of the illustrious missionary. He came into the class room while we were engaged in reading the first page of the 'First Instructor,'

—the first of a series of class books compiled by himself; and though forty-four years have elapsed since the occurrence of the incident, my recollection of it is as vivid as if it had happened only yesterday. I cannot say he walked into the class room—he rushed into it, his movements in those days being exceedingly rapid. He was dressed all in black and wore a beard. He scarcely stood still for a single second, but kept his feet and hands moving incessantly, like a horse of high mettle. He seemed to have more life in him than most men. He had his white pocket handkerchief in his hand, which he was every now and then tying round his arm, and twisting into a thousand shapes. He seemed to be a living personation of perpetual motion. . . . In our lesson there occurred the word ‘ox’; he took hold of that word and catechized us on it for half an hour. He then left our class and went into another, leaving in our minds seeds of future thoughts and reflection. Such is my earliest recollection of Alexander Duff.”¹

To such a teacher and his school pupils began to throng.

“Throughout the whole progress of these preparatory arrangements,” Mr. Duff afterwards wrote, “the excitement among the natives continued unabated. They pursued us along the streets. They threw open the very doors of our palankeen, and poured in their supplications with a pitiful earnestness of countenance that might have softened a heart of stone. . . . Such was the continued press of new candidates that it was found absolutely necessary to issue small written tickets for those who had suc-

¹ Day, “Recollections of Alexander Duff,” pp. 49-51.

ceeded ; and to station two men at the outer door to admit only those who were of the selected number."

Under Duff's personality, the method he had devised was doing its work. The students drank in his spirit and as they went about made but little effort to conceal the change of opinion through which he had led them. "At length, this undaunted bearing and freedom of speech began to create a general ferment among the staunch adherents of the old faith. The cry of 'Hinduism in danger' was fairly raised." Orthodox Hinduism resisted. The Bengali paper *Chundrika*, which had been established to resist the proposal that the government should deprive Hinduism of the privilege of burning living widows, had opened war on the college. The effects were insignificant. The institution was now solidly established and its success was already talked of far and near.

Duff followed up this success at once. He perceived the great opportunity there was for aggressive effort to reach the large body of men in Calcutta whose faith in the old Hinduism had been destroyed but who had nothing in its place. The Hindu college had produced in its students a mass of infidelity regarding "all religious principles whatever." Its students read licentious English plays and Paine's "Age of Reason" was one of the books constantly referred to. The conditions appalled the orthodox Hindus and the government alike. Yet they were the legitimate fruitage of their secular education. Duff faced the situation undismayed. "We rejoiced," he wrote, "in June, 1830, when, in the metropolis of British India, we fairly came in contact with a rising body of natives, who had learned to think

and to discuss all subjects with unshackled freedom, though that freedom was ever apt to degenerate into license in attempting to demolish the claims and pretensions of the Christian as well as every other professedly revealed faith. We hailed the circumstance as indicating the approach of a period for which we had waited long and prayed. We hailed it as heralding the dawn of an auspicious era,—an era that introduced something new into the hitherto undisturbed reign of a hoary and tyrannous antiquity.”

A course of public lectures was arranged to meet this situation. Mr. Hill, a Congregationalist, was to open the course with a lecture on the moral qualifications necessary for investigating truth. That was the only lecture given. The furore caused by the idea that students of the Hindu College had attended a lecture on Christianity at a missionary’s house led to a prohibition that such students should attend any “political and religious discussion,” and rather than imperil his plans, Duff gave up the course.

He soon projected another, however; this time to men whom no such prohibition could reach, and weekly from forty to sixty men listened to Duff’s great arguments and appeals. It was in the days when intellectual interests were few in India, before the government service had been made accessible to natives, before the telegraph, before the flood of Western literature. The minds of the people were just opening and Duff had a clear field. The result was that four of those who heard him in the course responded to his appeal and were baptized. They attached themselves to different churches, but Duff found no fault with that. He never proselytized

himself and he cared little to what church converts joined themselves. Years later, he declared: "I would as soon leap into the Ganges as go near Tinneveli, except as a brother to see the good work that is going on." And at the Conference in New York he declared: "If for a moment I could wield the wand of despotic power for a good purpose, I would go to the heathen field and there chalk out a separate district for every evangelical denomination. I would say to the Baptist, do you go there; to the Episcopalian, take this field; to the Presbyterian, labour in that district; go and convert them, and then baptize them all in whatever way you think best."

Indeed, he was prepared to carry his ideas of missionary comity much further and hoped to see a closer measure of coöperation in educational work. He proposed to a missionary conference in Calcutta a plan by which he would take the best pupils of the various missions and for a fee of ten shillings a month give them the highest Christian education, but home divisions prevented such coöperation upon the field, and various colleges came into existence, when one more powerful institution would have sufficed.

This method of argumentative lectures was kept up for years by Duff and his associates. It was a new method. The audiences had just been created and the experience of such discussions was novel to them. The alternatives presented to their minds were limited to orthodox Hinduism, evangelical Christianity, a Hindu Unitarianism, and atheism. The alternatives of scientific scepticism, agnosticism, and an ethical theistic faith were not present. It

was as it was later in Japan. Christianity came into the field at a time when old positions were untenable and before the new positions in which men satisfy themselves in the West outside of the Christian Churches had become known. There is room for such discussion to-day. It is going on all the time and its form is often just what it was in Duff's day, but the range of the discussion is immensely wider and more difficult.

It is worth noting in passing that some at least seem to have been drawn to Duff's institution because it was cheaper than the others. Nothing, however, was allowed to subordinate in Duff's view the supreme aim of his school. Mr. Day says that in spite of the advantage of the economy of education at Duff's institution, "there was one serious drawback. Mr. Duff was a most zealous missionary; he made no secret of it, but publicly avowed that his chief object in setting up the institution was to instruct Hindu youth in the principles of the Christian religion." Attendance at the Bible classes was rigorously required. Parents also were invited to listen to the hour's preaching to the boys each day. He believed in making the Bible itself "not only the principal book but to bestow upon the teaching of it the largest measure of their time and attention, so long as this could be done without occasioning that desertion of pupils which the more successful prosecution of general literature and science in other native seminaries must inevitably insure, if there be not a correspondent progress in such studies in the mission seminaries."¹ His aim was nothing

¹Smith, "Life of Duff," Vol. I, p. 422.

superficial or semi-religious. It was something definite and decided. In protesting against the idea that the institution would be a mere school for young boys, he wrote :

“ If it is to continue a mere school, then I say that all the time, money and labour hitherto expended on it have been thrown away for nought. Instead of being an apparatus which God might bless as the means of leading heathens to the way of salvation through Christ, it would be much more likely to become a machine for transforming superstitious idolaters into rogues and infidels.”

But were Duff's desires and expectations ever realized? Did higher English education as a missionary method accomplish the results he hoped for? (1) He conceived that he was preparing a mine and setting a train which would one day explode and tear up the whole of Hinduism. But it has proved, not a destructive mine, but a transforming suffusion. Hinduism is not destroyed. It is adjusted. The reform movements of Ram Mohun Roy and Debendranath Tagore and Keshub Chunder Sen ceased as a movement of separation from Hinduism. The old elastic system has simply once again opened to take back these elements of dissent and now makes room within itself for what in Duff's day found no comfortable abiding place there. “ Only use English as the medium,” said Duff, “ and you will break the backbone of caste.” This was seventy years ago. The backbone of caste seems almost as rigid as it has ever been. (2) Knowledge alone does not inevitably destroy error. Duff did not see how men could continue to believe or practice falsehood when they knew that

it was falsehood. But to know the truth, intellectually, is not to abandon the antagonistic error, especially in India, where as Mr. Meredith Townsend contends, one of the characteristics of the Hindu mind has been the failure to see any inconsistency or contradiction in a life lived in moral antagonism to intellectual opinion. Education does not necessarily destroy superstition. It has undoubtedly done so in many minds in India. That has been its net result but that, as Duff contended and as we shall see, is a very inadequate achievement. (3) The use of English has accomplished something of what Duff hoped and, as he foresaw, some of the first opposition to its use has disappeared. It was urged that to teach it would be merely to supply rogues. Young men would learn enough of it to put to use in some discreditable or small commercial way and then they would leave. Missions face that difficulty now in China. Duff replied that this difficulty would soon take care of itself, that the market for such men would soon be overstocked and that then those who wished to obtain better positions would have to remain longer. It is a long and expensive process, however, to overstock the market. (4) The history of Duff's policy suggests that it is not education as such which makes men Christian or indeed in any large number prepares them for the acceptance of other influences, which will make them Christians. Character is more a matter of feeling and will, of the personality, than of opinion. Opinion enters and to the extent that it does so, all processes which affect opinion affect character, but men are not simple thinking-machines, and they do not always see truth

when it is presented, or recognize it when they see it, or embrace it when they recognize it. To teach them the truth is a duty and a service, but it can only accomplish its full measure of good when to the intellectual presentation of truth are added the vital personal forces, which affect the whole personality and get for the truth a living acceptance. Duff's plan included the most powerful and persistent use of all these personal influences. He used them. "As the chief object of the General Assembly's Institution," says Mr. Day, "was to convert the students to Christianity, the course of studies pursued in it was thoroughly saturated with the spirit of that religion from the lowest to the highest class." And Duff poured out his own life among the students in his zeal for their direct conversion. There was no concealment of his purpose, no indirectness, no justification of his work by secondary consequences. The result appeared in such actual conversions of young men as we do not see to-day in Calcutta. The same change has been witnessed in Madras. In the different missionary institutions in Madras, Dr. John Murdock says there were thirty-nine baptisms in 1852-1856, ten baptisms in 1856-1861, five in 1862-1866, and one in 1867-1871. What produced this change? For one thing, unquestionably the conditions have altered greatly, so that students who hold Christian opinions can now remain in Hinduism without discomfort. They are not Hindus, but they feel no necessity of leaving Hinduism to become real Christians. For another thing, there is not the same consuming eagerness to make converts. Men go out to teach in missionary

educational institutions who do not work for the conversion of students as Duff and his associates worked, who say that the burden of their other work makes it impossible for them to do so. Mr. Day recognizes also the change that has passed over such institutions as Duff's in the matter of the supremacy of their distinct missionary aim, due to the influence of the government educational system, for whose degrees the missionary institutions prepare their students :

“As missionaries prepare their students for the degrees of the university, they adopt the curriculum of studies prescribed by that learned body ; they have, therefore, at present, less time for the Christian and theological training of their pupils than before ; while the students themselves naturally pay little or no attention to those studies which do not pay in the university examination. The state of things was different, however, in the pre-university days of which I am now speaking. The students were in those days thoroughly grounded in a course of natural theology, a course of the evidences of Christianity, a course of systematic theology, a short course of ecclesiastical history, besides a course of lectures on almost the whole of the Holy Scriptures, from the Book of Genesis to the Book of Revelation.”¹

Connection with the government system has its advantages. It secures certain grants in aid. It brings students to the mission colleges who would not come unless their course made them eligible to government degrees and so to political office. It ensures

¹ Day, “Recollections of Alexander Duff,” p. 138.

a high standard of educational work. But in many cases, it does involve a subordination of the distinctly evangelistic purpose of missionary education. It has done this so certainly that some leading missionary educators have dropped Duff's religious emphasis and justify their work on grounds wholly distinct from direct influence in making men Christians and leading them to baptism.

There was no such government educational system when Duff came to India. In its creation and the determination of some main features of its character, Duff exercised a great influence. Duff began his work in India just as the long struggle for a recognition of England's duty to the moral and intellectual life of India was ending. At each renewal of the East India Company's charter since 1783, there had been agitation as to the character of the company's rule, and the charter of 1833 completed its transformation. It destroyed the last element of a trading concern in the company and made it a purely governing body. It removed the last obstruction to the work of the missionary and to the free movement and activity of all foreigners. And it recognized the duty of the company to provide for the education and improvement of the people of India. The old era of subordination of Western standards and ideals to Hindu superstition and immorality was over, though it required the shock of the Mutiny to complete the transformation. But the question which now arose was as to the form of the educational schemes which should be undertaken. As has been already pointed out, there were three schools of opinion: (1) The Orientalists, who believed in teach-

ing the people their own literature and perpetuating the ignorance and superstition of the past, its false science and geography and immoral and philosophical error; (2) the Vernacularists, who occupied the middle ground of the Serampore College, who taught English but laid chief emphasis on the development of the vernaculars, the translation of proper books into them and their use as the chief instrument, and (3) the Anglicists who indeed used the use of the vernacular as Duff did, but laid the chief emphasis on English because it made available the literature not accessible in the vernacular, was itself an instrument of enlightenment and an agency of the truth, was the language of the government and so useful politically as a means of communication and as a bond of union, and would as a silent influence operate to the accomplishment of the ends of improvement contemplated by the new charter.

The third party prevailed. Its victory was due to three young men. The first was Duff, who was now twenty-seven years of age and who had demonstrated in his institution the value and success of the method advocated by the Anglicists. The second was Trevelyan, a young man of twenty-eight whose character and abilities and fearlessness of principle had made him already one of the foremost men in India. The third was Macaulay, who was thirty-three and who had been the chief factor in securing the new charter and was now in India as the law member of the Governor-General's Council. Trevelyan knew of Duff's work and had heartily accepted his principles and through him Duff's view came to expression in the famous minute of Macaulay which established and

determined the new educational policy of England in India. Trevelyan subsequently, after half a century, acknowledged Duff's predominant influence.

But ignorance and falsehood do not die easily. The teaching of English produced no such results as Macaulay dreamed of. It has bred much deism, little Christianity, and very much of what the Allahabad *Pioneer* calls "fetichism in patent leather boots." And its influence has also been far reaching for good. It has penetrated the educated mind of India with Christian conceptions. But popular Hinduism is not dead and it remains to be seen whether several centuries of English teaching, resulting some day in making India a practically English-speaking land, will present the phenomenon of an English-speaking country worshipping idols and phallic symbols and cows, in the language of the English Bible and Milton and Shakespeare.

But Duff's ideal was not an ideal of English teaching alone. He did believe that teaching English would destroy error and caste, but it was only by accompanying it with a powerful positive teaching of religious truth in English. The new education in India, however, was to be absolutely neutral in the matter of religion. As a matter of fact it was not neutral. It refrained from teaching Christianity, but it substantially allowed the teaching of Hinduism and Islam. Even the Mutiny did not make it wholly neutral. Atheism and agnosticism have been freely taught where Christianity has been barred. But against even a purely neutral system, Duff made earnest protest.

The first and last paragraphs of the decree of the

Governor-General in which the new policy was announced were as follows :

“His Lordship in Council is of opinion that the great object of the British government ought to be the promotion of European literature and science among the natives of India, and that all the funds appropriated for the purposes of education would be best employed on English education alone. . . .

“His Lordship in Council directs that all the funds which these reforms will leave at the disposal of the committee be henceforth employed in imparting to the native population a knowledge of English literature and science through the medium of the English language; and his Lordship in Council requests the committee to submit to government, with all expedition, a plan for the accomplishment of this purpose.”

The educational system was to be thoroughly English. In this Duff heartily concurred. He declared his belief in it as a wise practical expedient. It was what he had striven for and what now, due in large measure to his influence, was settled. But Duff was fundamentally and immovably opposed to the exclusion of religious teaching from the government schools. In his first missionary oration before the General Assembly after returning to Scotland in 1835, he referred to this matter, urging the political inexpediency of the government's plan of purely secular education :

“If in that land you do give the people knowledge without religion, rest assured that it is the greatest blunder, politically speaking, that ever was committed. Having free unrestricted access to the whole range of our English literature and science, they will

despise and reject their own absurd systems of learning. Once driven out of their own systems, they will inevitably become infidels in religion. And shaken out of the mechanical routine of their own religious observances, without moral principle to balance their thoughts or guide their movements, they will as certainly become discontented, restless agitators,—ambitious of power and official distinction, and possessed of the most disloyal sentiments towards that government, which, in their eye, has usurped all the authority that rightfully belonged to themselves. This is not theory, it is a statement of fact. I myself can testify in this place, as I have already done on the spot, that expressions and opinions of a most rebellious nature have been known to drop from some of the very protégés of that government, which, for its own sake, is so infatuated as to insist on giving knowledge apart from religion.”

The policy of the British government in India in the matter of neutrality in its educational system it is easy to understand. Historically, it is an inheritance from the old East India Company days. Then the company distinctly discouraged Christianity and subsidized idolatry. When later, this became impossible, the government ceased the “endorsement of error” as Duff called it but proceeded upon the theory of a colourless religious influence. And of course, the idea of purely secular education, neutral as to all religion, is a commonplace idea with us in America, where some states have even forbidden the Bible or anything Christian in the schools. Now apart from what can be said about such a policy in America, three things are to be said about it in

India. (1) The idea of an education religiously neutral is delusive. As Duff said in his speech to the General Assembly in 1835, "Do then let me crave the attention of this venerable court to the grand peculiarity, that if in India you only impart ordinary useful knowledge, you thereby demolish what by its people is regarded as sacred. A course of instruction that professes to convey truth of any kind thus becomes a species of religious education in such a land—all education being there regarded as religious or theological." To teach the truth about geography is to destroy some of the Hindu theology. On the other hand, to teach agnostic philosophy or materialism is to assail Christianity. Both of these things were done in Indian schools. (2) No public sentiment in India demanded such a course. Indeed, the idea was wholly novel. As a statement to one of the viceroys by the Home Department in 1872 declared: "That most remarkable feature in Indian education, the religious neutrality of the government, is no doubt a relic of the extreme apprehension which prevailed in 1793, and whether its original declaration was a wise one or not is far too deep and many sided a question to be discussed here. We must accept the fact as we find it. But it is, I believe, absolutely without precedent or parallel elsewhere, besides being entirely opposed to the traditional idea of education current in the East." (3) The results are calamitous. The statement just quoted refers anxiously to this fact and during the Mutiny, Duff urged that England should learn from it the futility of her secular agencies for reforming the character of a people :

“Railways and telegraphs and irrigating canals,” he wrote, “and other material improvements, alone will not do. Mere secular education, sharpening the intellect, and leaving the heart a prey to all the foulest passions and most wayward impulses, will not do. Mere legislation, which, in humanely prohibiting cruel rites and barbarous usages, goes greatly ahead of the darkened intelligence of the people, will not do. New settlements of the revenue and landed tenures, however equitable in themselves, alone will not do. Ameliorations in the present monstrous system of police and corrupting machinery of law courts, however advantageous, alone will not suffice. A radical organic change in the structure of government, such as would transfer it exclusively to the Crown, would not, could not, of itself furnish an adequate cure for our deep-seated maladies. No, no! Perhaps the present earthquake shock which has passed over Indian society, upheaving and tearing to shreds some of the noblest monuments of material civilization, as well as the most improved expedients of legislative and administrative wisdom, has been permitted to prove that all merely human plans and systems, whatsoever, that exclude the life-awakening, elevating, purifying doctrines of gospel grace and salvation have impotence and failure stamped on their wrinkled brows.”

And the firm corroborative views of John Lawrence we shall have to consider when we come to his life.

It is sufficient now to point out that the government educational system, secularized against the protest of Duff and later, after the Mutiny, of John Lawrence, has produced the results which Duff foresaw. “The present teaching in the government high

schools and colleges," said Robert Clark, a lifelong resident of the Punjab, "with its so-called religious neutrality is only throwing a thick veil over the land, hiding from it all true light and life, whilst it is sapping the foundations of all creeds and proving itself to be the most destructive to morality and to all good government in this life as well as all hope in the world to come." "Concerning teaching in government schools," said the late Bishop Parker of the Methodist Church, "the natural result of the system is to destroy religion, to break down feelings of moral obligation and to raise up a proud, unsatisfied, discontented, complaining class." "I found," said Sir W. W. Hunter, "from taking evidence of 193 witnesses through India, as president of the Education Commission, that those leaders were unanimous in lamenting the absence of religious teaching in our state schools in every province of the Indian Empire."

Neither its English character nor its religious neutrality has fulfilled the expectation of the founders of the educational system of India. Carey was wiser in his advocacy of "a permanent healthy naturalization" of education by a larger use of the vernacular. The Japanese have taught English and German, but they are doing their educational work in their own language. The Chinese, it is to be hoped, will do the same. In the Philippines, where the problem is more like India's, we are following England's example both in English and in religious neutrality. Already in the Philippines five times as many people speak English as ever spoke Spanish. In this we are nearer wisdom than the English

were. In our colourless religious policy, we are equally unwise.

Duff realized that the policy adopted was unchangeable and exerted his influence to develop the grant-in-aid scheme in the hope that it might result in the transfer of the work of education from the government with its neutral principles to other organizations which would supply the religious element which was required. This principle was adopted fifty years ago. "We confidently expect," said the government, "that the introduction of the system of grants-in-aid will very largely increase the number of schools of a superior order; and we hope that, before long, sufficient provision may be found to exist in many parts of the country for the education of the middle and higher classes, independent of the government institutions, which may then be closed."

Duff always pled for this principle. Its recognition has been very undulatory. Some viceroys have supported it, others ignored it. Dufferin cordially approved it: "In aided schools," he said in 1887, "religious instruction may, of course, be freely given, and the Governor-General in Council would be sincerely glad if the number of aided schools and colleges in which religious instruction is prominently recognized were largely increased. It is in this direction that the best solution of this difficult problem can be found." But Lord Curzon had less zeal for an increased support of such institutions, his political temper inclining to a secular system under state control.

Duff's influence helped to carry another useful re-

form, the Romanization of writing the Indian languages. Sir Charles Trevelyan gives Duff the credit for this.

Such services as these are directly missionary, but they are also the services which are of indispensable value to every agency, political and commercial, requiring communication with the people.

After five years' work in India, Duff was ordered home, on account of an attack of dysentery and the prostration which ensued. He protested against going. He had come to India to spend his life there as Carey had done. He urged that if he must leave, it might be only for a short sea voyage. "I devoted myself to the Lord," he urged, "to spend and be spent in His service in this land." The physicians were inexorable, however, and after nine months of suffering from tropical disease, Duff sailed to spend four years at home and to do in Scotland a work for missions as great as he had wrought in India. He found at first a cool welcome. For his first quiet presentation of the work to a little company at a private house, he was called before the Missionary Committee and taken to task for the irregularity of holding meetings without consulting and securing the consent of the committee. This small battle he won but the doors open to him were few until the meeting of the General Assembly. Friends tried to dissuade him from appearing before the General Assembly. He was still weak. The docket of the assembly was full of items of important business,—such routine business as is so important here on the earth and as is seen from heaven in its true pettiness. But Duff knew he must win the Church and he insisted on ap-

pearing before the assembly in connection with what had been the merely formal affair of the presentation of the report of the India Mission. He rose half sick to address the assembly which waited unawares for what he had to say. For between two and three hours he spoke to them. Indifference gave way to interest and interest to emotion. When he finished, the audience was weeping. Tears rolled down the cheeks of even the most callous and careless. At one bound, the missionary and his cause had been lifted into the front of the Church's interest and care. The next day, the *Scottish Guardian* presented the speech in full and said: "It will be long ere the assembly will forget his pleading. His appearance has thrown a sacredness around its meeting, and will give a Christian elevation and dignity to the whole of its procedure. His speech will yet tell in its moral influence, not only in the cottages of India, but in the cottages of our own land, and will send back our clergy to their homes smitten with the missionary and apostolic spirit that burns with sweet fervour in the breast of our devoted missionary." Indeed, the assembly became known for that one speech of Duff's, then twenty-nine years of age. The assembly ordered its publication and 40,000 copies of it were scattered over Great Britain and America.

Duff now had his hearing. Indeed, his first speech brought upon him appeals to stay at home and work there. To these he replied: "Were I to remain in my native land, it would doubtless be still in my power to do something by way of advocating the claims of poor, benighted India. In that case, however, methinks my tongue would not only falter, but

often 'cleave to the roof of my mouth.' Fearlessly and unsparingly have I reprobated the indolence and cowardice of those who kept lingering, lounging and loitering at home, in lazy expectation of some snug, peaceful settlement, instead of nobly marching forward into the wide field of the world, to earn new trophies for their Redeemer, by planting His standard in hitherto unconquered realms. Neither have I suppressed my honest indignation at the no less criminal supineness of others, who, having once obtained such settlements, ingeniously devise a thousand petty frivolous pretexts for continuing to wrap themselves up in the congenialities and luxurious indulgences of home, instead of boldly daring, though at an immeasurable distance, to tread in the footsteps of apostles and prophets and martyrs." From all such temptations, Duff turned to fling himself into the work of rousing Scotland to her missionary duty. He went everywhere systematically visiting and organizing the Presbyteries, inaugurating spiritual revivals; stimulating gifts to work both at home and abroad and enlarging alike the mind and the soul of Scotland, and then in 1839 returned to India.

There he found the college still a great success. Its work had been much advanced and its spiritual agencies perfected and it was steadily directed as an agency of great power to the direct winning of young men to Christian faith and life. He and his four associates from home constituted a band of singular power and as singular unity. One of his first acts was to propose a plan of reorganization, which placed the work under a council made up of the missionaries under the senior or whomever the home Church might

recognize as the head. The spirit in which the work was done is shown in Dr. Mackay's report to the home committee: "Dr. Duff will tell you of our meeting together regularly for consultation, and of what we have agreed upon; but I cannot refrain from saying that in all our new and complicated arrangements, arising out of our increased number and efficiency, there has been no difference of opinion; and we are all agreed as one man. Each is satisfied with his own peculiar work, and all are satisfied that everything has been done for the best. In Christ we feel that we have one Head, one end, and one mind; and believing, we pray that we may always labour together in peace, and unity and love."

Not long after this Duff felt obliged to open a public criticism of Lord Auckland, the new Governor-General, who had receded from Lord Bentinck's educational policy in the direction of Orientalism so far as to renew the subsidies to what Duff called "the shrines and sanctuaries of Hindu and Mohammedan learning with all their idolatrous, pantheistic and antichristian errors!" In publishing his letters, Duff was not deceived with the thought that he would accomplish anything, but he felt that he must justify his position and express his condemnation of a "reactionary measure of an incompetent Governor-General."

"These words are penned," he wrote, "in the full assurance that with your lordship and councillors they will have the weight of a feather. So let it be. Here, your lordship is everything. Here, politically and civilly speaking, your voice is all but omnipotent. But, my lord,

I must remind you that the greater the power, the more tremendous the responsibility! I must also remind you that—apart from the solemnities of the great assize to which the noble and mighty will be summoned, without respect of persons, along with the poorest and the meanest of the land—there is, even here below, another tribunal, of a different frame and texture from that of an Asiatic, time-serving, favour-seeking community, at whose bar the appeal of a gospel minister will be heard as promptly as that of the noblest lord. There is a great British public, and above all, a religious public in Great Britain, which heretofore hath been moved, and may readily be moved again, by the addresses and expostulations of a Christian missionary. A surer prospect of earning the garland of victory no Christian missionary could possibly desire, than the opportunity of boldly confronting, on a theme like this, the mightiest of our state functionaries, in the presence of a promiscuous audience of British-born freemen, in any city or district from Cornwall to Shetland. His march would be that of one continued conquest. The might and the majesty of a great people, awakened to discern the truth and import of things as they are, would increasingly swell his train. And from the triumph of indomitable principle in Britain would emanate, as in times past, an influence which would soon cause itself to be felt in the supreme councils of India, and thence extend, with renovating efficacy, through all its anti-religious schools and colleges."

This was characteristic of Duff's spirit. He was not afraid of governors. He had declined to attend a governor-general's ball in Lord Bentinck's time, because there was to be dancing, and his attitude

was appreciated ; and he and Mrs. Duff were invited to dinner by Lord and Lady Bentinck. And now he scored Lord Auckland for his folly and reaction. He spoke once of the bold position his mission had taken in the face of heathenism. He was quite as bold in the face of Christians. His force and magnitude of mind delivered him from timidity before strong issues or great situations.

He had to face one such soon in the problem raised by the disruption of the Church in Scotland and he did not hesitate. His mind, which he had not felt free to express, while the issue was undecided, had been already made up and when the schism at home made it necessary for the missionaries to choose, he at once took his place with the Free Church. The property of the college which he had built up and which would have had no existence but for him, went to the Established Church, so that he and his associates were forced to begin anew. Duff believed that he was entitled to retain the institution and he might have kept a spacious tract of adjoining land which was held in his own name and which was ample for a new institution if the old should be surrendered to the Established Church. But Duff had no mind to fight with Christian men for his rights when time was needed for saving souls.

“ Were any one at this moment,” he wrote, “ to offer me, in free gift, a library and apparatus, of ten times or tenfold ten times the extent of those now in debate, under the contingent condition of its possibly entailing, some years hence, half the loss of time and vexation of spirit which, from first to last, has been incurred by the present wretched

and unedifying discussion, I would fling the offer with loathing indignation away from me." And he even refused to use the land he held adjoining the old institution. He held to the Saviour's theory of rights,—that being ours, we have a right to surrender them. You cannot very well quarrel with such men.

As the Free Church was obliged to found both at home and abroad new institutions of its own, Duff urged that the opportunity should be improved to establish in the Free Church College at home a missionary chair. It was years before his ideal was realized. He had written of it to Dr. Gordon in 1844. It was in 1865 that he revived the project before the General Assembly.

"When passing through the theological curriculum of St. Andrews," he said, "I was struck markedly with this circumstance, that throughout the whole course of the curriculum of four years not one single allusion was ever made to the subject of the world's evangelization—the subject which constitutes the chief end of the Christian Church on earth. I felt intensely that there was something wrong in this omission. According to any just conception of the Church of Christ, the grand function it has to discharge in this world cannot be said to begin and end in the preservation of internal purity of doctrine, discipline and government. All this is merely for burnishing it so as to give light not to itself but also to the world. There must be an outcome of that light, lest it prove useless, and thereby be lost and extinguished. Why has it got that light but that it should freely impart it to others?" He succeeded in

getting this Assembly of 1865 to provide that the chair should be accepted by an experienced foreign missionary and that the appointment should be for life. To his surprise, the place was forced upon him, though he succeeded in preventing the chair from bearing his name and he never used the income from the endowment of £10,000, but devoted it to the establishment of a Missionary Institute.

Duff's second term of service in India covered eleven years. More than once during these years baptisms in the college shook its patronage but not its stability. In 1845, he faced a specially bitter opposition; this time due not so much to old orthodox Brahmans, as to wealthy upstart Hindus allied with the priests. The opposition expended itself in coarse threats of personal violence and more permanently in the establishment of a rival institution.

Towards the close of his second term of service in India, Chalmers died. The Church at home instinctively turned to Duff as his true successor, at the head of the New College which Chalmers had founded in Edinburgh. The General Assembly of 1848 formally called upon him to return. After expressing his appreciation of the honour of the great call, and of the way men and the press were congratulating him upon his contemplated "election" or "promotion," he wrote: "I deem it, therefore, an unspeakable privilege to have it in my power to do anything, however humble, towards magnifying my much despised office. The conclusion of the whole matter is this, that in some form or other, home or abroad, or partly both, the Church of my fathers must see it to be right and meet to allow me to retain, in the view of

all men, the clearly-marked and distinguishing character of a missionary to the heathen abroad, labouring directly amongst them ; at home, pleading their cause among the Churches of Christendom. . . . For the sake of the heathen, and especially the people of India, let me cling all my days to the missionary cause."

The protests which the proposal called forth from various classes in India showed how powerful an impression Duff had made. "His name," said one appeal from eleven learned Brahmans, in Sanskrit, "is in the mouth of every Hindu because of his transcendent eloquence, learning and philanthropy." While he declined the call from the home Church, he assented to the advice of friends that he should go home to do for the missionary work of the new Free Church what he had done for the whole Church before the disruption. What he had in mind to press as he came home on this errand, he set forth in a letter to Dr. Tweedie on May 3, 1850. He is speaking of the approaching General Assembly :

"Tuesday the 28th would do well for our missions. Could we not get the whole day for them? How often is a whole day given to the discussion of a case of discipline! And is it too much to give to that of the greatest cause on earth? There is your report; Anderson, Nesbit, perhaps Rajahgopal, will speak, why not some other members of Assembly? Then I would require at least two or three hours to be able to say anything at all. If the whole day were given to the mission, I would prefer to have the evening, so as to take up any matters that may have dropped during the day, etc. For yourself alone, at pres-

ent, let me state a few things that appear to me highly desirable to be done. First, to appoint a day of humiliation and prayer throughout the Church for past sins of negligence, with reference to the Redeemer's great command to evangelize the nations. This would, if done *con amore*, go much to the root of our evils, and mellow people's hearts and open the windows of heaven. Second: Substitute regular weekly subscriptions for the annual collections, as the only suitable and productive and becoming source of supply for a great and permanent undertaking. Third: Let the rule of proportion be better established with reference to men's liberalities towards different objects. Fourth: Cut me off a county or a synod in which to give fair trial to the new experiment. There is no other way of fairly testing it. Occasional addresses and appeals go for nothing. I should like to see a living machinery established as a specimen somewhere."

The General Assembly approved his plans and for three years and a half he "gave himself to the creating of his new organization—an association for prayer, information, and the quarterly collection of subscriptions for the missions in every one of the then seven hundred congregations of the Free Church of Scotland." Dr. Smith says that "before or soon after his return to Bengal, he had secured the establishment of five hundred associations, yielding a 'sure and continuous increase' of funds to meet 'the requirements of a continuous expenditure.' "

In addition to his direct missionary service while at home, Duff was able to do a work of great value in giving form to the reorganization of the government educational system in India. The last renewal of the

East India Company's charter came in 1853. The Mutiny swept it away. As preceding acts of renewal had steadily liberalized the charter, so this one continued the tradition of improvement. The civil service was improved and the educational provisions were changed for the better, though not to Duff's satisfaction. What improvements were made, however, were chiefly due to his testimony and advice before the Parliamentary Committee. He contended for a Christianization of the government education. "No amelioration in our legislative or judicial policy," he declared, "will reach the springs of some of those evils which I have attempted so inadequately to delineate. Their spring-heads are to be found in those deep-rooted superstitions which work so disastrously in deteriorating native society. Nothing can suffice but a real, thorough, searching, moralizing, and I should individually say, Christianizing course of instruction, which, by illumining the understanding and purifying the heart, will inspire with the love of truth and rectitude, and so elevate the whole tone of moral feeling and social sentiment among the people."

He advocated the discontinuance of the remnant of the policy of Orientalism which Lord Auckland had revived, the development of the grant-in-aid scheme, the establishment of universities and the introduction of the Bible in the schools.

Before returning to India, Duff visited America in 1854-1855, at the urgent invitation of George H. Stuart. His presence here was a continual ovation and the great missionary was nearly killed with kindness. In Philadelphia and New York he ad-

dressed great throngs. Some still living remember the fervour of his oratory. One who heard him has told me of the way in which he worked up his argument. He grew more and more intense, gathering up the tails of his long frock coat under his arms, until as he reached the climax, he had a tight rolled bundle of coat tail under each arm and was leaping up and down in his intensity. The reporters were so carried away that they lost his peroration. It was a style of oratory which has passed away, affluent, orotund, tropical, with immense sentences and periods, rather destitute of humour, but wonderfully rich and ample; and warm with the transfusing love and devotion of the man's soul. He would speak for hours—often three—and he told one Scotch General Assembly that it was easier for him to speak six hours than a shorter time. But he was not all words. His speech was to definite ends and in question of policy he came right down to hard and elemental essentials.

During his stay in New York, a missionary conference was held attended by 300 ministers at which a series of discussions dealt with various missionary problems and issued in a series of resolutions offered by Dr. Duff. The influence of that conference lasted in New York into our own day. The men whom it influenced, however, have now almost all passed away. It may be doubted whether any subsequent conference has produced more definite and abiding results. Duff's personality dominated it. He was the conference.

Duff returned to India just before the Sepoy Mutiny broke. He called it the Indian Rebellion. He wrote

a series of letters from the midst of it and about it to Dr. Tweedie which were published and which though coloured, of course, by the intensity of the situation, are far less so than the accounts of the Peking siege by those who experienced it. These letters are still in many regards the best account we have of the cause and character of the rebellion. Duff agreed with John Lawrence in attributing the rebellion to the policy of the East India Company and government; it had been a truckling, compromising policy from the beginning, a policy of treason to Christianity and of steady resistance to the claims of higher duty. The discontent which issued in the Mutiny had many causes and occasions but in the end the great cause was the traditional spirit of trimming, of selfishness, of no principle, wrong principle or half-principle. The Mutiny was the reaping from a long harvest. Even when in the midst of it, the Anglican Bishop appealed to the Governor-General to set a day of humiliation and prayer, he refused, but issued a proclamation which practically disavowed Christianity by ignoring it, and the same paper which published the proclamation announced, said Duff, "the closing of all government offices for about ten days in honour of the most celebrated of our idolatrous festivals—the Durga Puja." Duff found in the Mutiny as others did fresh evidence in support of his conviction that the Christianization of India was the only guarantee of the endurance and peace of its political relations to Great Britain.

Duff spent six years in India after the Mutiny. The college continued its prosperity and its influ-

ence. Yet such influences are both more and less than men suppose. Indeed, here is as good a place as elsewhere to suggest that Duff was far more the power of his work than his method was. The method itself minus such a personality as Duff's has proved relatively barren. Indeed, any method will. Vernacular preaching may be as futile a performance as pouring water through sieves. The real power is the power of personality possessed by the living Spirit of God. That will accomplish results through any method which provides for personal contact, and no method will avail much without that.

His last work in India was in connection with the University of Calcutta and the grant-in-aid system. "So long as he remained in Calcutta he secured fair play for the liberal and self-developing principles of the education despatch of 1854. When he and Dr. Wilson ceased to influence affairs and rulers, the public instruction of India began to fall back into the bureaucratic, anti-moral and politically dangerous system, from which Lord Halifax thought he had forever rescued it."¹

He was vice-chancellor of the university, the viceroy by his office being chancellor, and he was its ruler while he was in India. "To his gigantic mind," says Dr. Banerjea, "the successive vice-chancellors paid due deference, and he was the virtual governor of the university. . . . Dr. Duff was the first person who insisted on education in the physical sciences, and strongly urged the establishment of a professorship of physical science for the university." He was a Christian missionary,

¹Smith, "Life of Duff," Vol. II, p. 382.

but he was also a bigger man than the men about him. It was he who years before had been the means of introducing real medical education, when the government was afraid to touch it for caste prejudice. It was he now who secured for the physical sciences their right place. He had never been afraid of truth. Truth would never hurt truth, he held, and he was sure that the truth about God would never suffer from truth about His world. Sir Henry Maine, who succeeded him as vice-chancellor, referred to this quality in a convocation as he left India :

“I am not aware that he ever desired the university to refuse instruction in any subject of knowledge because he considered it dangerous. Where men of feebler minds or weaker faith would have shrunk from encouraging the study of this or that classical language, because it enshrined the archives of some antique superstition, or would have refused to stimulate proficiency in this or that walk of physical science, because its conclusions were supposed to lean to irreligious consequences, Dr. Duff, believing his own creed to be true, believed also that it had the great characteristic of truth—that characteristic which nothing else except truth possesses—that it can be reconciled with everything else which is also true. Gentlemen, if you only realize how rare this combination of qualities is—how seldom the energy which springs from religious conviction is found united with perfect fearlessness in encouraging the spread of knowledge, you will understand what we have lost through Dr. Duff’s departure, and why I place it among the foremost events in the university year.”

In 1863 at last, when he was fifty-seven, Duff realized that his duty to foreign missions required that he should return to Scotland to live and work there. But he would not have gone even then if his dysentery had not returned, and his condition of health become such that a sea voyage and rest had failed to restore him. His departure was the occasion of such a flood of grateful expressions as no governor or viceroy ever received. It took many forms, but none, with his approval, inconsistent with the missionary character which had been so supreme with him as to warrant Sir Henry Maine's words in the address already referred to: "It would be easy for me to enumerate the direct services which he rendered to us by aiding us with unflinching assiduity, in the regulation, supervision, and amendment of our course of study; but in the presence of so many native students and native gentlemen who viewed him with the intensest regard and admiration, although they knew that his every-day wish and prayer was to overthrow their ancient faith, I should be ashamed to speak of him in any other character than the only one which he cared to fill—the character of a missionary." "A few of the Scottish merchants of India, Singapore, and China offered him £11,000. The capital he destined for the invalided missionaries of his own Church, and for these it is now administered by the surviving donors as trustees. On the interest of this sum he thenceforth lived, refusing all the emoluments of the offices he held. The only personal gift which he was constrained to accept was the house, No. 22 Lauder Road, Edinburgh, which the same friends insisted on purchasing for him."

Duff lived to lead for eleven years the missionary work of the Free Church. He perfected the missionary organization of the Church. He believed in work, and the man of sixty went about with the same almost tornado energy of a generation before. He believed in expansion, and his administration saw the establishment of new missions in India and Syria and Africa and led to the mission to the New Hebrides in the South Seas. Home administration of missions is not a romantic thing and Dr. Duff did not accomplish anything remarkable. Governments with unlimited financial resources can plan with a free hand, but even the greatest missionary administrators like Duff are obliged to do things patiently and to be content to wait. He dealt with the difficult problems of missionary salaries and decided as all Boards decide. But his work at home was more than a work of missionary administration. He was foremost in the negotiations for the union between the Free and United Presbyterian Churches, which has only been consummated a generation since in our own day. For a second time, he was Moderator of the General Assembly, and in his own land and in England and on the continent, his ripened character and immense powers found opportunity for activity in behalf of the cause which he realized was the one cause above all causes. At the age of seventy-two he went peacefully on from his work here for that cause to the service in its behalf which is untrammelled by the limitations of earth. The whole of Edinburgh shared in his burial. "For the first time in Scottish ecclesiastical history, three Kirks and their Moderators, the representatives of the English and American

and Indian Churches through their missionary societies and officials, trod the one funeral march." And shortly after, Mr. Gladstone, after referring to the great missionary names of the Church of England, Selwyn and Patteson, and then the names of Carey, Marshman and Moffatt, added :

"But we must recollect Dr. Duff as one who not only stood in the first rank for intelligence, energy, devotion and advancement in the inward and spiritual life among those distinguished and admirable personages, but who likewise so intensely laboured in the cause that he shortened the career which Providence would in all likelihood have otherwise committed to him, and he has reaped his reward in the world beyond the grave at an earlier date than those whose earthly career is lengthened into a long old age. He is one of the noble army of the confessors of Christ. Let no one envy them the crown which they have earned. Let every man, on the contrary, knowing that they now stand in the presence and in the judgment of Him before Whom we must all appear, rejoice that they have fought a good fight, that they have run their race manfully and nobly, and that they have laboured for the glory of God and the good of man."

I have to add but a little regarding Duff's missionary view and personal character. He is often spoken of as representing the exaltation of education as a missionary method above direct evangelization. But his type of education was evangelization. His teaching of secular subjects was interpenetrated and indissolubly bound by requirement to the fullest religious teaching. He valued his school only as a direct

evangelizing activity. He had no conception of secular character. The only character of which he knew was Christian character. Yet he did recognize a distinction between education and itineracy.

The numerical results of both forms of work he admitted had been small in Bengal. Speaking of Lacroix, he said, "Though he laboured far more and far longer than any other man in the direct preaching of the Gospel to myriads in their own vernacular tongue, and though no foreigner, in this part of India, ever equalled him in his power of arresting and commanding the attention of a Bengalee-speaking audience, yet the success vouchsafed to his faithful, acceptable and untiring labours in the way of the conversion of souls to God, for which he intensely longed and prayed, was comparatively very small." Yet as he urged upon the General Assembly of 1865 both services must be carried on :

"Our plan never was intended to be—and, in point of fact, never actually was—a narrow, one-sided, fixed, exclusive plan. . . . From the very outset the two kindred and reciprocally auxiliary processes of training the young for varied future usefulness, and addressing the adults, through whatever lingual medium might be found most effective in reaching their understandings and their hearts, were simultaneously carried on, side by side."

He was an extreme advocate of centralization in mission work. But he had a good mediating mind, although in the matter of English higher education he defended the extreme view, and doubtless he would state almost as strongly as missionary experience would lead us to state to-day the importance of

balancing strong central stations with a wide country work, both to save the central stations from stagnancy and to utilize and conserve the results of their institutional work.

Duff did not see as clearly as we see—our bitter experience has taught us—the importance of self-support and the spirit of independence in the native Churches. His eyes were towards the Church at home, not towards the native Church as the source of support for native workers. There is a large, legitimate and necessary employment of native workers from home funds. But it is calamitous to allow a native Church to grow up in the view that the choice for young men qualified for Christian service must be either secular service or support from mission funds.

The importance of a native agency Duff saw clearly from the outset. He put it vigorously in his first General Assembly speech in 1835.

“Oh, there is that in the tones of a foreigner’s voice which falls cold and heavy on the ear of a native, and seldom reaches the heart!—whereas, there is something in the genuine tones of a countryman’s voice, which, operating as a charm, falls pleasantly on the ear, and comes home to the feelings and touches the heart, and causes the tenderest cords to vibrate. . . . And having the thousand advantages, besides, of knowing the habits, the manners, the customs, the trains of thought and principles of reasoning among the people, they can strike in with arguments, and objections, and illustrations, and imagery which we could never, never have conceived. How glorious then must be the day for India

when such qualified native agents are prepared to go forth among the people, and shake and agitate and rouse them from the lethargy and the slumber of ages !”

The missionaries, however, were hampered by their adherence to home requirements in the case of ordination and in several ways the practical carrying out of useful plans was impeded by stiffness of policy and system. In consequence some of Duff's first converts, who could have been useful assistants, went off into the service of other missions.

Duff's large-minded views on comity have been already referred to. He was prepared for a yet larger measure of coöperation and advocated joint missionary activities on the part of the Presbyterian Alliance. Such a plan, carried on side by side with the separate missions of the various Churches, would have been difficult of operation, but it might have given the Alliance an enduring life or made it the central agency in the movement towards Christian union.

Of Duff's character, what has been said has been sufficiently illustrative—of his modesty and humility, of his energy and power, of his devotion and singleness of mind, of his solidity of conviction and breadth of influence, of his large comprehension and yet sagacious directness of accomplishment. To our American mind, he dreamed a little too tropically, was a little too serious and universal. We play more with our grave purposes. But he liked no such play. It was “playing with missions” of which he accused the Church and he dealt with life and his work with a solemn conviction of which it were well for us to

have more, if we would complete the great task at which he wrought with such prodigious power and marked success. "It was the special glory of Alexander Duff," said Bishop Cotton, when the great missionary was leaving India for the last time, "that arriving here in the midst of a great intellectual movement of a completely atheistical character, he at once resolved to make that character Christian. When the new generation of Bengalees and too many, alas, of their European friends and teachers were talking of Christianity as an obsolete superstition, soon to be burned up in the pyre on which the creeds of the Brahman, the Buddhist and the Mohammedan were already perishing, Alexander Duff suddenly burst upon the scene with his unhesitating faith, his indomitable energy, his varied erudition, and his never-failing stream of fervid eloquence, to teach them that the Gospel was not dead or sleeping, not the ally of ignorance and error, not ashamed or unable to vindicate its claims to universal reverence ; but that then, as always, the Gospel of Christ was marching forward in the van of civilization, and that the Church of Christ was still 'the light of the world.' The effect of his fearless stand against the arrogance of infidelity has lasted to this day ; and whether the number he has baptized is small or great (some there are among them whom we all know and honour) it is quite certain that the work which he did in India can never be undone, unless we, whom he leaves behind, are faithless to his example." Where are the men of gravity and courage who will do this work in our day ? Viceroys come and go in India and new Pharaohs arise at home, but the need, which Alex-

ander Duff so nobly met, abides, the need of men who can drive the religious sanctions under the institution of government, who can keep the truth of God alive amid the truths which men discover about God's world and who can penetrate the intellectual and social life of their day with the purifying and saving power of the Gospel.

LECTURE IV

**GEORGE BOWEN, THE CHRISTIAN
MYSTIC AND THE ASCETIC IDEAL**

LECTURE IV

GEORGE BOWEN, THE CHRISTIAN MYSTIC AND THE ASCETIC IDEAL

IF any Christian of modern times is worthy to be set with Raymond Lull, it is George Bowen. There have been missionaries who accomplished more than either of these, but there have been few who combined in the same unique way the spirit of absolute self-sacrifice, extraordinary intellectual abilities, unresting energy and a love for the personal Christ as passionate as Peter's and as steadfast as John's, whose faith moreover stood unshaken against discouragement, and rested with confidence upon the certainty of things not seen. It is less of an injustice to these great men that they should be unknown to our generation than it is of loss to us that we should miss the courage and spiritual incentive to be found in their lives; lives, in Bowen's case at least and we may believe in Lull's also, as closely resembling the earthly life of Christ as any lives that men saw in their times, as any lives that we see in our time. The centuries remove Lull from our personal acquaintance but some are living who knew and loved Bowen, and his influence is still so clear and characteristic that many of us who never knew him have yet felt him and in the truest sense touched his soul.

“George Bowen was born in Middlebury, Vermont, April 30, 1816. His father in after years was a wholesale

merchant, an importer of dry-goods in New York. The father, much attached to his family, was of a literary turn of mind and collected an excellent library, by which his children greatly benefited as they grew up; but his desire for George was that he should acquire a good business training, succeed him in business and become a successful merchant.

“At the age of twelve, George was withdrawn from school and taken into his father’s counting-house. After that, he never attended either school or college. He read with avidity the books in his father’s library. At fourteen, he took lessons on the piano, and when about sixteen years of age a great passion for music took possession of him, and for a dozen years he cared for nothing more than Italian operatic music. During this period, his evenings were spent in the acquisition of French, Italian and Spanish, in which languages he became quite proficient.

“About the age of eighteen, he became very much dissatisfied with the career which his father had appointed for him, chafing under his repugnance to a commercial life and lamenting the meagre educational advantages which he had enjoyed. In October, 1854, he received his father’s grudgingly and ungraciously accorded permission to retire from the mercantile life which he had followed for nearly eight years, resolving to devote himself *con amore* to literary pursuits.

“It was a short time before abandoning forever a mercantile career that he became a sceptic, or, as he prefers to call himself, a disbeliever. An enthusiastic admirer of Gibbon, charmed with the dignity and suggestiveness of the great historian’s style, he easily persuaded himself that Christianity was destitute of all well-founded claims to be regarded as a divine system. In the course of a year or

two, he read the works of Volney, Voltaire, Shelley, Hume, Bayle, and others of kindred minds, the result being that he settled down into a calm and confident acceptance of an especially cold type of Deism."¹

We have in Bowen's reminiscences an account from his own hand of these sceptical days. "There was a young man," he writes, "very fond of reading, who at the age of seventeen was led to doubt the truth of Christianity by that chapter of Gibbon in which he attempts to account for the spread of the Christian religion in the world. He was acquainted with several modern languages, and read in these the principal works in which Christianity is assailed,—Volney, Voltaire, Diderot, and a number of others. He soon persuaded himself that Christianity was not a revelation from God, that there was no revelation, that there might be a God and probably was, but there was no life to come, and there could not be a more futile employment than prayer. His mind was made up on the subject, remained absolutely unshaken and unwavering in unbelief for eleven years. He occupied himself with literature all these years, and naturally read a great deal that tallied with his views; whatever did not, made no impression upon him, and he only wondered how people could be so simple as to believe things so preposterous and baseless. With a single exception, no one ever addressed him on the subject of personal religion, it being thought by those who knew him that the fixity of his views was such as to make the task hopeless. To a friend who once addressed him on the subject of re-

¹ "In Memoriam, George Bowen," pp. 11-14.

ligion, he replied by a letter the character of which may be gathered from the quotation which he placed at the head of it, 'Thinkst thou that because thou art virtuous, there shall be no more cakes and ale? Ay, by St. Anthony and ginger shall be hot in the mouth too.' At a later period, Strauss came in his way, and what surprised him was that the German should take such prodigious pains to disprove that, the falsity of which lay, as it seemed to him, on the very surface."¹

"In the summer of 1836, George, accompanied by the other members of his father's family, went to Europe, the period immediately preceding having been marked by sundry manifestations of his liberty-loving and enthusiastic spirit. . . . Several years of wide-spread travel in Europe, including a year or so of fast life in Paris, followed. His journal contains extensive notices of the places of interest visited. They are most attractive reading, fascinating at times in their exquisite depictive power. Freely interlarded we find reflections and philosophizings of a most audacious, irreverent and oftentimes blasphemous character. Here is a characteristic extract from his diary of April 16, 1837: 'Saw the sea at Terracina and ruminated on the beach, cigar in mouth, over the vicissitudes of human events and the nothingness and nonsense of my own existence. It is a great boon that God should have taken into His head to put this spirit, soul, essence of mine into a human body and make a creature of me,' and more in the same strain. A deep, overflowing pessimistic current flows through all his writings of those unhappy years of alienation from God. 'My destiny'! he says,

¹ "Daily Meditations," Preface, pp. v., vi.

‘inglorious and mean; a bubble that breaks from the flood in the night-time, no sun nor moon to paint it with gay hues.’ About fifty pages of his journals of these years of European travel are covered with notes of books, mostly German and French. . . . He states that he read eighty German volumes in six months of 1838, some of which he translated into blank verse as well as prose.

“In 1839, we find the subject of our narrative in Upper Egypt, greatly delighted with all that he there saw, with eyes ever open for the beautiful, the ancient and the humorous. Later in the same year, he passed over to Palestine, where he spent the months of August and September, and after visiting Turkey, Greece and Italy, we find him once more in Paris at the end of the year. The early part of 1840 welcomes him back to New York, whither his relatives had preceded him. . . .

“In a few months, we find him commencing and abandoning the study of law because of difficulties and disinclination, and finally beginning the composition of a work of fiction, the scene of which was Rome, the epoch the early part of the sixteenth century, and the principal personæ the distinguished artists and literati of that day. After returning to America, he continued to be an omnivorous reader. . . . At this time, he became enamoured of the pantheism of Spinoza and Goethe, chiefly on the ground that it shifted all the corruption of humanity over upon God, naturing and natured. His poetic genius found generous vent at this time also. The effusions, mostly in blank verse and covering a wider range of subjects, reveal a high order of imagination and a deep philosophic insight into the nature of things. In 1842 Mr. Bowen read no fewer than 150 large volumes, on 105 of which he made extensive notes !

"It was in this year also, that Bowen made the acquaintance of a lady who was destined to exercise a greater influence upon him than any other living person. Beautiful in person and endowed with rare charms of mind and manner, his whole life was bound up in her. In his journal for July, 1843, he speaks of four days of incomparable enjoyment spent in her company, '*les plus beaux jours de ma vie,*' and indulges in many daily flights of what he terms '*rhapsody and idolatry.*' In December of the same year, she was smitten with what proved to be a fatal sickness, and he was overwhelmed with grief. . . ."¹

Bowen's fiancée died on the morning of January 26, 1844, forty-four years almost to a day before his own death in Bombay. He wrote in his diary, "There remains nothing now but the constant, perennial, hourly necessity of such preparation as shall ensure the earliest meeting in that exalted sphere to which she has gone." On February 4th, he records that he received her dying gift, a copy of the Bible, "with words of benediction on the clasp and an injunction from her to read it daily and also to attend the house of God." He obeyed this injunction out of simple devotion to her but before long the great transformation came to him. It will be best to let him tell the whole story in his own words :

"After eleven years of profoundest infidelity [he says of himself] he had his attention drawn to the career of the apostles, and to the evidence afforded by the extraordinary labours, sufferings, successes of these twelve men, that Jesus of Nazareth had already risen from the

¹ "In Memoriam, George Bowen," pp. 14-19.

dead and ascended up on high. His attention had, however, been previously drawn to a remarkable fact which seemed to show that the same Jesus Who was crucified many centuries ago had power to accomplish things upon the earth at this day which no mere man could accomplish.

“There was a young lady dying of consumption in a certain city. She was surrounded by all that could make life attractive, and it seemed, especially to the one who was much bound up in her, one of the saddest conceivable things that she should go down to a premature grave. She herself would have gladly lived; there was a hope in life that death could not offer. There was in the same city, a lady in whose school she had been a pupil; this lady incidentally heard that her former pupil was dying and not prepared to die. She went to see her but was not allowed access to the invalid; she would not, however, be denied but persisted and almost forced her way to the sick chamber. The Lord blessed her ministrations, and she was enabled to show the patient her need of the Saviour and to lead her to Christ. Then was all fear of death removed; the desire to live left her; the hopes that seemed to irradiate this life shifted to the life to come, but elevated and enriched a thousandfold; a sweet peace possessed her soul, and she died rejoicing in the assured conviction that she was going to be with Christ. Whatever grace and beauty seemed to belong to her in health were eclipsed by the spiritual grace and loveliness that invested her last hours as with a halo. There was one who would have given all his interest in life to impart the least alleviation to her pain, to have diminished in the least the sting of death; but he was made most painfully conscious that this was utterly beyond his power to accomplish. Now the fact that arrested his attention was that Jesus of Nazareth Who had been so long

disregarded and scorned by him should come to the dying one and give her peace and sweet content and joy in the assurance of a blissful immortality; here was something marvellous and inexplicable. He was bewildered. The effect wrought corresponded only with that which only the sublimest truth, in connection with a present divine power, could accomplish; it was the removal of the sting from death, the bringing of life and immortality to light, the opening of a door into a glorious and holy heaven; and all this heightened by contrast with his own utter impotency and total penury of help. . . .

“ A Bible, bequeathed to him with a dying request that he would read it, he received with thankfulness and proceeded to obey the injunction. He read it and found much to admire in it; valued it for the comfort it had bestowed upon another; but he never for a moment doubted that he was right in his views regarding it, or suspected that it was really a revelation from God. One night, just before retiring—this was in March, 1844—he said aloud in his room, ‘ If there is a God that notices the desire of men, I only wish that He would make known to me His will, and I shall feel it my highest privilege to do it at whatever cost.’ He had been brought to see that there was nothing more desirable than for a man to be conformed to the will of an All-wise Creator, and also to feel that there must be some divine guidance in order that he might know that will. But immediately after that ejaculation, the thought arose, ‘ How foolish to suppose that God will occupy Himself with our desires ! ’ However, the sequel showed that God was pleased to hear that bewildered cry,—that could scarcely be called a prayer. Two or three days afterwards he went to a public library from which he was accustomed to take out books, asked for

a book, received one, put it under his arm and returned home. The distance was about two miles. When nearly home, he looked at the book and found to his surprise that it was Paley's 'Evidences,' a very different book from the one he had asked for. He could not go back to the library that day, and had to keep the book until he had an opportunity of returning it. He would not read it. He knew all about the evidences of Christianity. He had long ago finally settled that question. Before putting it away, however, he glanced at the first sentence and was arrested by it: 'I deem it unnecessary to prove that mankind stood in need of a revelation, because I have met with no serious person who thinks that even under the Christian revelation we have too much light or any degree of assurance that is superfluous. Let it be remembered, too, that the question lies betwixt this religion and none, for if the Christian religion be not credible, no one with whom we have to do will support the pretensions of any other.' He read one page and another and another, was pleased with the style and candor of the writer, and at last sat down and read a good portion of the book. To his surprise he found that he was beginning to take a new view of the evidences, and then shut up the book and put it aside, afraid of being surprised into any change of belief. He went away for a few days into the country, and on his return resolved to read the book carefully and calmly, and see if there was really any reason to believe the Bible to be from God. When about half-way through the book he offered the prayer 'Help Thou mine unbelief.' When he had reached the last sentence, his doubts were all removed; he was perfectly convinced of the truth of the Scriptures. He turned to Gibbon and read again the chapter which had first led him astray and saw its sophistries and the weak-

ness of its arguments most clearly. The Bible was now God's book, but he did not believe that it contained the doctrine that men pretended to find in it; he would read it for himself, and by himself, and see what it really taught. But he had had a great lesson and felt that humility best became him; he would read it in an humble spirit and whatever he found there, he would receive, no matter how repugnant it would be to his own ideas. Day after day, alone in his room, communicating to none the change he had experienced, he read it and by degrees found there the very doctrine that he had so much disliked. He found that he was a sinner, that he needed a Saviour; that a Saviour was offered him. He took this Saviour, yielding himself to His entire direction. He was led on publicly to confess his faith in Christ, and after some years he became a missionary in India."

It was within three or four weeks of his conversion that Bowen fully resolved to be a foreign missionary. He became an attendant at the Mercer Street Presbyterian Church, of which Dr. Skinner was pastor. He was not a man who delayed duty and he went on at once to public baptism on profession of faith, on June 9, 1844. There are great diversities among men in this regard of promptitude of character. Some suppose that there is virtue and especial assurance of divine guidance in delinquency. A man who has been moving upon one course of action preparing for the practice of law or contemplating some Christian service in America, regards his pursuance of this course as supplying so powerful a presumption in its favour that he cannot bring himself to make a change without long delay. In the colleges and seminaries,

one often hears warnings against the dangers of hasty decisions in the matter of missionary purpose ; as a matter of fact the contrary danger is ten times greater. After all, a decision is made in an instant. It may have taken weeks or years to come up to it and the consequences are eternal, but the decision itself was instantaneous. Bowen was no delinquent. When he saw, he did. Will instantly caught up the movements of conscience and moral judgment and solidified them in action. His later life in this regard resembled the beginning. He promptly obeyed every gleam of new and as it seemed to him larger duty.

When he had formed his missionary purpose, Bowen spoke to Dr. Skinner and others about it. He had supposed at first that "there would be nothing to hinder him from going at once just as he was, with his Bible under his arm." He was advised however to go to the Union Theological Seminary and accepted the advice. He had had no college course, so he studied Greek at once alone, and began without delay active Christian work. He led a Sunday-school class, and "worked a district as tract distributor and earnestly sought to equip himself for his life-work under the guidance of the Holy Spirit and judiciously advised by kind friends." It is worth while to observe George Bowen's readiness to receive help from others. All his life, he was a man of positive opinions, who saw his duty for himself and did it, but he was a man amenable to reason, who checked his own disposition by revelation of duty through others.

About the time of his entering the seminary, he

broke off the habit of smoking, without solicitation or suggestion from any one. The habit had become very firm and enthralling and he simply resolved to throw it off. He succeeded in delivering himself by using for a time, by set purpose, the cheapest and worst tobacco. With tobacco Bowen stopped profanity and cut off absolutely all use of intoxicants, "believing that the spirit of the New Testament favoured total abstinence."

He was constantly drawing his life up to the highest. That purpose will settle a hundred little questions of habit and practical living for men. Men who are of mediocre spiritual ambition can find adequate reasons for petty squalor of personal habit and can live with their moral self-approval on a plane that would be impossible to them if they asked not, "Must I give this up?" but "May I not free myself from this also and enter into a larger liberty?"

During his seminary course, Bowen was constantly at work. He did not postpone missionary service because his present sphere was not as large as the sphere he contemplated. He realized that the only possible preparation for many kinds of work is to do them, and to be a winner of souls in India ten years in the future he knew that he must be a winner of souls where he was. There is no spiritual alchemy in a sea voyage that will make a missionary out of a man who is not already one before he goes. During his summers, Bowen worked with his friend and fellow student Mr. Ford, afterwards a missionary in Syria, in Pike County, Pennsylvania, visiting the farmers, offering books in behalf of the Bible and

Tract Societies, talking about Christ and praying from house to house as there was opportunity. In the seminary also, he was a Christian of the sincerest type. He did not coquet with the world. He lived his religion. Young men in theological seminaries speak of the difficulty of maintaining warm spiritual life in the atmosphere of the seminary, and there have been students in such seminaries who seemed to regard the deeper religious duty as irksome and who evidently were postponing for a time the full practice of the Christian life. Men and institutions vary and one generation is worse or better than its predecessor, but there is room enough still in all our seminaries for men who will live as Bowen lived. He was not the type of theological student that he was through the calculation that he must be that type if he would later exert the largest positive spiritual influence, but later he was the profound spiritual power that he was in India and throughout the world because of that character in him which had expressed itself in sincere and earnest Christian living and working in the seminary. Men do what they do because they are what they are. Absolute freedom of the will is an untrue doctrine. We see around us every day its refutation in the determinism of character which we find whenever we will look in ourselves and in all men. And there is no greater folly than to suppose that men can prepare themselves flippantly for life and not enter life in consequence with flippant characters, destitute of the power of lofty sacrifice or spiritual sensibilities like George Bowen's. "While at the theological seminary," says the Rev. J. E. Robinson, "he was in

the truest sense a missionary, ever seeking the conversion of souls in the outside world, as opportunity served, and also helping many a fellow student into the full enjoyment of the gospel salvation. He was the leading spirit in the prayer and experience meetings among the students, in all things and at all times seeking first the kingdom of God, while at the same time a diligent and conscientious student."

In the seminary those deeper experiences of the Christian life began with Bowen which were to issue in the singularly powerful spiritual character of the future. The 4th of December, 1845, is noted in his journal as the beginning of a new era in his life and spiritual experience. Of this he writes in the third person.

"Nothing in heaven and earth astonished him more than the discovery made on that day that Jesus was his sanctification, and that all he had to do was to abide in Him as the branch in the vine, and the goodness of Christ would sway him moment by moment, and it would always be Christ's goodness and not his own, for there is none good save one, that is God. . . . When the discovery was made, he was filled with wonder, love and praise, but also with a sense of the need of perpetual vigilance, lest at any time he should forget his absolute dependence on Christ. He felt that he must watch against everything which could in any way weaken his sense of dependence. He felt that he was under law to Christ in eating, drinking, sleeping, study and conversation ; that he must habitually stand ready to cut off a right hand, to secure the continued realization of Christ's love. All self-denial now became easy ; the sense of God's love filled him with joy unspeak-

able, and he valued nothing more than the opportunity of expressing his own love in return."

On April 19, 1847, he preached for the first time in the pulpit and he preached three times. He "was sustained," he writes, "but was disappointed in the results . . . yet favoured in one respect—freedom from reflex acts." Throughout there was in him a rich combination of deep introspection and of calm faith in the objective facts of salvation in Christ.

Having been duly accepted and appointed by the American Board of Commissioners for Foreign Missions, Mr. Bowen left New York for Boston, July 27, 1847, and embarked from the latter port on an ice-ship four days later, Mr. and Mrs. Wood his only fellow passengers, being, like himself, bound for Bombay. At once he began to work among the crew, earnestly seeking to lead them individually to Christ. He also began the study of Marathi. These days on shipboard were days of prayer and heart-searching and growth in grace. God was girding him for his forty years' service in Bombay. His faith was growing exceedingly as he learned Christ. He read many works of religious biography and history during the voyage. In his journal for December is a remark which may be regarded as the key to his whole life. "It appears to me now," he writes, "that the highest style of Christian in God's sight is one who lives in the wise exercise of all his powers, sparing himself not at all, doing all to produce great and immediate results, yet esteeming that in God's favour is his life repining not, when there is no appearance of

fruit, and willing to be thought unprofitable by the Church."

Bombay was reached January 19, 1848, after a voyage of 172 days.

Bowen at once took up the language, employing two pundits, each of whom gave him an hour and a half daily. The spirit of the man is shown in a sentence or two from his letter of March 31, 1848, to his friend, the Rev. William Aikman.

"I was thinking this morning that here thirty-two years of my life had rolled away, and I had not yet begun to live. That is, to work—for to work is to live. All my past life has been a long and strangely circuitous avenue to my present position, a wandering maze whose issue God alone discerned. Only to think of it, thirty-two solid years cast away, and who knows whether my allotted time is not comprehended in them. Surely if any individual should resolve to do with might what his hand findeth to do, that purpose should be mine. But after all it is not time that we want so much. If the choice were now offered me to live twenty-five years with my present measure of grace, or to live six months with that measure of the Spirit's influence which I sometimes crave from God, I would certainly choose the last. Yes, I believe that three days with the baptism of the Holy Ghost will be of more value to this unhappy world than the longest life of mediocre piety."

From the beginning, Bowen's remarkable journals are full of reflections on all questions of missionary policy, especially one of great interest in which Bowen

was the forerunner of many later earnest missionaries who felt the constraint of the same spirit. "From the very first," he writes of himself, "the idea of a very simple style of living, approximating that of the natives, was before his mind, and he freely expressed his conviction that one way in which the gulf between the natives and Christian missionaries might be bridged was by the latter ceasing to occupy in worldly respects a superior position to the former. His conviction was deepened by the perusal of Edward Irving's famous missionary sermon, preached before the London Missionary Society some time previously."

At the outset of his missionary life, however, two temptations came to him to leave the work, before he had opportunity to develop his theory about the manner of a missionary's life. One temptation sprang from the sense of duty to his mother and sisters at home left unprovided for by the death of his father, who with his two sisters had become believers the same year with himself. Many men in Bowen's position would have seen in this providence a warrant for return to America, and some doubtless justly; but there are many to whom such temptations come merely as trials of faith and new discipline into robustness of character. Bowen felt this news to be just such a temptation to him and trusting God to solve the problem of the family's support, he remained in India, saving, however, about twenty dollars per month out of his salary to aid his mother and sisters. I believe myself, not that too much is made of family ties,—that would be impossible—but that they are allowed too much to hamper Christian

work and that many men and women plead as an evidence of exemption from missionary work claims that in God's sight and the light of such high moral principle as ruled Bowen's life are not valid claims at all.

The other temptation sprang from the condition of his health. In August, 1848, he "was prostrated by an affection of the liver and of the windpipe. He declined very rapidly, insomuch that he was given over to die by his physicians and all who saw him. He himself even wrote home announcing his approaching death. A few days after doing so, he began to mend, and his physicians urgently advised that he leave the country immediately. This, however, he refused to do, hoping that the Lord would eventually fully and permanently restore him. The Lord saw fit to order it so, and the one who was declared by able physicians, under their hands and seals, to be absolutely unequal to further residence and labour in India, lived and laboured with indefatigable energy for forty years, without lengthy sea voyage, furlough, residence in the hills, change of climate, or other means generally considered indispensable to prolonged stay in the tropics." The various means employed for the maintenance of physical health and spiritual tone in mission fields such as those just mentioned are wise and necessary, but they can be both under and overused, and it is a good thing often to turn back to the lives of men like Bowen and Judson and see how vigorously independent these men were of them, and how with them the work was supremely first, and puny questions of a few months' extra furlough, or this or that other small

comfort, beneath their world. Bowen was right and wise too, in refusing to leave lightly the work for which he had been sent out, even with a physician's certificate advising his return. When missionaries once reach their fields, at great expense to the home Church and presumably under the guidance of God, no light reason should bring them home. Often there must be physical readjustment, but as an old missionary lady in China once said to me, "Let the new missionaries go slowly. They may not be as well here as at home, but if they can live here at all, let them stay. They will get broken in if they have patience and courage." Bowen refused to leave the field and he lived and worked in India for forty years.

Remaining in India, he took up the two questions of the mode of life of the missionary and his spiritual example and influence. He wrote in his journal, "I want to have Christ walking about the streets of Bombay as He did about those of Jerusalem and living among this people as He did among the Jews. He was emphatically the friend of the people. They were His family, His home. . . . I want to have Jesus the Missionary in my mind's eye continually. It will be a blessed day when I feel at home in these streets and can linger in them without any desire save to continue preaching the Word. . . ."

"It was strongly borne in upon his soul," says Bishop Robinson, "that it was his duty and privilege to authenticate his divine commission to the ignorant people among whom he toiled with so little success by 'signs following.' The references in his journal are scanty and somewhat vague, but it seems that after

days and nights of prayer and study of the Word, he on one occasion essayed the healing of a sick or disabled person by a command of faith and was signally unsuccessful. He was greatly humbled and confounded, but God held him in the hollow of His hand, and he suffered no eclipse of faith. He never, however, abandoned the conviction that the miracle-working power was recoverable by the Church and ought to be an adjunct for missionary labours among idolatrous peoples; but we do not find any further attempts on his part to manifest or exert this power, though he appears to have sought it with prayer and fasting and many tears."

The practical measure which he soon came to believe it his duty to adopt was the surrender of his salary and the attempt to live among the natives in a style of simplicity and renunciation of earthly comforts to indicate the utter unworldliness of the motives of the missionary and the disinterestedness of his aims. In January, 1849, after having been in the country one year, he wrote a letter to the missionaries throughout India, urging his views. It would doubtless be regarded as more presumptuous now in a missionary yearling to do this than it was then. The body of missionary practice and precedent has grown and solidified greatly in these years, but Bowen's course was rash enough. His own actions, however, were not to be determined by what others did. As he writes: "By the grace of God! I will put myself in a position where all men shall see that I am the disinterested servant of Christ. By the help of God, I will honour the Gospel and conform myself to it with all strictness."

In accordance with this purpose, Mr. Bowen, on February 13, 1849, resigned his missionary's salary, amounting then to ninety rupees per month, left the mission house, and took up his abode in a little room of an old pensioner's mud-walled house near Waree Bunder, under Nowrojee Hill, in the midst of a community composed entirely of Portuguese and natives. The house has long since been swept away and the whole neighbourhood altered. His journal of this date has the following: "At length, thanks be to God, I am in that situation which I have so long desired to be in. The Lord did not more truly guide me to India than He has guided me to this humble spot. Were the Apostle Paul in Bombay, I should be far more content in receiving him where I now am than where I have hitherto been. . . . On opening my Bible the first text that met my eye was, 'Now there was found in it a poor wise man, and he by his wisdom delivered the city.'"

In his later reminiscences, he refers quietly to this self-denial, again speaking in the third person: "After spending about a year in India, he was led to believe that his influence would be greater if he were not in the receipt of a salary from a missionary society, and since January, 1849, he has received no salary from any quarter. For some years he earned his livelihood by giving an hour daily to private tuition; for a still longer period, he has trusted to the Lord to supply his need, without such occupation. It is unnecessary to say that he has enough and to spare." This work of private tuition lasted for twelve years. Thereafter, he depended as he says upon the Lord, the earnings of his editorship of

the *Bombay Guardian* probably not relieving the Lord greatly !

But what a curiously un-Christian conception this is ! It was the Lord supporting Bowen through the American Board as truly as through small charities in Bombay. The fact that the sparrow goes out and gathers his food does not in the least alter the fact that it is the Lord Who feeds the sparrows. What Paul earned from the sale of his tents, it was the Lord Who gave him. The use of means and effort on the part of believers does not diminish at all the reality or the immediacy of the Lord's influence and active present care. George Bowen depended no more on the Lord than Bishop Thoburn, or Dr. D. L. Anderson of Soochow depended on Him. Bowen may have *felt* that he was more directly dependent upon God, but many other men may have as great a feeling of dependence who yet see the Lord's hand giving them what comes through the missionary agency with which they are connected. Doubtless, many do not depend upon the Lord who use means and organization ; but the use of means and organization is not responsible for their want of dependence. That is their inner spiritual deficiency. Dependence upon the Lord makes some means unjustifiable, but not the use of means. The missionary organization which most emphasizes the thought of direct dependence upon God and which shows forth the beauty and sufficiency of such dependence is probably the most diligent society in the world in making known its work, publishing books about its history, and setting forth the vast needs of the field which it is endeavouring to reach. Instead of doing wrong in

this, it is pursuing the most Christian course possible. It does not believe that depending upon God requires cessation of effort or disuse of means.

In Bowen's case, doubtless the surrender of all regular support did help to strengthen the sense of immediate dependence upon God, although it did not increase at all his real dependence. He hoped also that it would greatly increase his missionary influence. In later years, he often confessed that he was greatly disappointed as to the effect which he had expected would be produced upon the natives by his course. And as a general rule of missionary conduct, Bowen's plan is not practicable. "Living as the natives" is not a clear proposal. Which of the natives? In mission fields in Asia and South America, there are all sorts and grades of natives. As to naked Africans, the rule is obviously impossible. But as to India, it is equally so, if by native is meant the poorest class. The physical constitution of the Western man cannot live on that level. Centuries of heredity lie back of the Indian villager who lives with his family on two or three dollars a month, and whose household furniture and wearing apparel could be purchased for five dollars. "Living on the level of the native is not a matter of consecration," as one missionary put it; "it is a matter of stomach," and it simply cannot be done. At the other extreme, of course, there are Hindus and Mohammedans in India who live as princes. It is true that the missionary lives far above the level of the class of natives with which he associates, but that is a simple physical necessity.

Furthermore, as a rule, the effect of trying to live on the level of the poorest of the Indian fakirs or

holy men is not what the theory assumes. Bowen, who never got down to that level, admitted this. It is not manner of living that wins or repels in Bombay or in Nashville. It is the spirit of heart and life represented in the manner of living, and Hunter Corbett, living simply but as the requirements of health and efficiency necessitate, draws nearer to his Chinese and has won more of them to Christ than Bowen did in Bombay among his Hindus and Parsees.

"Living like the natives" is a much used phrase among missionary critics and independent missionaries. There is a good deal of unreal fetichism about it. The germ of truth which it contains is the truth at the bottom of the whole missionary enterprise, the truth of the Incarnation itself. To reach people, we must go to them, love them, win their love, draw as close to them as we can. To do this, simplicity, frugality and perfect sincerity of life will be necessary. That is all. And this is a lesson for us as well as for the missionary. For the question of moral and economic principle involved is the same here as there. In America as truly as in India the doubt must often come to men as to whether they should not literally forsake all and follow Christ, give away their property, lay up no money, adopt the practice of St. Francis of Assisi and go out absolutely free, without a burden or a tie, save the love of Christ. The inequalities in the distribution of wealth are so glaring and so terribly unfair. The economic system is so obviously unsatisfactory. The multitudes of the needy are separated so widely from the affluent and the luxurious. Even among the merely well-to-do

the scale of living is so overwrought and the complexity of life so confusing. Why not cut loose from it all by the one decisive surrender of asceticism? "Let us do it," said Tolstoy. Well, we cannot, for a score of reasons. We have our children. We have no right to exact of them the toll for our spiritual and economic vagrancy. We are in an order and we must redeem the order and not run away from it. We have a whole world of nature and of men to save and we may not go off alone to save our own souls. The solution of asceticism is too selfish, too irresponsible. But if we reject this ideal, we can do so only by the more earnestly accepting the law of service which sees in all that we have, not a personal possession, but a means of human ministry and a trust to be administered for Christ and for men.

Bowen's renewed consecration of his life and his effort to lay himself completely upon the divine care was followed by days of anxious inquiry and earnest desire for unequivocal manifestations of the power of God to accompany his preaching of the Gospel to the nations. "He spent hours of the nights in prayer, fasted for long periods—in one instance for a fortnight—and in response to a profound impression, made upon his mind in meditation on the character of Christ, he gave away every penny he had in the world. The 20th of March of this year, 1849, proved another important era in his spiritual life. He writes of it as the greatest day in his whole life. 'I entered,' he says, 'upon a religious experience far higher than any before attained to. Its characteristic is self-annihilation and a wonderful revelation of God in the place of myself.' The immanence of

God in his natural creation, the absolute dependence of the creature upon God, the power, wisdom and goodness of God as exhibited in the works of His hands, were unfolded to his mind in a manner that filled him with unutterable joy, peace and love."

In the spirit of this new experience, deepening year by year, Bowen carried on his work in Bombay. In 1851, the *Bombay Guardian* was established with Bowen as an associate editor. After three years, he undertook whole charge. The paper was discontinued for a time, but later was revived and his singular abilities made it a paper of great power, his connection with it continuing through the rest of his life.

His literary work included much more than the *Guardian* though that was enough. My friend, Mr. Henry W. Rankin, in sending me a valuable set of the bound volumes of the *Guardian* for the last ten years of Bowen's life, wrote of them, "They not only contain the reminiscences (of Bowen's early life which he wrote under the pseudonym of Homunculus in the third person) but his invaluable editorials on an immense range of subjects, political, philosophical, ecclesiastical, discovery, the ethnic religions, the Brahmo Samaj, and all other experiments of eclectic religion in India. The papers contain," added Mr. Rankin, "a consecutive commentary on all of John's Gospel and all of Revelation. They are crowded with the richest ore of gold and seamed with beds of diamonds."

How rich Bowen's comments on Scripture were, all know who have read his three best known books, "Love Revealed," "The Amens of Christ," and "Daily Meditations." Many books of devotion have

blessed the Church, but few have blessed more hearts or helped them more deeply than these sincere, noble-minded outpourings of Bowen's experience of the love and life of the loving and living Christ.

Beside his literary work and doubtless transcending it in importance, in Bowen's view, he was constantly preaching. In 1854, he wrote to Dr. Anderson of the American Board, "I continue to preach in the streets and wherever the people so congregate that I can quietly talk to them. Occasionally, I am maltreated or am mobbed. But I do not suffer my mind to dwell on those occasional unpleasantnesses."

In 1871, William Taylor, known all over the world as Bishop Taylor, began his mission in India and Bowen at once gave him his hearty support, becoming one of the leaders of the Methodist Episcopal Church and devoting to it his great talents thenceforth until his death in 1888. Bishop Taylor regarded Bowen with deep reverence, saying once regarding him to Dr. Aikman, "George Bowen was the Lamb of India." And whether or not the people for whom he lived and died, always with the spirit of the Lamb of God, responded to his message, they did respond to his love. The editorial which appeared in *The Times of India*, on February 11th of the year he died, 1888, though coming from Englishmen and appearing in the leading secular English paper of Bombay, yet expressed the general feeling of the entire community. The editorial is too long to allow the quotation of more than a few sentences.

"The death of the Rev. George Bowen, the tidings of which passed rapidly through our city on the 5th inst., has

deprived this community of one of its oldest and most widely honoured members. The sorrow awakened by his unexpected removal is not confined to any one section of the Christian Church, or to any one class of the community. One who has for forty years occupied a unique place as a missionary among us has passed away, and the sense of loss is intensified by the feeling, present doubtless to the minds of all who knew him, that the place of George Bowen will always remain empty. His was a work and a personality *sui generis*, and in the ordinary acceptance of the word, he can have no successor. The removal of George Bowen marks the close of an epoch in the history of our community. . . .

“George Bowen was a man of rare individuality. In any community this individuality would have asserted itself, but in a community like ours, in which the conditions of society so manifestly tend to the levelling down of all men to the same tone of thinking and action, a man who could stand alone, who could mould his life according to his own high convictions of responsibility, and who felt bound by no artificial standards, could not but stand forth as a conspicuous personality. Hence it was that many a visitor passing through our city, intent upon noting not merely the outward features of our life in Western India, but also the moral forces which are at work among us, sought out before all things the humble dwelling of this saintly man, that they might be brought in contact with something of the inward movements that are silently moulding the life of the community. . . .

“He retained throughout his new life all his breadth of culture, and no circumstance or surroundings, however humble, could dwarf the moral and spiritual dignity of the man; on the contrary, they only served to render it more

conspicuous. In his most humble dwelling, he could entertain the humblest and make him feel welcome ; but in the same dwelling the highest had no consciousness of the exceptional surroundings and no feeling of condescension in the presence of one who received them with true gentlemanly courtesy and dignity. The same breadth of nature was conspicuous in his relations with men and with churches. . . .

“Mr. Bowen’s whole life was a testimony to the disinterestedness of his aims ; but special instances of it were of frequent occurrence. . . .

“The life of such a man could not fail to make a deep impression on all earnest minds in this community. We have no doubt that many of our native fellow citizens have felt its influence, and some of them have not been slow to acknowledge it. We know of many amongst our own fellow men who owe all that is best in their lives to their contact with him and of others who were made better through their reverence for his character. . . .

“Reality and self-forgetting sympathy were the most marked features of his character, and these are the qualities which most inspire confidence and affection. His was a nature incapable of affectation and free of all self-consciousness. . . .

“Through forty years, that life has been amongst us, from its very character mingling little with the busy currents of public movement that have been flowing onwards, guided by other aims and other plans ; and yet we cannot but feel poorer that a life so rich in noble purpose and lofty aim has passed away from among us. Gladly and ungrudgingly, therefore, do we offer this tribute of honour to the memory of one who neither loved nor sought it while he lived.”

George Bowen's method of life and work was not an absolute method. There is nothing in the Scriptures which makes it prescriptive and while the spirit of his life is the right spirit for all workers for Christ and for men, experience did not demonstrate that his methods were the only methods or the most effective methods. They were probably much more effective than Bowen himself believed. He referred with some despondency at times to the apparent fruitlessness of his work, but at his funeral, Mr. Hume, speaking of the great indirect influence he wielded over the natives of Bombay, mentioned "cases which had come under his own observation of heathen who had been brought to Christ through the holy life of him who for forty years had been before the people as a living example of the saving, keeping, sanctifying power of Christ as no other man had been."

Those who deny the absoluteness of Bowen's method are in a position of real peril, however. We may easily turn back from such self-sacrifice into a spiritual easiness and self-indulgence which are fatal to the highest power. It may be feared sometimes that over-reaction from the ascetic ideals of earlier days will carry us too far. Those who say, "We will not fast with the outward fast," easily forget that fast of the heart which is the gate of God. Those who would "use this would without abusing it" find that road, though the right road, very slippery. After all, it is better to err on the side of robust sacrifice, of completeness of self-denial, and to give up all literally, rather than under the plea of moderation to cover over a love of the world, or of pleasure, or of ease which is the death of holiness and of the might of God in a man.

Bowen was no narrow-minded ascetic recluse. "It is too common in these days," says Dr. Mackichan of the United Free Church of Scotland, in his preface to a little sketch of Mr. Bowen, "to look upon every form of high devotedness as the offspring of a certain one-sidedness, verging on fanaticism, the result of excess or defect in some emotion or faculty in minds otherwise rational and well furnished. We have little doubt that the popular conception of George Bowen's life amongst those who had but slight contact with it was not very different from this. The study of this sketch of the life which it unfolds will show how far such conceptions fall short of the realities of the Christian life. It exhibits the development of a mind singularly free from the enthusiasm of mere emotion, broad enough to be able to assimilate the best elements of the culture of other times and other lands, and strong enough to retain its own originality in the midst of all the influences which crowded in upon it." Bowen was a man of rarest intellectual and moral strength of character, large natured, easy, conscious of balance and poise, yet so humble and modest that these qualities were continually hidden so far as their possessor could hide them from conspicuous gaze. "We had Bowen dining with us last night," says a Bombay English civilian,¹ "and I only wish some reporter had been behind the scene to take a note of the 'droppings.' . . . Oh, I wish you had been with us. You would have been elevated when listening to Bowen discoursing on these wondrous themes. A meek, lowly, despised man, but oh, how happy ! living in that miserable hut in the bazaar, holding

¹ "Memorials of Robert Brown, Esq.," p. 268 f.

converse with his God. Hunter is greatly enamoured of him, the more so because he is very musical. Last night, before going away, he played an accompaniment on the piano to Hunter's violincello—"Weep not for sorrow." You need not be surprised if you hear of both of us taking up our quarters with Bowen in the bazaar at ten rupees a month."¹ And one who knew Bowen long and intimately in Bombay is quoted by Dr. Hanna in his biographical preface to the Scotch edition of "Daily Meditations" as writing :

"If expressions of the deepest reverence, admiration and affection were all that is required, I should not be found wanting ; for, taking him all in all, I have always thought him the most delightful and remarkable Christian man I ever met. He was at one time an infidel. Afterwards he gave up friends, country, fortune (his father was a rich man), and consecrated himself and his whole life to the service of Christ among the heathen. You know how he has laboured for so many years, night and day, in Bombay ; how he preaches every day to the native population ; and you also can tell how great has been his influence for good on the Europeans there. For many years, he actually lived in the native bazaar, and among that sadly degraded population, until asked to become secretary to the Religious Tract Society, at whose depot he now resides, managing the affairs without fee or reward, in addition to his other labours. Probably it has added to his weight in the consideration of the English section of the community, that he is a most accomplished and highly intellectual man, having travelled much in Europe at one time ; knowing

¹ Quoted in Dr. Hanna's preface to "Daily Meditations," Edinburgh, 1891.

French, German, Spanish, Italian and I don't know how many other European languages, in addition to Hindustani and Marathi. Many years ago he used to try and enlighten my dear brother in the mysteries of astronomy; and his musical powers are quite remarkable. It is seldom any one has an opportunity of testing them; but on meeting him one evening quietly, after hearing him play a long and difficult piece of music, I asked him for a repetition of part, when I was surprised to find that the whole had been impromptu improvised as he went along. Perhaps one should add that in spite of Mr. Bowen's abundant labours, little visible fruit has been the result. His standard is scrupulously high and rigid. Other missionaries have frequently baptized natives instructed and impressed by his teaching. I asked him once if he did not feel discouraged. 'Thank God,' he said, 'I can truly say I have never experienced such a feeling. This thought, "In Thy favour is life," swallows up all others. It is enough for me.' I believe eternity alone will reveal the amount of his unconscious influence and reveal the bearing his noble self-sacrificing life has had on the hearts of others."

A good deal of this wider range of life in his missionary days was doubtless due to the manner of his life in his youth. God would surely prefer to get His men unmarred, but if they come marred, He takes all that was innocent in their past and turns it to power. It was so with Raymond Lull and it was so with George Bowen. He kept much from those early days and he let much go. Mr. Rankin sent him a copy of a romance which Bowen had written as a young man, entitled, "The Pupil of Raphael" and which he had published through Putnam. "I am reading it," Bowen wrote,

"but have no desire that anybody else should read it. Not a single incident or a single character remained in memory. There are portions of it that I regret exceedingly, showing the effects of Balzac's writings. I am glad that the Lord so completely snuffed the book out. Above all, I am grateful that He has saved me from myself."

An outstanding characteristic of Bowen was his reality. The *Times* editorial emphasized this. All who knew Bowen felt it. Dr. Mackichan refers to it: "George Bowen's conversion from unbelief to faith was a spiritual movement to which every part of his nature gave consent, and the life which followed was the harmonious expression of his whole being thus raised to a higher plane by the revelation of God in Christ. That reality which is referred to in this sketch as the leading characteristic of all his religious life, was the result of this transformation. All he did in the service of the Saviour Who had revealed Himself to him was done with the calmness, the resolution, the rationalness of one who found in the atmosphere of a consecrated Christian life his soul's true element. . . . And this reality was the secret of the joy and beauty of his self-sacrifice. There is a kind of self-denial which is ever conscious of itself. But his was true and beautiful in proportion as it was free from this selfish taint."

No faintest shadow of uncandour, of hypocrisy, of professionalism, darkened George Bowen's life. He was what he appeared. He appeared what he was. And he tried to be and to appear what he ought. A bad man may claim to possess the virtue of reality because he is really bad. But Bowen be-

lieved that the only reality of life is the right adjustment of itself to God and goodness and he strove thereto. And men were influenced by him through his reality. The missionary finds sincere men among Mohammedans, Hindus and Buddhists, not men who are living up to all the light they have, but men who honestly believe what they profess and in human measure live by it. The same thing in the missionary will not convince them that he is right and themselves wrong. His type of reality must be larger and fuller. He must be sincere and honest and true but the truth which he represents must be the complete truth, the divine element, and his reality must mean the adjustment and coördination of his life to that.

Bowen's spiritual fervour and devotion did not blind the accuracy of his intellectual judgments. There is a pious goodness, which desiring to speak evil of no man, is derelict in its testimony to the truth and defective in its defense of righteousness. Bowen was the soul of charity but he was the servant of the truth and he did not sacrifice truth to amiability. "I am convinced," he wrote to Mr. Rankin, "that Chunder Sen was more intent on his own glory, throughout, than on that of Christ. He honoured the Christ of his own conception, the Christ that was plastic in his hands, to be moulded as the Hindu national pride demanded. There was no unconditional surrender to Christ at any time. The Christ that he favoured was one that would give greatness to Chunder Sen." This was Bowen's spirit in the study of comparative religion. He was not deceived. He saw the truth clearly, unobscured by the immoral tolerance of a false liberalism, and the truth he saw

he spoke. Because he was good, he was not "glib," to use Vivakanda's adjective in expressing his judgment of the American people. All religious expressions were not the same to Bowen. Some of them rested as he had told his pundit at the beginning on a foundation of untruth. There are false religious elements as there are true and they are not to be mixed indiscriminately.

As with all great religious leaders, so with George Bowen, his doctrine grew out of his experience. I have spoken of this in Lull. It was equally noticeable in Bowen. "You will have seen," he writes to Mr. Rankin, "that I wrote something about the Trinity. The Bible does not undertake to explain it to us. What it most positively teaches us is the Trinity of God, and what is said about the manifestation of God in Christ is never treated as though it conflicted with that in any way. We get at the right conception of these things not so much by intellectual effort, as experimentally. As we grow up into Christ, we apprehend Christ. There should never be a shadow of a doubt in the mind (there never has been in mine) that in honouring Christ we honour the Father." On the same subject, he writes later, "I have no trouble or confusion as that you speak of in regard to the persons of the Godhead. I conceive of God as absolutely one, yet have no difficulty in apprehending God in Christ and God the Spirit in me. Without this trifold manifestation I have never known God. There is more approach to a mystery in the distinguishing between the Christ of God and His brethren fully redeemed, in whom too is all the fullness of the Godhead. John fell at

the feet of one of these. But I suppose there will be practically no difficulty. He is always the Saviour and they are always the saved. John xvii. and Ephesians iii., etc., show that we must get where Christ was when about to ascend. The more fully we are conformed to Him, the better we shall understand all things." Bushnell solved the mystery of the Trinity in the same way and in the end we shall find that what theology is unlivable will be difficult of permanent propagation in mission fields. Religious value is not the right criterion of truth, but the truth whose religious value is not known and evidenced in our own life we shall find it hard to communicate to others.

His deep Christian experience, his attempt to make his Christian life real and his shrewd knowledge of the heart, led Bowen to anticipate by many years that form of Christian teaching identified now largely with the Keswick convention for the deepening of the spiritual life held annually in the English lake-country. Whatever excrescences there may be, the main teaching of the Keswick conference is simply the Gospel of the redeemed life in Christ. As Bowen put it in his "Daily Meditations" (for December 30th), "You believe in Christ and not in yourself; in His goodness, not in yours; in His power and wisdom, not your own; in His word, not in yours; in His work, not in yours; in His sufferings, not in yours; in His prayers, not in yours. When a man believes his vessel to be on the point of going to pieces, and is hailed by another that is seaworthy, you will quickly find him removing all his goods from the first to the other one. His faith finds

unequivocal utterance in his conduct. And he that believes in Jesus Christ makes haste to get everything that he values transferred to Him." And he writes in 1880 in a personal letter: "The best use we can make of our past sins is to turn from them to Christ. Anything that diverts our attention from Christ does us harm. This and that sin may appear very odious to us, and are so truly, but with God the most odious sin is that of not accepting His offer of love. . . . There is not the slightest use in trying to correct anything amiss in our mental habits by direct efforts. We get the victory by faith, *i. e.*, by ceasing to combat them and making them over to Christ. Do not even be impatient with these evils. Nothing so discomfits Satan as when you praise the Lord (2 Chron. xx. 20)." It is an intensely interesting thing to see in church history how the teaching of Christian men regarding the higher spiritual life repeats itself from age to age and how the heresies of the earlier days arise recurrently, and especially in both matters in connection with missions.

The wisest and most practical attempts of to-day to feed the hungry human soul, Bowen anticipated. It is to be feared that sometimes the technical theological schools little realize how deep the hunger is or for what it longs. The summer conferences which testify to its existence and attempt to allay it are too often left beyond the sympathy and interest of the school. But we may be sure that these conferences exist because of a need and to some measure succeed in meeting it. It is not so much clear theological doctrine that these hearts crave as the sense of assurance, the secret of peace, the way of a larger life,

something more than the conventional teaching gives, or the conventional standard requires. What Bowen said is just what those who attempt to meet these higher spiritual demands are saying to-day. "I live in hope," he writes to a correspondent, "that you will send me word some day that you are believing these words of God to 'whomsoever' and banish that sense of condemnation and all vain thirsting. Whatever your nature really demands for its highest development is in that word 'eternal life.' I wish that you could make up your mind that nothing more is ever to come to you from God than has come to you, and give your attention to what has come to you and is ignored by you. It was a blessed hour for me when I lost faith in the future and began to interrogate the present. I think I see a prisoner in a cell. On a table a letter has been lying many days which he fancies for somebody else and not for him. It authorizes him to claim the right of egress and to go out of his jail and to go to a comfortable dwelling provided for him. But, he says, it is not for me ; if it were for me, it would not leave me here. He is there because he has not the faith. Why should you make 'light of all that God has done to inspire you with faith ? You do this when you fail to recognize what God offers you. The lying spirit of unbelief will say to you, this does not suit your case. Let not that spirit continue in his post of doorkeeper of your heart. How glad I should be to hear that you have decided to let God be true, though every man a liar. All happiness is in the recognition of Him Who sits upon the throne, whose nature and whose name is Love, Who gives Himself and is Himself Love

Almighty to every atom, and is excluded only by man's unbelieving heart. God has never done anything for me, or will do, that He is not offering to every creature, for He offers Himself and He is Love. You have only to let God be true, let Him be Himself, and you will find yourself in Paradise. The New Jerusalem comes down from God out of heaven when men discover this. But it is hid from them by the great concern that they have for self. Do not allow your heart to cheat you out of the blessings contained in this truth. . . ."

Again he writes, "I deeply feel that what you want is not that God should take up some new attitude towards you or do anything, or be anything but what He is, but that you should recognize Him as revealed at the Cross. What makes heaven to be heaven is that the truth which you fail to see is there seen by all." Bowen counselled thus out of his own experience. "As you would wish your own word to be honoured," he wrote (August 11, 1885), "honour God's. Salvation is in that very thing. I was just on the border of despair in 1845, till on the 4th of December I saw that all I had been seeking in myself, I had in Christ. I had been tormenting myself by looking hourly to my own heart for the dawn of a brighter day, looking (if you please) for Christ in my heart rather than for Christ in the Word, and I found life, joy and peace when I let go my own heart and looked for Christ alone, as the Israelites looked to the brazen serpent." The path he urged upon others he had trod himself and he knew whither it led.

One supreme test George Bowen met. Little chil-

dren loved him and felt that in him they had a friend without dissimulation or suggestion of distrust. Can a man ask more than that? When he died, says Prescott of the great William, in "The Rise of the Dutch Republic," a whole great nation mourned for him and the little children cried upon the streets.

I suppose to some of you this sketch has introduced an altogether new character. Men fall fast out of memory and George Bowen would not have lifted a finger to prolong his fame. But he is a man whom we cannot afford to forget. In reviving his story, I am conscious of the danger to which Dr. Mackichan referred just after his death: "To those of us who were intimately associated with the departed missionary leader, the sense of loss has day by day grown deeper. Christian work with which he was associated and Christian assemblies which he was wont to frequent, have seemed almost less Christian by reason of the absence of one who gave the high tone of his own spirit to everything with which he was identified. As we contemplate the end of his conversation we are not strangers to the danger of resting satisfied with a vicarious devotion. It was inspiring and strengthening to know that one lived and worked so nobly in the midst of us. But to admire and describe this life is the least part of that which it requires of us. In every department of Christian service the same spirit of reality and consecration is needed, and if this brief record of his life shall in any measure help to keep alive the memory of this man of God, and lead those who have a part in the same work to become partakers of his higher faith, it will be contributing to the accomplishment

of no unimportant part of the work for which George Bowen lived and laboured and died."

It is easy for us to be content with looking at such sacrifice and total devotion in a missionary of a past generation. But there was no standard of duty or ideal of character before George Bowen that is not before us. If he utterly denied himself and wholly sought to live unto God in all things, it was in response to no call that does not also sound in our hearts and summon us to the same task of the world's evangelization and to the same life of Christlike candour and reality. In the quiet of this hour can we not hear this Voice saying to us, "And you, why do you too not follow Me as he followed whom men called 'The Lamb of India'?"

LECTURE V

JOHN LAWRENCE, THE CHRISTIAN
STATESMAN AND THE PROBLEM
OF RELIGION AND POLITICS

LECTURE V

JOHN LAWRENCE, THE CHRISTIAN STATESMAN AND THE PROBLEM OF RELIGION AND POLITICS

THE Hon. John W. Foster has remarked with reference to the failure of the American Senate to confirm President Grant's nomination of Caleb Cushing to be chief justice of the Supreme Court, that it was not the only time the American people have pronounced defect in moral character a bar to the highest honours. And the world has never been content to ask mere moral rectitude in its greatest heroes. Many other men of their generations were as honest and pure as Chinese Gordon and John Lawrence, but these men smote the imagination of the world with a peculiar power and won its love and regard, and the unique and distinctive thing about them was their religious character. The remarkable thing is that men should so admire and revere the highest religious character in their heroes and so neglect its development in themselves. One is tempted to ask whether men have a right to praise in Gordon and Lawrence a religious principle and life which are as open to every man as they were to them, unless they are conscientiously following after these things for themselves.

There were wide differences between John Lawrence and Chinese Gordon, as will appear, but in this

one thing they were alike. They had one passion. It was to do their duty in the fear of God, and in no other fear, and in the simplicity and faith of Christian men. On the gravestone of John's brother Henry at Lucknow are the simple words prescribed by Lawrence himself, "Here lies Henry Lawrence, who tried to do his duty." The last words Chinese Gordon wrote to his sister were the postscript to his letter of December 14, 1884, "I am quite happy, thank God, and with Lawrence have tried to do my duty." "I never cared for honours," said John Lawrence, as he drew near the close of his term of office as Viceroy of India; "I do not regret the resignation of all the state, pomp, power, or patronage which appertain to the office. It was a proud moment to me when I walked up the steps of this house, feeling as I then did that, without political interest or influence, I had been chosen to fill the highest office under the crown, the Viceroy of the Queen. But it will be a happier moment to me when I walk down the steps with the feeling that I have tried to do my duty." It was every man's judgment about him that he had in sincerity and honour done his duty his whole life through. When Sir Charles Wood resigned the Secretaryship of State for India to become Lord Halifax, he wrote to Lawrence, "It was a great satisfaction dealing with so honest and straightforward a person as you are." "After ten years' witness of his private life," said Miss Gaster, his private secretary during the last years of his life, "I believe from the depth of my heart that God never made a purer, nobler nature than his. Faults, of course, he had. But to those who knew him well

they were only spots in the sun of his goodness, inappreciable in the warmth and life he diffused around." And Lord Derby summarized the spirit of the man in his words at the "Lawrence Memorial" meeting at the Mansion House, "Without claiming any special intimacy with Lord Lawrence, I may say, as the world goes, that I knew him well, and the impression that his character always left on my mind I can only describe as that of a certain *heroic simplicity*." And Mr. Bosworth Smith begins his biography with a recognition of the man's rough humour and giant vigour, but relates it as it was related in his life to his fearless veracity and honour. "John Lawrence," says Mr. Smith, "was nothing, if he was not truthful; he was transparent as the day, and my highest aim has been to render to so 'heroically simple' a character that homage which is its due—the homage of unalloyed truth. . . . He always said . . . exactly what he thought. He always acted . . . exactly as he spoke. He raised against himself, as every strong ruler, as every vigorous reformer, as every great man must inevitably do, not a few enemies; he attached to himself by the selfsame processes, and for the selfsame reasons, troops of most devoted and most loyal friends. . . . The rugged lineaments and the deep furrows of his grand countenance—

For his face

Deep scars of thunder had entrenched, and care
Sat on his faded cheek, but under brows
Of dauntless courage—

were a picture, which he who runs may read, of the grand and rugged character which lay beneath it."

John Laird Mair Lawrence was born of a North Ireland family in Richmond in Yorkshire on March 4, 1811. His father was a soldier who cherished a sense of unjust treatment, and his mother was a daughter of a Donegal clergyman named Knox, descended from the great reformer. The calm religious conviction of his ancestry ran in the boy John. "I should say," he wrote years later, "that on the whole we derived most of our mettle from our father. Both my father and mother possessed much character. She had great administrative qualities. She kept the family together, and brought us all up on very slender means. She kept the purse, and managed all domestic affairs. . . . When I was coming out to India, my poor old mother made me a speech somewhat to the following effect:—'I know you don't like advice, so I will not give you much. But pray recollect two things. Don't marry a woman who had not a *good* mother, and don't be too ready to speak your mind. It was the rock on which your father shipwrecked his prospects.' " John Lawrence married later the kind of wife that suited his mother, but he never lacked in readiness to speak his mind, and on that rock of unswerving courage of conviction and the expression of conviction he won his name.

John and his older brother Henry were schoolmates as boys, as they were workfellows as men, and they got the characteristic schooling of the day. "I was flogged every day of my life at school except one," said John, "and then I was flogged twice." "For my part," said Henry Lawrence, "my education consisted in kicks; I was never taught anything." John was no great student, and his life was

an ordinary boy's life, with an unusual amount of daring and courage, perhaps, but nothing more. He wanted to be a soldier and go out to fight in India as his father had done but he was obliged instead to go to the East India Company's college, for the training of its civilian agents, and then went out to India as a civilian in September, 1829, with Henry. It was already foretold of Henry that he would come back Sir Henry Lawrence, but no one expected great things of John. J. H. Batten tells of a visit in 1859, the year of the Mutiny, to Mr. Le Bas, who had been principal of Haileybury College and Le Bas asked him, says Mr. Batten, "'Who is this John Lawrence of whom I hear so much?' to which I replied, 'Don't you remember a tall thin Irishman with whom I much consorted, who once kept an Irish revel of bonfires on the grass plot opposite to Letter C; and whom you forgave on account of his Orange zeal and his fun?' 'Aha!' said the old dean, 'I remember the man; not a bad sort of fellow;' and then he burst into one of his fits of laughter, ending with the dry remark, 'but what has become of all our good students?'" He had got, however, some training in character, and he had won some friends among his fellows who were later in the terrible days of the Mutiny to stand solidly about him when he was to be the great rock in the midst of the tempest, upon whom it was to break and be broken.

The boys reached Calcutta in February, 1830. Alexander Duff came in the same year. Henry was sent at once to join his company of Foot Artillery at Kunol, beyond Delhi, on what was then the north-west frontier. John was sent for the study of the

native languages to Fort William College. He soon became thoroughly sick of India; the Calcutta climate did not agree with him. He had no taste for the empty social life. He longed for home, but when he passed the language examination in Urdu and Persian, and was sent off, at his request, to frontier work at Delhi, "there was now no more inaction, no more halting between two opinions. He had put his hand to the plough and there was no looking back. He shook himself, like Samson, and awoke to his work. From the present moment to the very end of his official life, we shall find no parallel to the inaction of the four months spent in England before leaving it for India, or to the depression which seems to have dominated him during the ten months he spent in Calcutta before embarking in his active work. There was, henceforth, no nervous looking forward to what might be, or backward to what might have been. To do the thing that lay before him, to do it thoroughly, to do it with all his might, not regarding the consequences and not turning either to the right hand or the left—this was henceforward the ruling principle of his life." This is the right prescription for homesickness, for all indolence—work. That is the first great missionary lesson from his life. Trollope held that the best cure for the disposition to shirk in writers was beeswax on their chairs, and for missionaries, as for all men, no medicine is so good as the medicine of stiff duty crowding them hard and driving them to unselfishness, whether they will or no.

The town and district of Delhi were under the control of a British officer entitled "Resident and

Chief Commissioner." Lawrence's first work was as "assistant judge, magistrate and collector" of the city and its environs. He was here for four years and at once put all that he was into his work. The principles of the Delhi administration were to save as much of the native institutions as possible, especially of the old village communities, and from the outset Lawrence entered with hearty sympathy into the life of the people. He had much greater opportunity for this upon the close of his apprenticeship and his consequent transfer to the charge of the Paniput district. In Delhi he had been associated with others and had made the friendship of Charles Trevelyan, one of the best men in India, to be ever his friend and later his valued helper, from whom even now we could learn lessons of fearless honour and resolute opposition to corruption in all places high and low. In Paniput, Lawrence was practically alone. How he did his work is best indicated by the description given by Mr. Charles Raikes, who went out to help him :

"Over some 400,000 of a population, scattered in large villages through an area of 800,000 acres, John Lawrence ruled supreme. He himself in those days had very much the cut of a Jat, being wiry, tall, muscular, rather dark in complexion, and without an ounce of superfluous fat or flesh. He usually wore a sort of compromise between English and Indian costume, had his arms ready at hand, and led a life as *primus inter pares*, rather than a foreigner or a despot, among the people. Yet a despot he was, as any man soon discovered who was bold enough or silly enough to question his legitimate authority—a despot, but

full of kindly feelings, and devoted heart and soul to duty and hard work. . . .

“First, he was at all times and in all places, even in his bedroom, accessible to the people of his district. He loved his joke with the sturdy farmers, his chat with the city bankers, his argument with the native gentry, few and far between. When out with his dogs and gun he had no end of questions to ask every man he met. After a gallop across country, he would rest on a charpoy, or country bed, and hold an impromptu levee of all the village folk, from the headman to the barber. ‘Jan Larens,’ said the people, ‘sub janta,’ that is, knows everything. For this very reason he was a powerful magistrate, and, I may here add, a brilliant and invaluable revenue officer.

“Secondly, he was never above his work. I have an indistinct recollection of his arresting a murderer, on receiving intelligence of the crime, with his own hand. At all events, when the report of a murder, an affray with wounding, or a serious robbery came in, John Lawrence was at once in the saddle and off to the spot. With greater deliberation, but equal self-devotion, he proceeded to the spot to investigate important disputes about land, crops, water privileges, boundaries, and so forth. The Persian proverb, ‘Disputes about land must be settled on the land,’ was often on his tongue.

“Thirdly, owing to this determination to go about for himself and to hear what everybody had to say about everything, he shook off, nay, he utterly confounded, the tribe of flatterers, sycophants, and informers who, when they can get the opportunity, dog the steps of the Indian ruler. What chance had an informer with a man who was bent on seeing everything with his own eyes? . . .

“I was younger than Lawrence, and had been only

three or four years in India when I went to join him at Paniput. For very good reasons I shall never forget my first interview with my chief. He was, I was going to say, in his shirt-sleeves, only I am not sure that he wore a shirt in those days—I think he had a chupkun, or native undergarment—surrounded by what seemed to me a mob of natives, with two or three dogs at his feet, talking, writing, dictating—in short, doing cutcherry. . . .

“Lawrence trusted me and taught me to trust myself. From that hour my fortune as a public officer was made. I learned my work under the ablest of masters, and shall ever gratefully remember the day which saw me installed as assistant to the young magistrate and collector of Paniput.”

Here Lawrence found opportunity and necessity alike for the complete identification of his life with the native people. That is the second great missionary lesson from his life. He talked their language as he talked his own. At times among them he could speak their tongue even better than he could his own. A young civilian, who went to call one day as he passed through, reported that he could hardly understand him, his conversation had been so full of Persian words and expressions.

These principles became the characteristics of the school of officials whom John Lawrence raised up, who were known as the Punjab School. They knew the people and loved them. They made it their business to understand and sympathize with their institutions and ideas. In the Paniput days Lawrence began the easy, familiar relations with the native people of all classes which put him in a position of almost irresisti-

ble power among them. He knew them, and they knew that he knew them.

From Paniput Lawrence stepped down to an inferior appointment at Gorgaon. But he was ready for any service, and while he had probably more pride of position than Gordon had, yet as Gordon was ready to descend from a governor-generalship to be a private secretary, so Lawrence wanted not honor but work. From Gorgaon Lawrence was called off in 1838 to be settlement officer at Etawa under Robert Bird. Bird was one of those great geniuses, of rare character and of rare tact and intellectual power, who do great and unnoticed work in the world and die in quietness unremembered. He untangled the land confusion in the northwest provinces, and settled in equity the most vital problems in the lives of a great agricultural community like India. To be chosen by Bird as one of his helpers was considered by men a form of advancement and mark of honour. Associated with him was James Thomason, another of the greatest and best of the East India Company's servants who rose to be Lieutenant-Governor of the northwest provinces during Dalhousie's viceroyalty. The two men were of the same Christian type. Thomason wrote that he found Bird "so instructive and communicative on subjects which regard another world," and they discussed together how to carry out their Christian principles into their daily walk as public servants.

After this illness, Lawrence was ordered home for a three years' furlough, and arrived in England at Christmas, having risen steadily in his work in India, "half a head above his fellows," as one expressed it,

and yet having given no full promise of what was in him and to come out of him. One of his desires on furlough was to get married, to find a "calamity," as he expressed it. He went to Bath, but that summer resort was not his style. "His manners and appearance," says one who was a young girl in the house where he was visiting, "were utterly unlike the ordinary young men we met in Bath. It was difficult not to feel a little shocked at first by his roughness and absence of conventionality; still there was so much force and originality apparent in his whole character that one soon forgot the defects of manner, and became interested in his conversation. As I remember him he seemed to me to embody Prof. Henry Morley's notion of the qualities which have given to Englishmen their proud position in the world, namely, 'the determination to find out the right and get it done; find out the wrong and get it undone.' " In August, 1841, he was married to the sort of wife his mother had prescribed. After the honeymoon he was seized with a long illness which made the doctors tell him that he must give up all idea of returning to India. "If I can't live in India," was his reply, "I must go and die there," and he sailed for Calcutta in October, 1842.

The first Afghan war, fought, as were all the Afghan wars, in behalf of a principle to which he was always opposed, occurred during Lawrence's furlough. When he returned, his first post was as Magistrate of Delhi. It was the same kind of work he had done before, only in a more conspicuous place, and the man himself was grown to match. While he held this post the first Sikh War was fought, ending in the annexa-

tion of the Jullundur Doab and the location of a resident at Lahore, the Punjab to remain independent. Henry Lawrence was sent to Lahore and John was placed in charge of the territory that had been annexed. John Lawrence plunged in with characteristic energy, and during his tenure of the Jullundur Doab, he set it in order, established in it a model rule and hewed out those principles of administration which were to prevail through the entire Punjab, and make it the rock of defense and deliverance in the storm of the Mutiny. It was under these men that John Lawrence's school—Montgomery, McLeod, Thornton and others—was formed. They were men who did not separate, as one of our great public characters had bidden us to do, their religious faith from their public service.

Lawrence never spoke slightly of any post of duty to which he was assigned, save in the case of Etawa,—“that hole Etawa,” he called it. His biographer is sure that this was “not because it brought him too much discomfort or difficulty or work, but because it brought him too little.” In connection with Lawrence's work at Etawa, several incidents are related illustrative of his capacity for anger and his iron will.

“Like Cromwell, John Lawrence was rough and downright in all he said and did. Like Cromwell, he cared naught for appearances, spoke his mind freely, swept all cobwebs out of his path, worked like a horse himself, and insisted on hard work in others. The natives, if they did not love him, regarded him with veneration and with trust, at all events, as somebody to be obeyed. They respect a

man who will be down upon them in a moment for anything that is wrong, provided only that he is scrupulously just, and this John Lawrence always was. His voice was loud, his presence commanding; his gray eye, deep-set and kindly as it was, glared terribly when it was aroused by anything mean or cowardly or wrong. His temper—the Lawrences were all naturally quick-tempered—was generally well under control; but when he felt, like Jonah, ‘that he did well to be angry,’ there was no mistake at all about it.”¹

In 1839 he fell seriously ill of jungle fever.

“He had often been heard to say, in the abounding and jubilant strength of his youth, that he was sure that many a man need not die, if he made up his mind not to do so. But he was now rapidly becoming worse and appeared to be in a state of collapse. One day the doctor who had been attending him told him that he feared he could hardly live till the following morning, and took leave of him accordingly. No sooner was he gone than his patient roused himself to the emergency. Now was the chance of putting his favourite maxim to the test. He determined not to die, and bade his servant give him a bottle of burgundy which lay in a box beneath his bed. He drank it off, and next day when the doctor called, by way of form, expecting to find that all was over, he found John Lawrence sitting up at his desk, clothed and in his right mind, and actually casting up his settlement accounts.”²

There may be some who will think that such exhibitions of wrath and self-will poorly show forth the

¹Smith, “Life of Lord Lawrence,” Vol. I, p. 101.

²*Ibid.*, p. 112.

Christian character of Robert Bird's disciple. But such power is better for God's use than tame imbecility.

"He as He wills shall solder and shall sunder
Slay in a day and quicken in an hour
Tune Him a music from the sons of thunder
Forge and transform my passion into power.

"So with the Lord : He takes and He refuses
Finds Him ambassadors whom men deny
Wise ones nor mighty for His saints He chooses,
No such as John, or Gideon, or I."

Lawrence ruled the Jullundur Doab with a just hand but it was very strong. The infanticide, suttee, and living burial of lepers and punishment for witchcraft he put down, and it was no easy task. He did all the other things which he had done at Paniput but these things especially I refer to because we have heard so many smooth Swamis and soft Americans declaring that the tales of infanticide and suttee in India came from missionary slanders.

"So wholesale," says Mr. Bosworth Smith, "was the destruction of female infant life that, when the attention of philanthropists was first directed to it, whole village communities were found to be without a single girl. . . . Nor was the practice confined to the Rajputs. It was still more universal among the Bedis, who were a subdivision of the Khuttri caste and traced back their descent to the Guru Nanuk. They had never allowed a single female child to live, and when the Bedi of Oona, the head of the tribe—in fact, the spiritual head of the Sikh religion—was warned by John Lawrence that he must forbid infanticide

throughout his jagheer, he replied that if the Sahib so willed it he would never enter his harem again, and would influence, so far as he could rightly do so, others to do the same, but it was impossible for him to command his dependents to give up so treasured a custom. 'You must do it or give up your lands,' rejoined John, and the stiff-necked old Levite acquiesced in the lesser of two evils, and did give up—his lands. Those who have never seen John Lawrence, but have accompanied me thus far in my efforts to reproduce the living man, can imagine the grim patience with which he would listen to a solemn deputation from the whole priestly race whose most cherished practice he was thus rudely threatening, and who based their petition on the proclamation issued by the Governor-General that all their rights and customs would be respected. 'These Bedis,' he writes to a friend, 'are an extraordinary people. You will scarcely believe it when I tell you that they publicly petitioned me for permission to destroy all their female children; which it seems they have hitherto invariably done. I sent for some of the most respectable of them, and set forth the enormity of the crime, and our detestation of the practice, before some hundreds of people, and ended by telling them that government would not only never consent to such a villainous crime being perpetuated under its rule, but that we should certainly hang every man who was convicted of such a murder.'"¹

At the time such conduct was denounced by many as an unwarranted interference with the religions of the people. Now that the practices are stopped by such measures as John Lawrence used, it is denied that they ever existed. If it had not been for such a

¹ Smith, "Life of Lord Lawrence," Vol. I, p. 184 f.

strong hand in India, those practices deep seated in the religion of India would be going on to-day. The special patron of female infanticide in the Punjab was the Sikh high priest, Bedi Bikrama Sing.

John Lawrence used plain speech in other matters than the suppression of the slaughter of baby girls. He spoke his mind to British officials as well as to native offenders. Indeed his practice was to speak his praise behind a man's back, but his reproof or condemnation to his face. "I have received your letter," he writes in one case. "As I do not agree in any respect with the views you there lay down, I think it kinder and fairer to write to you privately on the subject before I take any public notice of the matter." He was as plain with native chiefs.

And he would speak as plainly to his friends. Writing once to Lord Canning, he said, "Your lordship may depend that I will write nothing but the truth. My feelings are so strongly enlisted in my public duties, that I may almost say that I have no friends independent of such considerations. My best friends are the officers of whom I think best in my public relations." This was a pretty high claim, but it was not undeserved, and on just this account he had the firmest and best friends, men whose friendship rested on solid grounds beyond the reach of all self-interest and who loved him the more because he was a man who would never "job for his friends, who never failed to tell them of their shortcomings, however ready he may be to bear with them; who is chary of his praise to their face, however lavish he may be of it behind their backs; who thinks nothing done while aught remains undone; who regards the

performance of duty by himself and by others as a matter of course, rather than as requiring 'generous recognition.' " He would have laughed at that melodramatic jewelry known as the Carnegie medal.

Lawrence's work in the Jullundur Doab melted into his larger work in the Punjab. He had to act for his brother in Lahore but wanted to go back to his own work; "I am ready to do what government wants," he writes, "but, personally, I prefer my work there. It is a new country, and my assistants need looking after; and I want to put my stamp on it, that in after times people may look back and recall my Raj with satisfaction. No portion of our empire promises better than it does."

He put his stamp on Jullundur, and on all north-western India, and on all India and on the English race; for the second Sikh War led to the annexation of the Punjab, and to the establishment by Lord Dalhousie of a Board of Three Commissioners to govern it. Henry Lawrence was chairman and John was one of the two other members. What this Board accomplished in the three years of its existence is one of the wonders of wise and capable administration. The directors of the East India Company were "good masters but very chary of precious words," but even they were warm in their appreciation of what had been done.

Perhaps the best aspect of the work the Lawrences did was the recognition of the rights and spirit of the native races. Henry Lawrence was ever the champion of the native chiefs and people, and lived and died probably the best beloved Englishman who has ever worked in India. He and John developed the sense

of local responsibility and self-government, and began the work of riveting the Punjab to the British rule which was carried forward under John Lawrence's firm hand until it bore the terrific strain which fell on it in the Mutiny.

After the Board of Commissioners was dissolved, for seven years John Lawrence ruled the Punjab as Chief Commissioner. At the very end he was given the title of Lieutenant-Governor, but for all these years he had been its actual governor and great and assertive as Dalhousie was, he did not overshadow John Lawrence or destroy the force of his personality or the grip of his distinct influence in the Punjab. And the one word that best characterizes Lawrence and his work in the Punjab is, truth : true work, true dealing with men and true motive and aim.

In the first place he was a terrific worker. He wrought like an engine himself and he expected the same thing in others. He wrote to Edwardes :

"Our officers should be young men, rough-and-ready fellows, fit to put their hands to any work against time and tide. I cannot believe that the treasury can take up the time of one officer. If I were deputy-commissioner I would be bound to prove that it did not take up one-half his time. I don't speak without cause ; I had charge of a treasury for six years, unaided, and the time it occupied was hardly appreciable. For instance, if I had to see money counted, I took my work to the treasury, and while my ears were hearing reports and cases, my eyes were looking at money being counted. I signed and checked bills while evidence was being taken by my side. Half an hour a day sufficed to look over the

accounts, with perhaps a couple of days' extra work in a couple of months." What would such a man say of some of our puttering half men? He despised procrastination. When he was Governor-General of all India he got through each day's work before he went to bed. Delay was the one blemish he saw in Donald McLeod, the "Cunctator" as he called him, of whom he complained that he spent too much time on polishing work that was no better done by reason of the polishing, and he wrote frankly to McLeod :

"If you only firmly resolve to postpone nothing that can be disposed of at the time, daily getting through what comes before you, there will be nothing further to desire. You do not, I think, give yourself fair play. You are like a racer who, instead of starting off directly the signal is given, waits until the others have got well ahead before he commences his running ; or, perhaps, what is nearer the mark, you only consent to make play when you have packed a good mound of traps on your back. Now pray excuse these ungracious remarks. There is no man who regards and respects you more than I do, or who could be better pleased to have you as a colleague. I see but one speck on your official escutcheon, and, like an officious friend, desire to rub it out."

In his work he never thought of his own ease or pleasure. He was no friend of indolence and for some years in the Punjab no vacation at all was allowed. Lord Dalhousie hinted at a leave for a relative of his, employed in the Punjab. Lawrence replied : "He has to stay . . . for the public interest." This was an emergency, of course, but it represented his principle,—work before all things else.

And he was very simple and frugal. "Always liberal with his private funds," said his wife, "and ready to help others, my husband spent as little as possible on himself, and was ever sparing of the public money, anxiously impressing on every one the necessity of strict economy in the management of the new province." And his private secretary, at the end of his life, tells of his going into a fruit shop in London when ill, to ask about some tempting strawberries which were displayed. When told the price he exclaimed, "Spend ten shillings on myself for such a purpose! I never did such a thing in my life," and he marched out without them. He did his work in truth and he handled men in truth.

"John," said Sir Henry Daly, "never deserted any friend of Henry's if he could possibly keep him, and hence his wonderful forbearance with Nicholson. He knew perfectly well that Nicholson did not like him, and spoke against him. But such things never made the slightest difference in his behaviour to him or to any one else. He had nothing mean or small in his nature; no spite or malice. He was the *biggest* man I have ever known. We used to call him 'King John' on the frontier, and it is as such that I still love to think of him."

He was stern and strong towards men, fearless in speaking his mind to them but no cross-grained or imperious character. "I had, in my imagination, pictured Lawrence," says Sir Richard Temple of their first meeting, "as an iron-looking man, somewhat severe in tone and aspect, with a massive brow, straight features, and compressed lips, uttering few

words, and those only of a dry and practical import. His conversation, I expected, would tend towards statecraft or political economy, and would proceed to the point and nothing but the point. Great was my surprise, then, on finding that he had an open countenance, an expansive forehead, a frank, genial bearing, and a vivacious manner of conversation. The lips, so far from being closely set, were parted constantly by smiles and laughter." He handled men by commanding their respect and by revealing a superior sense of duty and strength in its performance.

And once again he had the true motive and aim. He loved the people and ruled for their good. That was his ambition. His last word on leaving India was, "Be just and kind to the natives." We are told nowadays that we ought not to call the people of any mission land natives, much less heathen. The word "native" can be spoken meanly, but it is a good word. No one resented it on John Lawrence's lips. On one occasion, when he was Viceroy, "a young officer in the army, who was talking, after the manner of his kind, contemptuously of the natives, happened, in Sir John's hearing, to speak of them as 'those niggers.' 'I beg your pardon,' said Sir John, 'of what people were you speaking?'" The rebuke did its work. Lawrence, like Gordon, was able to trust men of whom others would have been afraid. The mutiny was put down by native troops whom Lawrence organized, calling in even old Sikh leaders who had been among the greatest sufferers from the Sikh wars. He did not despise the race he ruled. No man can really help and win the love of

those whom he does not respect, whether he be statesman or missionary.

And in the larger problem of British government in India Lawrence kept his head and followed true ends. He was opposed to the Afghan wars and was no Russophobist. He believed that Peshawar was the right limit of India to the northwest, and that India should be governed by itself, for its own good, not for British aggrandizement or as a base for British dominion over Asia. And for all his great services he sought no reward. He accepted quietly such honours as came to him, all belated, the K. C. B. for his great services in the Punjab and the baronetcy for his work in the Mutiny, and the peerage when at last his work in India was done. And he went on, the same frugal, simple, straight working man as before.

The time has come to speak a little more fully of the Mutiny. Its causes were varied and complex; the annexation of Oudh and the consequent discontent of the Sepoys, almost all of whom were from Oudh and who lost by the annexation some of their most valued privileges, the plotting of dissatisfied Mohammedan leaders, unrest at the settlement of land problems, fear of enforced conversion to Christianity due to the greased cartridge scare and other things which had preceded it, longing for the old days of easy going government and disorder and Oriental corruption, the dictatorialness of the British rule, the slow absorption of the native states and the destruction of native rights and above all the truckling compromisingness of the government—these were among the principal reasons for the Mutiny. As a matter of fact, the Madras and Bombay Sepoys did not mutiny

and there was little trouble in Bengal. The difficulty was in the Northwest Provinces, as they were then called, the United Provinces of Agra and Oudh as they are called now. The insurrection cut off Calcutta from the Punjab and left John Lawrence practical ruler of northern India, and from the Punjab, which his work had made secure and his strong arm held in quietness, he poured out the men, British and native, who retook Delhi, which was the storm centre of the Mutiny and thus broke the insurrection at its heart. It is a very wonderful story worth reading in all its detail, but it only shows John Lawrence in a time of great crisis as the man we have already found him to be in times of quietness. He was in the mighty hour of trial what he had been in the calm and orderly work of administration for eighteen years. The Mutiny found him at Rawal Pindi. It struck the mass of British officials dumb, but not John Lawrence. It released the measureless power of the man. He left details at Lahore and other points to the men he could trust, and like an eagle from its eyrie he looked out over the whole field and sent his enkindling messages everywhere while he drove the machinery of his government like a giant. He did not fear scanty resources. "It is want of action rather than want of means," he wrote to Edwardes, "which may prove disastrous to us." When the commander-in-chief hesitated and was going into entrenchments at Ambala, he telegraphed to him, "Clubs, not spades are trumps; when in doubt, take the trick," and wrote in a long letter, "Pray only reflect on the whole history of India. When have we failed when we acted vigorously? When have we succeeded when

guided by timid councils?" The whole Punjab thrilled with Lawrence's zeal, and steady but impetuous resolution. "I like issuing orders by telegraph," he said, "because they cannot give me their reason, nor ask me for mine." When others faltered he was firm; when others were restless he was calm and patient. When others were confused, he saw with clear vision, and holding unwaveringly to the intrepid policy he had originated, he swung the great cause through to success. The world recognized when the Mutiny was put down that it was Lawrence who had done it. "There remains," said the Viceroy, Lord Canning, in his minute on the services of civil officers and others during the Mutiny, "the large and important province of the Punjab. The merits of the officers to whose courage and ability the preservation of that country is due have been set forth by their distinguished chief, Sir John Lawrence, with a fullness which leaves little to be added. Of what is due to Sir John Lawrence himself no man is ignorant. Through him Delhi fell, and the Punjab, no longer a weakness, became a source of strength. But for him, the hold of England over Upper India would have had to be recovered at a cost of English blood and treasure which defies calculation. It is difficult to exaggerate the value of such ability, vigilance, and energy at such a time."

When the Mutiny was over Lawrence was given charge of Delhi as well as the Punjab, and pacified and consolidated what had been the violent centre of the tempest. He sided with Canning in the policy of clemency towards the people who had been involved. He loathed the spirit of revenge and cruelty with

which British sentiment in India and at home urged that the mutineers should be hunted down. We forget in our judgment of the Eastern people that their atrocities have been matched, at a distance, at least, by ours, and that if they have been guilty of massacre and violence, we have had no small part in schooling them to regard wrong and injustice as proper qualities of international intercourse.

For two years after the Mutiny, John Lawrence continued his work in India and then in 1859 went home with no purpose of ever returning again. He had done a real work. He had left his stamp not on the Jullundur Doab alone, not on the Punjab alone, but on the whole of India.

Temple was the secretary who wrote his reports. Asked years later whether the reports of the Board at Lahore were not too highly coloured, he replied, "There is not a word in the Punjab reports which I would wish unwritten. On the contrary, I should feel justified in speaking now even more strongly of the achievements of the Board than I did then. I have borne since that time a part in the government of nearly every province in India, and now, looking back upon them all, I declare to you that I have seen no government to be compared with that of the Lawrences in the Punjab."

When Sir John Lawrence left the Punjab for home he received a long address signed by two hundred and eighty-two civilians, four hundred and seventy-four military and naval officers, fifteen clergymen and eighty-three gentlemen unconnected with the government, in which these men who had known him well declared :

"All of us, of whatsoever class or profession, are conscious of the untiring energy, unflinching firmness, unswerving honesty of purpose, with which you have devoted yourself to promote the public service. We all believe, from personal knowledge or common fame, that you have been an instrument in the hand of Providence for the preservation of British rule in Upper India, by your good management and resolute bearing during a period of unexampled difficulty."

The address was a testimony which few other men who have ever ruled their own or other races could have received. But Lawrence had only done what he saw was his duty. That was his spirit and the atmosphere which he created around him. Robert Napier described this in a noble picture of the Punjab as it was under the two men of God who have left their stamp on it forever :

"There was a glow of work and duty round us all in the Punjab in those days, such as I have never felt before or since. I well remember the reaction of feeling when I went on furlough to England, the want of pressure of any kind, the self-seeking, the want of high aims which seemed to dull and dwarf you. You went back again lowered several pegs, saddened altogether. The atmosphere was different."

And now Lawrence went home to the meaner atmosphere. He found, as Lord Stanley had written him he should, his name and service in every one's mouth. Oxford and Cambridge each gave him a D. C. L. The Queen and the nobility made a lion of him, but nothing could spoil his plain simplicity or make him other than the free and frugal man he

had always been. For the next four years he served in the Indian Council at home. Our own Civil War broke out during this period and John Lawrence was one of the Englishmen who from the first firmly desired the preservation of the Union.

Lawrence's work for India in India, however, was not yet done, and in 1864 he was sent back as Viceroy to the land where as an unknown Irish lad he had begun his work thirty-four years before. The condition of the viceroyalty had greatly changed since Lawrence went to India. The telegraph had brought Calcutta into immediate contact with London, and the Mutiny had resulted in the destruction of the East India Company and the absorption of India under the British crown. These changes took away from the Viceroy his old imperial power, and gave Lawrence as Governor-General no such free hand as he had had in his old days in the Punjab as a commissioner. But he did not lose his character and reputation. He bent all his energies to carry on the great works which Dalhousie had originated. The railway and telegraph systems were largely extended; sanitary reforms were pushed forward, in the teeth of much native prejudice; barracks fit for British troops were erected on a large scale, with reading-rooms, workshops, gardens and *prayer-rooms*; education was vigorously fostered, especially for the poor villagers who most needed it. Dr. George Smith, then the leading editor in India, often criticized Lawrence's political views; but of these measures for the benefit of the people he wrote: "He is great in the work he has done; he is great in the moral spirit in which he has done every act; in the lofty

principle which has guided him ; in his noble private character, which towers above that of any of his predecessors.”¹

As Viceroy of India, Lawrence was the same unconventional, unresting character as ever. He worked in his shirt-sleeves. He contemptuously trod upon the slaveries of viceregal etiquette. He toiled on the “double barrelled principle of ‘no arrears,’ and ‘what you do, do thoroughly.’” And the great Durbars at which he spoke to the native chiefs in their own language—a thing no other Governor-General has ever been able to do—showed that he could put on the viceregal clothes when he wanted to show the viceregal character, which he never needed to put on because he wore it always. And after five years of noble service, as great probably as any man could have rendered under the conditions which prevailed, he went home for the remaining ten years of his life.

Within a few weeks of his return to England, the long delayed elevation to the peerage which was his due came to him, and amid cheers from both sides he made his maiden speech in the House of Lords on April 19, 1869. He served two years as chairman of the London School Board with Huxley, putting himself into this work as he had ever done in all his work. He took a large part in settling the question of religious instruction, and while he was on the Board the resolution was adopted “that the Bible should be read, and that there should be given such explanations and such instructions therefrom on the principles of morality and religion, as are suited to

¹ “History of the Church Missionary Society,” Vol. II, p. 484.

the capacities of children." His eyesight failed him towards the close and his secretary had to read to him, but he was full of activity and love, of firm principle and tenderness to the end.

He had some last work to do in fearlessly warring against the government's policy in Afghanistan, which later events sufficiently condemned, confirming the old sage's every judgment and prediction, and then on June 25, 1879, he passed on. "Do you know me?" whispered his wife. "To my last gasp, my darling," he replied quite audibly; and as she bent down to give him her last kiss, and felt the last pressure of his lips and hands: "I am so weary," he said. Such were the words which those who stood around his bed heard him murmuring to himself as he was entering the land where the weary are at rest. So lived and so died John Lawrence.

I have said enough about John Lawrence's character and methods, and their lessons to missionaries and all workers in the living movement of the world are sufficiently obvious. It remains to say something, first, about his religious nature; secondly, his attitude to the non-Christian religions, and thirdly, his relation to the missionary work.

1. John Lawrence and the Punjab School were men of devout and uncompromising Christian life. They were such in their private character. "And it is to be remembered," says Mr. Bosworth Smith, "that in India the private character of a public man is a more important element in estimating his general influence even than it is in England. Indeed, I question whether the example set to his countrymen at large in this respect by Sir John Lawrence, and I

might add, in their measure, by all the members of the Lawrence School, is not among the most valuable of all the services which he and they rendered to India. Throughout his life, even in the early Delhi and Punjab days, John Lawrence had set his face strongly against practices which it is easier to understand than to describe, and which were then all too common among our countrymen in India. No one whose character was not above suspicion in these respects could hope to stand well with him, even in early times. Still less could he have obtained access now to his viceregal court. Vice of all kinds stood abashed in his presence. Men, aye, women too, 'saw how awful goodness was.' The gambler, the profane, the profligate, the flippant, the self-indulgent, felt that his court was no place for them. No one ever dropped an impure word or made an impure allusion in his presence. No one ever scoffed at religion, whether his own or that of the natives. No one ever spoke contemptuously or harshly of the natives themselves without receiving from him a stern, and sometimes a sledge-hammer rebuke. On one occasion a lady who was sitting at the viceregal table allowed herself to sneer at the Bible. Sir John Lawrence looked sternly on her and said, with all his dignity, but with more of sorrow than of anger in his words, 'How can you speak like that of God and of God's Book in the presence of these young men?' The next minute he was talking with her of other subjects as if nothing had happened. But the rebuke had done its work on her and on the assembled company." ¹

¹Smith, "Life of Lord Lawrence," Vol. II, p. 511.

"Amidst his great successes," says Mr. Cust, "and his unparalleled good fortune, he had the grace given him to remember the Hand that gave, and while mindful of things temporal, not to forget the things eternal. He set the example of a bold, independent, and yet Christian ruler, an uncrowned king of men by grace and election."

When he became Lord Lawrence, both Lord Shaftesbury and the Duke and Duchess of Argyle used the name of God in congratulating him. He and they were living unto Him.

"From the earliest period of my acquaintance with him," says Captain Eastwick, "he was a decided Christian; a simple, God-fearing man, who, to the best of his ability, translated into daily practice the precepts of the Bible, of which sacred volume he was, to my certain knowledge, a daily assiduous and meditative reader. I have often seen him, when his sight had grown too dim to allow of his reading other books, spelling out slowly, with his finger on the page, a few verses from a New Testament printed in large type."

He believed in prayer and prayed. As Viceroy, he instituted family prayers in government house, and gave orders that the servants and others connected with it should, as far as possible, be released from labour on Sundays.

"He never omitted having family prayers for the household; and he and I hardly ever missed our daily reading of the Bible together, even when he was at his busiest," said his wife. And when he was losing his sight, his wife says "his gentle patience never failed," and she recalled "how earnestly he

prayed with me that God would help him to submit with resignation to His will."

He ever observed Sunday, as the whole Punjab School did. At the great Durbars at Lahore and Agra all festivities ceased on Sunday, and there were only the quiet ordinary services of the church. In his speech to the Lahore Durbar he openly recognized God.

"I will now only add," he said in closing, "that I pray the great God, who is the God of all the races and all the people of this world, that He may guard and protect you, and teach you all to love justice and hate oppression, and enable you, each in his several ways, to do all the good in his power. May He give you all that is for your real benefit."

He was not a Viceroy after the style of some who went before and came after him. His simple presence at the head of the government was a help to all missions and a preaching of the Christian Gospel by character and act and word also from the highest pulpit in India.

2. John Lawrence was never soft in his thoughts or words about the immoralities or superstitions of the Indian religions. He spent a deal of strength fighting the practice of infanticide which was imbedded in the native religion, and in those same earlier days he was wont to chaff the people upon their foolish beliefs. When he was Viceroy the Hindus were forbidden to throw their dead bodies into the Hoogly, and after the first Sikh War he declined to continue feudatory arrangements in support of temples, converting all such into money payments. After the Mutiny, however, he refused to allow any

injustice. When it was proposed to destroy mosques or temples, "I will on no account consent to it," he said. "We should lawfully abstain from the destruction of religious edifices, either to favour friends or to annoy foes." And he insisted on the restoration of temples and mosques to the people for their original uses. "He did not appear to have turned his attention," says Temple, "towards the philosophical, ideal, and metaphysical phases of Hindu thought. But of the priestly and fanatical classes among Mohammedans he had a vivid and exact appreciation, which he would embody in forcible language."

The great problem in this sphere, however, on which the position of Lawrence is most interesting was the question of religious neutrality on the part of the government. The East India Company had started out by opposing Christianity and fostering the native religion, idolatry and immorality and all. Gradually it was forced by public opinion at home, and the character of some of its agents in India, to discontinue all its support of the heathen religion, but varying views prevailed as to what government officers might or might not do in supporting Christianity in their personal capacities. In the Punjab Lawrence, Edwardes and their school had always been perfectly fearless in their position, supporting Christian missions and living and avowing their Christian principles. When the Mutiny broke out it was realized that if the Sepoys had known what Christianity was they would never have been played upon as they were. As a matter of fact, the Sepoy armies in which there were Christians did not mutiny. It was only the Bengal Sepoys among whom the government had

practically prohibited Christianity. After the Mutiny, accordingly, there were many who argued that the time had come for the government to be openly and avowedly Christian. Some held that the Mutiny was a judgment of God for England's perfidy to Christianity in the past. Among the most outspoken advocates of a Christian policy was Sir Herbert Edwardes. He addressed to Lawrence a long and able "Memorandum on the Elimination of all Unchristian Principles from the Government of India," to which through his secretary Lawrence made a reply of which the following were the concluding paragraphs :

"The various points named for discussion have now been reviewed. Before concluding this letter I am to state that Sir J. Lawrence has been led, in common with others, since the occurrence of the awful events of 1857, to ponder deeply on what may be the faults and shortcomings of the British as a Christian nation in India. In considering topics such as those treated of in this despatch he would solely endeavour to ascertain what is our Christian duty. Having ascertained that according to our erring lights and conscience, he would follow it out to the uttermost, undeterred by any consideration. If we address ourselves to this task, it may, with the blessing of Providence, not prove too difficult for us. *Measures have, indeed, been proposed as essential to be adopted by a Christian government which would be truly difficult or impossible of execution. But on closer consideration, it will be found that such measures are not enjoined by Christianity, but are contrary to its spirit.* Sir John Lawrence does, I am to state, entertain the earnest belief that all those measures which are really and truly Christian can be carried out in India, not only

without danger to British rule, but on the contrary with every advantage to its stability. *Christian things done in a Christian way will never, the chief commissioner is convinced, alienate the heathen. About such things there are qualities which do not provoke nor excite distrust, nor harden to resistance. It is when unchristian things are done in the name of Christianity, or when Christian things are done in an unchristian way, that mischief and danger are occasioned.* The difficulty is, amid the political complications, the conflicting social considerations, the fears and hopes of self-interest which are so apt to mislead human judgment, to discern clearly what is imposed upon us by Christian duty and what is not. Having discerned this, we have but to put it into practice. Sir John Lawrence is satisfied that within the territories committed to his charge he can carry out all those measures which are really matters of Christian duty on the part of the government. And, further, he believes that such measures will arouse no danger; will conciliate instead of provoking; and will subserve the ultimate diffusion of the truth among the people.

“Finally, the chief commissioner would recommend that such measures and policy, having been deliberately determined on by the supreme government, be openly avowed and universally acted upon throughout the empire; so that there may be no diversities of practice, no isolated, tentative, or conflicting efforts, which are, indeed, the surest means of exciting distrust; so that the people may see that we have no sudden or sinister designs; and so that we may exhibit that harmony and uniformity of conduct which befits a Christian nation striving to do its duty.”

Although this reply did not accept Edwardes' po-

sitions, it went far towards doing so. Edwardes wrote of it: "It is a noble expression of the duty of the Indian government to do whatever Christianity requires, at whatever cost; and it only differs from mine as to what Christianity *does* demand of us, and what it does not." In other words, the principles were identical, but the application of them different. Again he wrote, "It is a fine manifesto, and I rejoice to have elicited it." Of a paper by Donald McLeod, with which Lawrence agreed, Edwardes generously said, "I rejoice to have fulfilled the office of a pump, and drawn so much sweet water to the surface."

Lawrence's view was not a negation of any positive change. He was strongly in favour of a positive course of action on the part of the government.

"There is now," he wrote to Trevelyan, "a great dispute growing up as to whether the Bible shall be introduced into our schools or not. I think that it should, and that—provided only it be done with prudence and tact—the people will never raise an objection. All that we have to do is to take care that the study of the Bible be optional with the children."

And to his friend, William Arnold, a son of Dr. Arnold of Rugby, and the able Director of Public Instruction in the Punjab, who took strongly the opposite view, and argued that the Founder of Christianity would Himself have disapproved of the measure, he writes as follows:

"I believe that, provided neither force nor fraud were used, Christ would assuredly approve of the introduction of the Bible. We believe that the Bible is true, that it is the only means of salvation. Surely we should lend our

influence in making it known to our subjects. . . . As a matter of policy, I advocate the introduction of the Bible quite as much as a matter of duty. I believe that, provided we do it wisely and judiciously, the people will gradually read that book. I have reason to suppose this because the missionaries are successful. On the other hand, nothing will more surely conduce to the strength of our power in India than the spread of Christianity. You seem to think that we violate the principles of toleration by attempting to convert the people. I think you might just as well assume that we violate such principles by preferring in a public office a respectable man to a reprobate, a wise man to a fool, and an industrious man to a lazy one. The whole question seems to me to resolve itself into what is the just interpretation of the term toleration. I consider that it means 'forbearance.' That is to say, that we are to bear with and not to persecute mankind for their religious opinions. But this cannot mean that we should not strive by gentle means to lead those in the right way whom we see to be going wrong."

In answer to the strong objection of Arnold, Lawrence wrote a despatch to the Governor-General, curiously omitted from his biography, which is deficient in its knowledge of Lawrence's religious life and activity. In this despatch he said :

"Our government is, as all other governments are, or ought to be, established for the good of the people. But while with other governments the popular will is generally the criterion of the public good, such is not always the case with us in India. If, by being trustees for the people, we are supposed to be bound invariably by the will of the

people, then we are not, the chief commissioner thinks, trustees in that sense. We have not been elected or placed in power by the people, but we are here through our moral superiority, by the force of circumstances, by the will of Providence. This alone constitutes our charter to govern India.

“In doing the best we can for the people, we are bound by our conscience, and not by theirs. Believing that the study of the Bible is fraught with the highest blessings, we, of course, do desire to communicate those blessings to them if we can. We desire this not only as individuals, but as a government; for Christianity does truly go hand in hand with all those subjects for which British rule exists in India. But this can only be effected by moral influences voluntarily received. Anything like ‘proselytism’ or ‘quiet persecution’ of any kind, or the application of secular motives, direct or indirect, is, in the first place, absolutely forbidden by the very religion we profess, and, in the second place, would be worse than useless for the object in view.

“Therefore, we have nothing to do with such means. Nor do we, as a government, undertake to found and maintain Christian missions, because the thing can be done better by private effort, and because our doing so might tend to introduce those secular means for propagation of Christianity which we wish to avoid. But, as we have schools, there arises a fair opportunity of offering the Bible to those who may choose to receive it; and, in the chief commissioner’s opinion, it is just, politic, and right that we should avail ourselves of that opportunity. Such, briefly stated, is the real argument for the formation of Bible classes in government schools.

“To say that we have no right to offer Christian teach-

ing to government schools because we do not allow the native religions to be taught there, is to misapprehend the fundamental relation that in this country subsists between the government and the people. We are to do the best we can for them, according to our lights; and they are to obey us. Mr. Arnold writes: 'What answer am I to give to Hindus and Mohammedans if they say that, after having excluded their religions, I have introduced my own? Shall I say that I am master, that I am the officer of a conquering government, and will do as I please?' That answer would indeed be arbitrary. The proper answer would be thus: 'We offer you the Bible in our government schools because we believe it to be for your inestimable good, if you choose to listen to it. We do not wish you to study it unless you do so voluntarily. But you cannot expect us to help in teaching your religion, which we do not believe to be true. That you can do for yourselves.' " ¹

Lawrence never abandoned this view. Let those who oppose it be sure that they can speak with Lawrence's authority, his knowledge of the people and his honest Christian character. It is a very different attitude of mind from that of General Kitchener when he forbade missionary work in the Soudan, and proposed that the Gordon Memorial College should be absolutely non-Christian. Well might the House of Laymen of the province of Canterbury reply to such proposals:

"That this House, while welcoming the noble effort now making to elevate and instruct the people of the Soudan and Upper Egypt through the means of the Gordon Col-

¹ "History of the Church Missionary Society," Vol. II, p. 250 f.

lege at Khartoum, is nevertheless of opinion that no effort to perpetuate the memory of General Gordon can be considered adequate which does not include the direct proclamation of the Gospel of Jesus Christ to all the races inhabiting the upper basin of the Nile, which has recently been brought under the control of England. They would express their earnest hope that at the earliest moment consistent with public safety the government of the Soudan will remove the restrictions at present existing upon the entrance of missionaries to Khartoum."

In the course of the debate over this proposal, the duty of the Church of Christ to preach the Gospel to every creature, including Mohammedans, was insisted on by Lord Cranborne, eldest son of Lord Salisbury, with the eloquence of genuine conviction. And Sir Richard Temple, one of the last representatives of the Punjab school of Christian officers and statesmen, expressed his surprise that any British ruler should put any obstacle in the way of a Christian mission. "I cannot understand it," he said. The policy of Lawrence and Edwardes and Montgomery, who so soon as cities like Peshawar and Lucknow were occupied, Mohammedan cities then as dangerous as Khartoum now, encouraged missionaries to come in and preach openly to the Moslem population, was a nobler and higher policy than that now enunciated. But it needs a man of rare faith and Christian experience to adopt such a policy and act upon it fearlessly; and judging from the lower standpoint of the average Christianity of England, the caution of Lord Kitchener in present circumstances cannot be pronounced unreasonable. Never-

theless, Herbert Edwardes' never-to-be-forgotten words remain supremely true: "*Above all, we may be quite sure that we are much safer if we do our duty (to proclaim Christ as the Saviour) than if we neglect it; and that He who has brought us here, with His own right arm, will shield and bless us, if, in simple reliance upon Him, we try to do His will.*" That also was John Lawrence's view.

3. It remains, accordingly, to say just a word upon his attitude and relation to missions. In this matter, as in his Christian character, he undoubtedly learned and grew with time. The first reference to missionary work in his biography occurs during his stay at Etawa.

"They (the people) had often heard me laugh at different absurdities of their religion, on which occasions I had reasoned with them, but in vain. 'No, no,' they said; 'you English are very wise, we will allow, but you do not understand our religion.' In fact, as far as my experience goes, time and labour are utterly lost in such discussions. The only way that will ever bring the natives to truer and more enlightened ideas is the gradual progress of infant education. The attempts to change the faith of the adult population have hitherto failed, and will, I am afraid, continue to fail."

On his first furlough, Caroline Fox reports in her diary that in a talk about India,

"Lawrence spoke of the stationary kind of progress which Christianity was making amongst them. When a native embraces this new creed, he retains his old in-

veterate prejudices and superadds only the liberty of the new faith. This Lawrence has repeatedly proved, so much so that he would on no account take one of these converts into his service. All his hope is in the education of the children, who are bright and intelligent. The Indians will, from politeness, believe all you tell them, and if you speak of any of Christ's miracles, they make no difficulty, but directly detail one more marvellous of which Mohammed was the author, and expect your civility of credence to keep pace with theirs. If you try to convince them of any absurdities or inconsistencies in the Koran, they stop you with, 'Do you think that such an one as I should presume to understand it?' "

But in later years he had grown far beyond this superficial view. At the Lahore Durbar the one prince who received the order of the Star of India was a Christian, the Rajah of Kaparthala. Lawrence exonerated missionaries from any blame for the Mutiny, in reply to arguments which preceded by half a century the arguments which laid the Boxer uprising at the door of missionaries. He always spoke of missionaries with respect. When he was Viceroy he invited a Moravian missionary to come and visit him at Simla. "You have given us life and health," the man said as he went away loaded with presents. Lawrence was not a neutral in any sense. He believed in toleration not neutrality on the part of the government, and he believed in earnest aggression on the part of the Churches. When he returned to England he became a member of the Committee of the Church Missionary Society, and at a meeting of the Wesleyan Missionary Society he declared :

"I believe, notwithstanding all that the English people have done to benefit that country, the missionaries have done more than all other agencies combined. They have had arduous and up-hill work, often receiving no encouragement, and sometimes a great deal of discouragement from their own countrymen, and have had to bear the taunts and obloquy of those who despised and disliked their preaching; but such has been the effect of their earnest zeal, untiring devotion, and of the excellent example which they have, I may say, universally shown to the people, that I have no doubt whatever that, in spite of the great masses of the people being intensely opposed to their doctrine, they are, as a body, remarkably popular in the country. . . . It seems to me that, year by year and cycle by cycle, the influence of these missionaries must increase, and that, in God's good-will, the time may be expected to come when large masses of the people, having lost all faith in their own, and feeling the want of a religion which is pure and true and holy, will be converted and profess the Christian religion, and having professed it, live in accordance with its precepts. . . . I have a great reverence and regard for them (the missionaries) both personally, and for the sake of the great cause in which they are engaged; and I feel it to be a pleasure and a privilege to do anything I can in the last years of my life to further the great work for which they have done so much."

And Lawrence saw nothing wrong in such a position. His firm conviction was that the men best served Christian governments who were most Christian in their own lives, and who most unfalteringly supported the religion in which they and their fa-

thers believed, and the event showed that he was right. Lord Northbrook once emphatically expressed his opinion that the natives of India respect an Englishman who is not ashamed of his religion, and have no fear of his infringing the principles of religious equality

“I believe that they do not honour a man the less, or love him the less, because they see that he is in earnest in his own religious convictions. If proof were wanted of this, it would be sufficient to recall to your recollection that some of the noblest deeds that had been done in British India had been done not only by earnest Christian men who never for a moment concealed their zeal in favour of the spread of Christianity. It is only necessary to mention the names of Herbert Edwardes and Lord Lawrence to prove that what I have said is true. I will add one thing more. Among those whom I have known in high office in India there are none who have so conciliated the respect and affection of the people of India as those very men who have never concealed their desire to extend the Christian religion. I will mention Sir Donald McLeod, Lieutenant-Governor of the Punjab, Sir William Muir, who is now present, and my friend Sir Richard Temple, now Governor of Bombay, and one of the men who was foremost on all occasions to join in everything which he considered to be of advantage to the welfare of the Church of England, and the spread of the Christian religion in India.”¹

And then he pronounced a hearty eulogy on the missionaries. “They are worthy,” he said, “of all support, encouragement, and admiration.”

¹ “History of the Church Missionary Society,” Vol. III, p. 70.

When will the day come that we may hope to have such men representing our nation abroad and their spirit ruling our public life? Are we sure that we ourselves are such men, and that our religious profession and Christian service rest upon as great a reality of faith and character as all men felt in the warm heart and the granite will of John, first Lord Lawrence of the Punjab?

LECTURE VI

CHARLES GEORGE GORDON, THE
CHRISTIAN KNIGHT ERRANT AND
THE POWER OF PURE DEVOTION

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FOUR of the five men whom we have thus far considered were professionally related to the missionary cause. The other one of the five and the one whom we are to consider now belonged to a class of men who have done as much as many missionaries have been able to do, soldiers, administrators and diplomatists, who have themselves followed Christ, who have done their work in His Spirit and lived their lives by His principles, and have advanced the mission cause through their open sympathy with its aims and their cordial support of its enterprises, and not less through their recognition of the missionary ideal as the right ideal in all contact, political and commercial as well as religious, between the advanced and the backward peoples. A sad proportion of the secular representatives of the West have not belonged to this class. A steady stream of merchants and soldiers and political agents has moved across Asia and Africa and South America, and the men in it who have lived the aggressive Christian life, and sought to advance the Christian cause have been the honourable minority.

In this minority, however, have been the greatest

men of their time, and they have been great by their Christian character and Christian purpose not less than by their governing genius, their military power or their diplomatic skill. I have spoken of one of these and am now to speak of another, Charles George Gordon, the most heroic character of his day. "There was no figure during our generation," said Lord Salisbury, "to which the popular feeling and sympathy were so attached." "The most restless spirit of the nineteenth century," Sir William Butler calls him,— "the bravest, the purest and the most truthful Englishman of his time." "The marvelous career is now ended," said the *London Times*, on the news of his death. "The life is over. . . . That life has done much for this generation. It has served conspicuously to remind us that the age of chivalry is not dead; that chivalry in the highest sense is rare, indeed, but that its influence is as great and as far reaching as of old. . . . If we cannot but deeply mourn the untimely end of so much genius and so much devotion, it is some consolation to feel that Gordon's heroic death has lifted him to a height of glory which renders him the most conspicuous Englishman of our time." He and Charles Darwin, Mr. Huxley said, were the two greatest men he had ever met. There was about him an unselfishness that was more than human, and a faith lived in him which even Huxley wished he might share. And above all these tributes and explaining them, he was, Mr. Lawrence Olyphant wrote of him to his sister, "the most Christlike man I ever knew."

We are more concerned in these studies with the character and principles of men than with the in-

cidents of their life, but it is the latter which give us our understanding of the former, and Gordon's life was a knight errant's if ever a knight errant lived.

He was born at Woolwich, January 28, 1833. For a hundred and fifty years his fathers had been fighting men. His Highlander great grandfather had been taken prisoner at Prestonpans, and his military soul, as well as his independent will, came to him out of his father's loins, who "spent a long life in the service, and like his son, was less fitted to obey than to command. More than once, well as he knew the value of discipline, it was his to resist his superiors, and to protest against dictates which he would hold to be superfluous and unjust."¹ His mother was an Enderby, whose father, Samuel Enderby, owned whaling ships which wandered everywhere over the world. Their owner's name is left on remote lands, and two of them were the vessels from which the tea was thrown overboard in Boston Harbour. The lad came naturally by the disposition that made him easy in any land. He never needed a long preparation to move.

He was educated at Taunton and at the Royal Military Academy at Woolwich. He had a temper and a free mind then as always. "Once during his cadetship at the academy he was rebuked for incompetence, and told that he would never make an officer; whereupon he tore the epaulets from his shoulders and flung them at his superior's feet." Not a nice story by itself. And his father knew the explosives wrapped up in the boy. "While he is

¹ Hake, "The Story of Chinese Gordon," p. 7.

in the academy," he said, "I feel I am like one sitting on a powder barrel." "Then after Woolwich," says General Butler, "came Chatham, where the young officer was still further to learn the lessons of the Master of the Art of Defense, so great in greater things than siege or safe-keeping of cities. Perhaps in the library of the barracks or on the old lines of Brompton the young sapper read the lesson that Vauban has left in his letter to Louvois—a lesson of deeper meaning in the safe-keeping of the citadel of self than any of his famous 'systems.' 'For I dare plainly to tell you,' wrote the veteran (marshal of France at last after his fifty-three sieges) to the all-powerful minister, 'that in a question of strictest honesty and sincere fidelity I fear neither the king, nor you, nor all the human race together. Fortune had me born the poorest gentleman in France, but in requital she honoured me with an honest heart, so free from all sorts of swindles that it cannot bear even the thought of them without a shudder.'"¹

In 1852 he got his commission as a second lieutenant of engineers. At the outbreak of the Crimean War he was in garrison at Pembroke Dock employed in drawing plans for the forts and defenses of the haven, but at the end of 1854 he was ordered to Sebastopol, and arrived at Balaklava on New Year's Day, 1855. This was his first schooling in war, but he had already achieved such victory over himself, as Milton said of Cromwell, that he was a practiced warrior before ever he came to the battle-field. "One day in going the round of the trenches, he heard a corporal and a sapper of engineers in violent alterca-

¹ Sir William Butler, "Gordon," p. 14.

tion. He stopped to ask what was the matter, when he was told that the men were engaged placing some fresh gabions in the battery, and that the corporal had ordered the sapper to stand up on the parapet, where he was exposed to the enemy's fire, while the former, in the full shelter of the battery, handed the baskets up to him. Gordon at once jumped up to the parapet, ordering the corporal to join him, while the sapper handed them the gabions. When the work was done, and done under the fire of the watchful Russian gunners, Gordon turned to the corporal and said, 'Never order a man to do anything you are afraid to do yourself.' " ¹ He already had become a gravely religious man who did not believe in chance, and his prescience as well as his utter fearlessness was discerned. "He attracted the notice of his superiors," said Colonel Chesney, "not merely by his energy and activity, but by a special aptitude for war; developing itself, amid the trench-work before Sebastopol, in a personal knowledge of the enemy's movements, such as no other officer attained. We used to send him to find out what new move the Russians were making."

After the Crimean War he was occupied for two years on boundary commissions in Bessarabia and Armenia and the Caucasus. In 1859 he was stationed at Chatham as Field Work Instructor and Adjutant, and in 1860 was sent to North China, to join the British force then engaged with the French in war with China. He was present at the wanton destruction of the Summer Palace upon the capture of Peking. Those who speak of missionary loot in

¹ Macaulay, "Gordon Anecdotes," p. 291.

connection with the capture of Peking in 1900, and who think the missionary disregardful of the traditions and treasures of the Asiatic people, should read Gordon's description of this vandalism.

This poor war over, Gordon was occupied at Shanghai in a military survey of the surrounding country, and from that work was loaned by the British government to China to take command of the "Ever-victorious Army," of foreign adventurers and Chinese, which had been organized by an American named Ward who had been killed, and indeed to assume direction of the independent Chinese force endeavouring to suppress the Taiping Rebellion. The biographies of Gordon describe the Taipings in the blackest colours, and account for the monster movement by the most fantastic explanations, unless, like Sir W. Butler, they pass the matter over by stating, which is not the fact, that its origin "lies still in oblivion." There is wide difference of opinion as to whether it would have been desirable to have the Taipings overthrow the present Manchu dynasty. The general judgment of students is against the Taipings. Gordon, who had no personal knowledge of their leaders and no acquaintance with China, its history and institutions, and none but the Shanghai highly interested view of the Taipings as disturbers of commerce, and who saw that the rebellion had dragged on for years, and that it might go on for years to come, felt that he was justified in endeavouring to suppress the disorder and reestablish the authority of the central government. As he accepted the position offered to him, he wrote home, "I have taken the step in consideration. I think

that any one who contributes to putting down this rebellion fulfills a humane task, and I also think tends a great deal to open China to civilization." Perhaps if Gordon had known Hung Siu-tsuen and the rebellion in its early years, and had seen the better elements in its leaders and its principles, his knight errant spirit might have led him to feel that he fulfilled a humane task who let the movement alone, or who sought to correct and direct rather than to suppress it. Griffith-John, who knew the movement well, told me once that he deemed this the great error of Gordon's career.

But he acted as always on the impulse of his conscience, and the four years which he spent in China in the repression of the rebellion constitute one of the most wonderful stories of military genius, personal power and high-minded unselfishness and courage of which the world knows. In summing up the lessons of his service in China, Sir William Butler says :

"Indomitable resolution, inexhaustible resource, sleepless activity, were his master qualities. The distrust of English leadership, which previous experience or prejudice had engendered in the minds of his officers and men, soon gave place to complete confidence, and, no matter how difficult the labour or how stubborn the resistance, the belief in the final triumph of the undertaking which their leader had conceived never wavered. The indirect advantages to China which the presence of Gordon as a leader gave were scarcely less important to the cause of order. The scandal of the sale of English munitions of war to the rebels received a check, and although it would have been too much to expect that a commercial activity

which had for its chief commodity the sale of a slow-consuming poison would have drawn the line of conscience strictly at any more sudden form of death, still there can be no doubt that the open supply of the rebels with arms and ammunition became, in the latter stage of the Taiping war, at least an unfashionable form of commerce. But the lesson of this brilliant episode was neither to China nor to Gordon, it was to us. China might well be trusted to go on its way for two thousand years. Gordon knew himself and his qualities before he ever pointed with bamboo cane the road of victory to his army, but to us the revelation was given that there was among us a man gifted with all the faculties of true kingship, a leader of men and a master, a veritable 'spear,' sharp and swift and self-wielding, and with a flash in its steel that could kindle the light of battle in ten thousand duller weapons. It was but a few years since, for the want of such a leader, that we had lost everything but honour on the plateau above Sebastopol, and now, having found this captain, it was for us to guard him with safe-keeping, to hold him for the evil day as the most precious of our treasures. Assuredly it was from no plethora of such possession that we could afford to waste or spend him in distant outside enterprise."

From China he returned to England, and from 1865 to 1871 was engaged at Gravesend in the direction of works connected with the defense of the Thames. It was a time of panic in England, worked up by the imaginary fear of French invasion. It was a time also of transition in military science, and defenses of one year were antiquated the next. Immense sums of money were being spent and wasted and Gordon chafed under it bitterly. The six years at Gravesend

were, however, as he said, among the happiest of his life.

“To the world, his life at Gravesend,” says Mr. Hake, one of his biographers, “was a life of self-suppression and self-denial; to himself it was one of happiness and pure peace. He lived wholly for others. His house was school and hospital and almshouse in turn, and was more like the abode of a missionary than of a colonel of engineers. . . . He always took a great delight in children, but especially in boys employed on the river or the sea. Many he rescued from the gutter; cleansed them and clothed them, and kept them for weeks in his home. For their benefit he established evening classes, over which he himself presided, reading to and teaching the lads with as much ardour as if he were leading them to victory. He called them his ‘kings,’ and for many of them he got berths on board ship. One day a friend asked him why there were so many pins stuck into the map of the world over his mantelpiece; he was told that they marked and followed the course of the boys on their voyages—that they were moved from point to point as his youngsters advanced, and that he prayed for them as they went, day by day.”

“His benevolence embraced all,” writes one who saw much of him at this time. “Misery was quite sufficient claim for him. . . .

“We used to say he had no *self*—in that following his Divine Master. He would never talk of himself and his doings. Therefore his life never can and never will be written.

"It was in these years that the first book about him came out. He allowed the author to come and stay at Fort House, and gave him every facility towards bringing out his book (all the particulars about the Taiping Rebellion) even to lending him his diary. Then, from something that was said, he discovered that personal acts of his own (bravery, possibly) were described, and he asked to see what had been written. Then he tore out page after page—the parts about himself—to the poor author's chagrin, who told him he had spoiled his book. I tried to get at the bottom of this feeling of his, telling him he might be justly proud of these things; but was answered that no man has a right to be proud of anything, inasmuch as he has no native good in him—he has received it all."

It would be easy to gather many anecdotes about these days. The great Christian was unfolding his own rare and original reality here and preparing, not for greater, but for more conspicuous service. Indeed, it is here really that his wonderful letters to his sister begin; not one is presented from the Taiping war period of his life. He himself never was deluded by confusion as to small and great. "Napoleon," he writes to his sister, "in a book lent me by Watson, says, 'The smallest trifles produce the greatest results.'" Again he writes to her, "In 'Gold Dust' is this paragraph, 'May I pass through this world unnoticed.'"

From the unnoticed years at Gravesend he went out in 1871 as the English member of the International Commission for regulating the navigation of the mouths of the Danube. On this service he met

Nubar Pasha, the ablest of the ministers of the Khedive Ismail who was at once attracted to the man and secured his appointment as Governor of the Equatorial Provinces of Egypt, succeeding Sir Samuel Baker. Gordon accepted, refusing to take more than one-fifth the salary of his predecessor. To his office as Governor of the Equatorial Provinces was added later the charge of the Soudan. Here Gordon's career was more wonderful even than in China and the magic of the man's personality worked like a spell over the turbulence of the land.

"His rule in the Soudan was glorious to himself, satisfactory even to the Khedive and gratifying to Englishmen, as a practical demonstration of the qualities which they must wish to see most common among their countrymen. . . .

"When at Khartoum he was on one occasion installed with a royal salute and an address was presented, and in return he was expected to make a speech. His speech was as follows: 'With the help of God, I will hold the balance level.' The people were delighted, for a level balance was to them an unknown boon. And he held it level all through his long and glorious reign, which lasted, with one small break, from February, 1874, until August, 1879."

The man's great work during the governorship was a work of personal inspiration, of the taming of men by love and trust and justice. He collided with falsehood and selfishness at every step and hated at times the very contact with civilization because it meant fresh warfare between his principles and those of schemers and self-seekers and conventionalists who were not serving a living God and His world.

Mr. Hake is not too enthusiastic in his description of Gordon's work, written while Gordon was living in Palestine.

“The work he had begun and was bent on finishing was fraught with peculiar perils. It demanded a tact, an energy, and a force of will almost superhuman. He had to deal not only with worthless and often mutinous governors of provinces, but with wild and desperate tribesmen as well; he had to disband 6,000 Bashi-Bazouks, who were used as frontier guards, but who winked at slave-hunting and robbed the tribes on their own account; he had to subdue and bring to order and rule the vast province of the Bahr Gazelle, but now beneath the sway of the great slaver Sebehr. It was a stupendous task: to give peace to a country quick with war; to suppress slavery among a people to whom the trade in human flesh was life and honour and fortune; to make an army out of perhaps the worst material ever seen; to grow a flourishing trade and a fair revenue in the wildest anarchy in the world. The immensity of the undertaking; the infinity of details involved in a single step towards the end; the countless odds to be faced; the many pests—the deadly climate, the horrible vermin, the ghastly itch, the nightly and daily alternation of overpowering heat and bitter cold—to be endured and overcome; the environment of bestial savagery and ruthless fanaticism—all these combine to make the achievement unique in human history. . . . Like the adventurer in Browning's magnificent allegory, he had everything against him, and he was utterly alone, but he stood for God and the right, and he would not flinch. There stood the tower of evil—the grim ruined land, the awful presences, the hopeless task, the anarchy

of wickedness and despair and wrath. He knew, he felt, he recognized it all; and yet—

“ And yet
Dauntless the stag-horn to my lips I set
And blew: *Childe Roland to the Dark Tower Came.*”

Undismayed, Gordon did his work, and then when he felt that he had done it and that he had got as far as the opposition of the world with which he was always fighting would allow, he laid it down.

In 1880 he was in Ireland and there his restless, sympathetic, penetrating mind devised his remedy for the Irish problem; but neither his remedy, nor what was more valuable, the man, was employed on that six century old entanglement, and the next year receiving no appointment in his own profession and chafing at being unoccupied, he accepted the post of private secretary to Lord Ripon, the new Governor-General of India. Before he reached India he had resigned, not because he felt it was a humiliation to descend from a Governor-Generalship to be a private secretary, but because he realized he could never stand what he deemed the horrors of British life in India.

Not once, but many times in his life, Gordon made changes like this. During the Taiping Rebellion he once resigned his position in justified anger and disgust, but reconsidered. In the Soudan he resigned his place and reconsidered his resignation. And now he left England as private secretary to the Indian Governor-General and resigned in Bombay. This was not vacillation. It was the ingenuous child nature at work in him. It was response to the sen-

sitive discernment of duty plus that moral courage which, whatever he says, he never lacked. He was not the sort who feared to move, to change, lest he should lose his bread and butter. When he heard his call or saw his gleam he rose up straightway and followed. To those people whose ethics are derived from their daily bread he was a very fanciful man.

The next three years Gordon spent in China advising the Chinese government not to go to war with Russia and what to do with its armies to prepare for war; in Mauritius, where he went because a fellow officer assigned there did not want to go; in South Africa where he seemed utterly impracticable to the Cape Colony officials because he was so fantastically upright and honest and just in his dealings with the Basutos, and in the Holy Land where with his Bible in his hand he studied the holy places and meditated upon the truth of God, and on his return to England he was asked by the King of the Belgians to go to the Congo as Governor of the territory occupied by the International Association. He accepted this call and went to Brussels to receive final instructions. A friend has told me of meeting him there at the time, a little, quiet, very quick and living man. How different the history of the Congo Free State would have been if he had gone, but he was called back to London to consider the proposition that he should return to the Soudan. After his preceding service there Raouf Pasha had been appointed Governor and the old abuses had returned. Corruption in government, connivance at slavery, incapacity and cruelty on one hand and the Mahdi uprising on the other had made a hell of the Soudan. Either it would

have to be subdued at great expense of money and life or it would have to be abandoned. The British government chose the latter course and selected Gordon to go up and withdraw the garrisons, bring down to Egypt all who wished to come, and leave the land to the old ways. But this was easier said than done. What were its old ways? Was it to be abandoned to the Mahdi who was making anarchy, or to be returned to the old chiefs who had ruled before Egypt had taken over the country? There is no use, however, in entering the controversy as to whether Gordon disobeyed his instructions. Lord Cromer has stated the case against him. But something more can be said. It is argued on the other side that there was sufficient disagreement and conflict between Egypt and England, and inside the British government, between the permanent departmental government and the temporary liberal ministerial government to account for the pulling and backing. The firm, true man at Khartoum was going to do his duty as he saw it, whatever his orders might be. Whether he disobeyed his instructions or not, he was entirely capable of doing so, and would have done so in an instant if he had thought that God and the Soudan required him to do so, and he would have taken the consequences, as he would have put it, "without a twitter."

The end, as every one knows, was that he was killed in Khartoum on January 26th, three days before the relief expedition reached the city. He was the only Englishman left. He had already come to feel that there would have to be some such atonement made for the sins of England against Egypt and the

Soudan. "I feel that all these wrongs can only be washed out in blood," he wrote from Jerusalem in 1883, and after reaching Khartoum he wrote on March 4, 1884: "May our Lord not visit us as a nation for our sins, but may His wrath fall on me, hid in Christ. This is my frequent prayer, and may He spare these people, and bring them to peace." And beyond this he felt that he must bear the sins of the poor Soudanese, too. Four days after this last letter he wrote to some friends in Jerusalem:

"I am ever interceding for you and yours; for when Job (xlii. 10) prayed for his friends, God turned his captivity. Make little Julia ask our Lord to help me. In vain, indeed, is the help of man. Yet I have dared to ask that the sins of these peoples fall on me, *hid* in Christ. . . . Good-bye. Many thanks to you and Mr. — for your prayers."

His last letter to his sister was dated December 14, 1884:

"This may be the last letter you will receive from me, for we are on our last legs, owing to the delay of the expedition. However, God rules all, and, as He will rule to His glory and our welfare, His will be done. I fear, owing to circumstances, that my affairs pecuniarily are not over-bright."

And so "the purest Englishman of his time," "the most restless spirit of the nineteenth century" died alone; save that God was with him.

This hasty sketch of Gordon's life has already

indicated some of his qualities of character. It has suggested that he was a religious man. I say "suggested," because no account of the man could do full justice to the depth and earnestness and permeating reality of his religious faith. The best expression of it is in his "Letters to His Sister." He himself was very fond of à Kempis' "Imitation of Christ," and in one of his last letters thanks his sister for some copies she had ordered for him. But for us his own letters are as good and stimulating a manual of devotion as à Kempis. These published letters are almost altogether on religion. They show no perfunctory and conventional religious life but a man of fearless and original spiritual experience.

He calls himself, quite frankly, "a religious fanatic." Religion was real to him: "To be like Christ: what a deal it means, and how very feebly the mass of the world realize it! . . . What an unsatisfactory religion it is that takes God's promises as mere talk!" On his road once to the slave den at Shaka he wrote: "I do not believe in you all. . . . The Christianity of the mass is a vapid, tasteless thing, and of no use to anybody. The people of England care more for their dinners than they do for anything else, and you may depend on it, it is only an active few whom God pushes on to take an interest in the slave question. 'It is very shocking! Will you take some more salmon?'"

His religion was the absorbing thing in life. He believed in God the Father Almighty and in Jesus Christ His only Son our Lord. And this faith was so complete that many regarded him as a fatalist and indeed he never sought to tone down his language.

He believed in the sovereignty of God and the war between soul and flesh.

“Man’s happiness,” he wrote to his sister, “consists in present peace even in the midst of the greatest trials, and in more than hope of a glorious future. It comes by trust in the Lord Jesus Christ, by realizing that His atonement, as Head, suffices for the members of His body, and cannot be cancelled by any acts or affected by any merit of theirs, and that it is a finished work for the past as well as for the future. This being the foundation of peaceful happiness, it is experienced according as the sovereignty of God is acknowledged *in everything* even our sins.”

Elsewhere he writes to her,—and it would be easy to find illogical and contradictory things in his theology :

“I do not believe in man’s free will ; therefore, if my actions are right, they are His actions ; if evil, they are the inevitable produce of the corrupt body in which I am placed by Him.

“My opinion is that the Brussels Conference is doomed to fail. It is too mighty for God to use in His work. He never has done any of His great works by great men.

“Those who hold by man’s free will must consequently be more or less elated if they do well. Now, if you accepted what I think is the truth, namely, that man has no free will, you would never be elated, for you would not arrogate to yourself your actions ; neither would you be depressed by your evil acts. To the one you would say, ‘Thanks to God for that’ ; and to the other, ‘This is nothing more than the outcomings of my corrupt nature.’ Paul says this in his ‘*Wretched man that I am !*’ I feel

sure no one can be happy till he has come to this knowledge.

"You and I are flies on the wheel; try and realize this, that you do not move the wheel." ¹

And again from the Soudan in 1875 :

"The moral I draw . . . is that my earthly shrine is ordained to certain evil works, and my spiritual shrine is ordained to certain good works; that neither in one case nor the other can I alter the decree." ²

And the same year :

"I consider we have not the least ground for supposing our clever thoughts are from ourselves, and, in spite of my feelings, I think we are in reality only spectators of the unrolling of the record of events." ³

He loved especially, of course, to dwell on the strengthening conviction that God was ruling things, and that he needed only to believe on Him as his "Governor-General."

"No man ever had a harder task than I, unaided, have before me," he said during his first period in the Soudan, "but it sets as a feather on me. As Solomon asked, I ask wisdom to govern this great people; and not only will He give me it, but all else besides. And why? Because I value not the 'all besides.' I am quite alone and like it. I have become what people call a great fatalist, viz. : I trust God will pull me through every difficulty. The solitary grandeur of the desert makes one feel how vain is the

¹ "Letters to His Sister," p. 114 f. ² *Ibid.*, p. 85. ³ *Ibid.*, p. 85.

effort of man. This carries me through my troubles, and enables me to look on death as a coming relief, when it is His will. . . . It is only my firm conviction that I am only an instrument put in use for a time that enables me to bear up; and in my present state, during my long, hot, weary rides, I think my thoughts better and clearer than I should with a companion.”¹

On the way to the Soudan he had written to his sister :

“Even really religious people are more or less infidels. . . . I have a Bank, and on that I can draw; He is richer than the Khedive, and knows more of the country than any one; I will trust Him to help me out of money or any other difficulties.”²

His rest on God did not make him inactive. His notion was that we could not say God had ordained until the thing was done.

“It is a delightful thing to be a fatalist, not as that word is generally employed, but to accept that, when things *happen* and *not* before, God has for some wise reason so ordained them to happen—*all* things, not only the great things, but all the circumstances of life; that is what is meant to me by the words ‘you are dead’ in St. Paul to Colossians.” Again: “We have nothing further to do when the scroll of events is unrolled than to accept them as being for the best. *Before* it is *unrolled* it is another matter; and you could not say I sat still and let things happen with this belief. All I can say is, that amidst troubles and

¹ Hake, “The Story of Chinese Gordon,” p. 403 f.

² “Letters to His Sister,” p. 62 f.

worries no one can have peace till he thus stays upon his God; it gives a man a superhuman strength.

"Everything that happens to-day, good or evil, is settled and fixed, and it is no use fretting over it. The quiet, peaceful life of our Lord was solely due to His submission to God's will. There will be times when a strain will come on one; and as the strain, so will your strength be."¹

And it was strength and action to Gordon.

"The people were amazed by his daring, his firmness, his irresistible energy. To tell a lazy functionary that if he did not get on with his work the Governor-General would be after him, was better than the whip itself. Everywhere the cry, 'The Pasha is coming,' became a signal for action. At such a pace did he traverse the continent he ruled that his camels, which under another rider could have gone for ten days, gave in at the sixth. More than once, when the sun was at its fiercest, they dropped dead beneath him. When this happened, he took a new mount and rode on."²

This faith, or fatalism, as you please, delivered him from the fear of death or other harm. "I would that all could look on death as a cheerful friend who takes us from a world of trial to our real home." He once told Huxley that death would only be an incident to him, that it would simply mean a larger government to administer.

"I wish, I wish," he wrote from the Soudan in 1878, "the King would come again and put things

¹ Hake, "The Story of Chinese Gordon," p. 404f.

² *Ibid.*, p. 326.

right on earth ; but His coming is far off, for the whole world must long for Him ere He comes, and I really believe that there are but very, very few who would wish Him to appear, for to do so is to desire death, and how few do this ! Not that we really ever die ; we only change our sheaths.”¹ So he carried no weapons. He did shoot men at times with his own hand, but his idea of warrior’s work was not destruction but constructive government, and he would not have men killed if it could be helped. He constantly released those whom ordinary principles of warfare would have removed. In China he simply carried a little wand of bamboo in his hand and when he went to the Soudan in 1884 among his first acts were the removal of the whips and the release of the prisoners. He governed by loftier means. “I do not carry arms,” he wrote in his playful way from Africa, “as I ought to do, for my whole attention is devoted to defending the nape of my neck from mosquitoes.”

But if he did not carry weapons he did carry his Bible.

“The chief proof, after all,” he said, “that the Bible is good food is the eating of it ; the healing efficacy of a medicine, when it is used, is a demonstration that it is good. I believe the origin of evil is disclosed in the Bible, and I have notes on it but it is not yet clear to me. ‘He that is of God heareth God’s words : ye therefore hear *them* not, because ye are not of God’ (John vii. 47). I like my religion ; it is a greatcoat to me.”²

“Why then,” he writes in a letter from Mauritius in 1881, “do we not progress ? It is because we do not look

¹ “Letters to His Sister,” p. 135.

² *Ibid.*, p. xii.

for the answer to our prayers—we pray and leave it ; we read the Scriptures and do not follow the connection between prayer and this reading, between our request and its answer ; so we are barren, lame and dull. Now God has opened my eyes to the truth, that it is by the Scriptures that He will speak to man, and *rarely* will He speak in any other way ; I would almost say *never*. Any one who asked a favour or help from a fellow man, and who, through ignorance or carelessness, did not trouble himself to listen to the answer, would be considered wanting in worldly wisdom.”¹

And his letters are full of comments on the Bible and of exegesis, which, however fanciful, at least was authenticated by his own life. Fanciful exegesis lived in the life is better than the sound exegesis of spiritual death. The Bible used at Gravesend, on the Danube, and in his first stay in the Soudan, was presented by his sister to Queen Victoria, who wrote in the warmest gratitude in reply. It was a well-worn book. Gordon found time to read his Bible. As he said :

“There is a material, actual study of the Scriptures necessary in order to know them, which we cannot have if we do not give the time to such actual study, which few of us do.”²

And sixteen years before he had written to her :

“Make Him your Guide ; you do not want any other. He has said, ‘I will teach you all things’ ; and, depend on it, you will find it the shortest course to pursue.

¹ “Letters to His Sister,” p. 177 f.

² *Ibid.*, p. 266.

"You say you have little time to read ; you have from six to eight every morning. I own it is not pleasant to flesh and blood ; but, if this trouble is much, the corresponding growth in grace is far greater. We must not deceive ourselves ; we have plenty and plenty of time during the day for ourselves. If we aspire to walk in the power of the new life, we must cast away all hindrances, and it must cost something we really value.

"I own it is dull when we do not feel much in common with those we pray for, but after a time it will grow into love, and at any rate it is honouring to God and keeps us from thinking of things of no import ; it also tends to make us less selfish. Take the Holy Spirit for your teacher, and you will never want another word from man on questions of doctrine." ¹

He had no high opinion of systematic theology.

"I think the veil is thickened," he said, "by the doctrines of men, and that to rend it is more difficult when these doctrines have been accepted and found inefficient. Had you not been imbued with them—had God not willed it in His wisdom—you would not have had such suffering in learning the truth.

"I believe when we begin life we are far more capable of accepting those truths than afterwards ; when we have imbibed man's doctrine we must unlearn and then learn again—a child has only to learn. . . .

"I feel sure that no study without trial is of avail ; life must be lived to learn these truths. I believe, if a man knows his Bible fairly and then goes forth into the world, God will show him His works." ²

¹ "Letters to His Sister," p. 14 f.

² *Ibid.*, pp. 95, 97.

I think we begin to see that this was a rare sort of warrior and Governor-General. We have never known much of this sort of Governor-General. We had such warriors in Stonewall Jackson and Robert E. Lee and O. O. Howard and S. C. Armstrong. There was a good deal of Gordon's quality in General Armstrong, more "patient continuance in well doing," and as much of the child faith.

Gordon was both a mystic and a man of prayer. From the Cape he wrote in 1882 :

"How different things look now that we see that union with God is rest, disunion is unrest ! that is the whole secret ; it is not what we do or leave undone, it is not this or that sin, all is summed up in *union or disunion*." ¹

From Jerusalem in 1883 :

"There is no doubt, I think, that very many know the fact of the indwelling of God, but how little do they meditate on what that indwelling is ! I would fain attain to the realization of Christ as sitting by me." ²

And the same year from Jaffa :

"There is nothing that may not be perverted ; the Sacrament, reading the Bible, intercessory prayer, all may be made gods of—of this there is no doubt. We tend to make gods of everything we do. If we can, as it were, get a pull over our neighbours, then we think we are better than they. We should look on everything as a means of realizing *His indwelling* ; the Scriptures only do *that*, the Sacrament only does *that*, intercessory prayer only does *that*."

¹ "Letters to His Sister," p. 209.

² *Ibid.*, p. 235.

Prayer did it with him. From Gravesend he wrote: "Prayer is spiritual labour." So Armstrong regarded it as "the best work he ever did." Gordon's letters are full of references to prayer, discussions of prayer, requests for prayer. In Khartoum he gave to the correspondent of the London *Times* whom he liked, his copy of Newman's "Dream of Gerontius." This was his only possession which came out of Khartoum. It was worn and underlined, and these were some of the marked lines:

"Pray for me, oh, my friends."

"'Tis death, oh, loving friends, your prayers,
'tis he."

"So pray for me, my friends, who have not
strength to pray."

"Use well the interval."

"Prepare to meet thy God."

On his last journey to Khartoum, meeting the Roman Catholic Bishop Sogaro, he said to him, "Do not forget me in your prayers." And as he coveted prayer in his behalf, so he was himself ever praying for others.

"Praying for the people ahead of me whom I am about to visit," he wrote from the Soudan, "gives me much strength, and it is wonderful how something seems already to have passed between us when I meet a chief (for whom I have prayed) for the first time. On this I base my hopes of a triumphant march to Fascher. I have really no troops with me, but I have the Shekinah, and I do like trusting to Him and not to men. Remember, unless He gave me the confidence and encouraged me to trust Him, I could

not have it; and so I consider that I have the earnest of success in this confidence."¹

And he knew well the purifying power of prayer. "I pray for those I envy," he wrote, "and the feeling leaves me at once."

A man who had such an inner life as this of course uttered it in his conversation. Gordon was not a reticent soul who never spoke of those things in which he most deeply believed. His letters to his sister were a prolonged unfolding of his religious ideas and his spiritual life.

He lamented that so few people took any interest in the subjects of conversation that were most worth while to him. He, nevertheless, always fearlessly spoke his own mind. He told Huxley his opinion about personal immortality. He concealed his religious opinions from no one. And when he could he worked for Christ.

"I hope, D. V.," he wrote to his sister in 1881, "to put myself in communication with some of our Scripture-reading people, and shall try and visit Christ who is in the East end in the flesh (Matthew xxv. 34). I feel this is what I shall like; these truths were not given to make a man idle."²

"Addresses in vague terms from the pulpit," he said, "do not arrest attention; it is only by direct attack in simple, plain words; and to do that we must use a boldness not of our own but of the Holy Spirit in Whose name we would speak. Let us strive to make those around us

¹ Hake, "The Story of Chinese Gordon," p. 298.

² "Letters to His Sister," p. 163.

know *that* truth, and the way to do so is to live in it ourselves, for 'if ye *abide* in Me ye shall ask what ye will and it shall be done.' " ¹

He felt the temptation to envious and evil speech. He was no untempted saint. "We are all dreadfully prone to evil speaking, but God is all-powerful against it; it is opposed to His nature, so He hates it." From Jerusalem in 1883 he wrote :

"Sins in act are not nearly so frequent as sins in word; both are sequences of the breach of communion, of the non-realization of the indwelling of God.

"Comfort yourself as to the actual fact of death; I have a great desire for it." ²

"Most people," he complained, "only nibble at these great subjects." And again, "This is one great reason why I never desire to enter social life, for there is very great difficulty in knowing people and not discussing others."

And he knew more about temptation and sin than this :

"Who could bear to be known as God knows us? When all is quiet, our friends agree that 'in the flesh dwelleth no good thing'; but if a fall takes place, they belie by their acts what they have said in words, and they are surprised, showing that they do not really believe it practically, but only theoretically." ³

¹ "Letters to His Sister," p. 15.

² *Ibid.*, p. 240f.

³ *Ibid.*, p. 27.

Again,

"We are all lepers. Some have their leprosy covered with silk, some with tattered rags; take off the silk and take off the rags, there are the lepers! Cover the face and cry, 'Unclean, unclean!' The leper in rags shows more to the fleshly eye of his leprosy than the leper in silk."¹

And again,

"Every one who pretends to be better than his fellow is a hypocrite (Isa. ix. 17)."

He lived resolutely at his life, however. As he grew in knowledge of Christ, he grew, of course, in knowledge of his own shortcomings.

"One thing I find is that, as we advance in union with Christ, we get more and more sensitive to our deficiencies; this is only natural, but it certainly does keep one very much alive—I expect, because one is unused to the frame of mind."²

As he discovered temptations he attended to them. From the Soudan in 1874, he wrote :

"Keep me from writing and talking, and then I am humanly safe. As a rule, Christians are really more inconsistent than 'worldlings.' They talk truths, and do not act on them. They allow that 'God is the God of the widows and orphans,' yet they look in trouble to the gods of silver and gold: either He can help altogether, or not at all. He will not be served in conjunction with idols of any sort."³

¹ "Letters to His Sister," p. 44.

² *Ibid.*, p. 199.

³ *Ibid.*, p. 70.

And from Mauritius in 1882:

"I have come to a conclusion; may God give me strength to keep it! *Stop all the newspapers.* It is no use mincing the matter; as the disease is dire, so also must be the remedy.

"These are the words which have done this: 'My son, *unglue* thyself from the world and its vanities. Put on the Lord Jesus Christ, find Him thy wisdom, righteousness, redemption, thy riches, thy strength, thy glory' ('Christ Mystical'). Somehow I thought it must come to this ere long. If I ever need to know, or give my opinion on any subject, then Christ will give me the necessary wisdom. Newspapers feed a passion I have for giving my opinion; therefore, as we have no right to judge and have nothing to do with this world (of which we are not) this feeding must be cut short."¹

He did not attempt to escape from life by any seclusion. He realized that in the open we must do our work and learn our lesson.

"The giving up the papers," he says, "may cause the starvation of my passion for politics, and that scab may drop off. God has shown me what the scabs are:—Evil speaking, lying, slandering, backbiting, scoffing, self-conceit, boasting, silly talking, and some few more.

"I would not wish to be rid of these, *unless God so willed it*, for they keep me down, and prevent my treading on my weaker brethren, which I should be sure to do if I was rid of them. The Canaanites (Ex. xxiii. 28, 29) are good, for otherwise the beasts will increase. So that

¹ "Letters to His Sister," p. 193.

I cannot say, any more than Israel could, what I shall do or where I shall go. I believe I shall not be confounded. I asked to know Christ's life; it may be, it is being taught me, in actual experience as far as my measure. We would like to know Christ's life in our rooms, *from the Bible*; God teaches it to us actually *by the trials of this life.*"¹

"And yet," wrote he, "it is quite impossible to be with the world and to be spiritually minded; the conflict, when one tries to do so, is enough to rend one in two."²

The world calls such a man a fanatic, but at the same time it erects him into its noblest hero because of his fearlessness, his unselfishness, his sincerity. He was a religious man through and through but as human and unconventional as man could be. Pearson, of the party of Church Missionary Society missionaries who went in 1878 to Uganda by way of Khartoum, wrote of these qualities in Gordon in his account of their interview:

"August 8, 1878.—On going to the palace at two o'clock, of course the guard turned out, and several kavasses ushered us up-stairs, and in a large corridor we saw a table laid for lunch, and a little man in his shirt-sleeves walking about. I took him for the butler. On looking through the open doors opposite saw a very splendid divan with a round table in the middle, on which was a bunch of flowers; several looking-glasses on the walls. But on catching sight of us the 'butler' rushed up and said, 'How d'ye do? So glad to see you; excuse

¹ "Letters to His Sister," p. 194.

² *Ibid.*, p. 161.

shirt-sleeves, so hot ! awful long voyage. I'll make a row about it. Are you very angry with me ?' " ¹

He hated formality and fuss. When he first went to Khartoum, "to his disgust he had to live in a palace as large as Marlborough House. Some two hundred servants and orderlies were in attendance ; they added to his discomfort by obliging him to live according to the niceties of an inflexible code of etiquette. He was sternly forbidden to rise to receive a guest, or to offer a chair ; if he rose, every one else did the same ; he 'was guarded like an ingot of gold.' This formality was detestable to him ; but he made a good deal of fun of it, and more than once, while certain solemnities were proceeding, he would delight the great chiefs, his visitors, by remarking in English (of which they knew nothing), 'Now, old bird, it is time for you to go.' " ²

His journals and letters show his very real humanity.

"A very real and human man he was," says Mr. Stannard in his personal recollections of him,— "as great, as good, and as true as any have described him ; not a colourless saint without a flaw or fault to retrieve his goodness from monotony—as some would apparently have us conceive him—but a man whose genius was too brilliant, and whose parts were too strong to be without corresponding weaknesses, and prejudices almost as marked as his talents. If I describe his peculiarities as well as his goodness, it will not be to detract from his reputation, but

¹ "History of Church Missionary Society," Vol. III, p. 103.

² Hake, "The Story of Chinese Gordon," p. 292.

rather to enhance it; for who could have loved Gordon as we did if he had been nothing more than a model of all the virtues? ”¹

Well, he was model enough of the virtue of plain speaking, untrimming veracity. These were his principles in that matter as he wrote them to his sister in 1869 :

“I cannot say what very quiet, relying comfort there is in doing everything quite openly and irrespective of the consequences. We are weak and uncomfortable when we act for man’s view of things ; it is humbugging God in reality, not man, and as surely as we do that we shall reap the reward. The things may be comparatively small, but a very immense principle is involved in them. It is most wonderful what power and strength are given to us by living for God’s view and not man’s. I do many things which are wrong, and I can say truly that, thanks to God, I am comforted in all the troubles, because I do not conceal them from Him. He is my Master, and to Him alone am I accountable. If I own in my heart that I am culpable, I have such comfort that I do not care what my fellow man says. We are most awful liars, every one of us, utterly false ; it is no use mincing it. ‘Trust in the Lord with all your hearts and lean not on your own understanding.’ ”

“A lie is told either to gain something or to conceal something. By telling it the person trusts more to what he may say having an effect on the person he addresses, than he does to the fact that God knows what is in his heart and can actuate as He wills the heart of the hearer. He

¹ “Gordon Anecdotes,” p. 141.

reasons thus: 'There is no God; I am quite free, and it is in my power to say this or that. . . .'

"If you tell the truth, you have infinite power *supporting you*; but, if not, you have infinite power *against you*. The children of kings should be above all deceit, for they have a mighty and a jealous Protector. We go to other gods—Baal, etc.,—when we lie; we rely on other than God. We may for a time seem to humbug men, but not God. It is indeed worldly silliness to be deceitful. Who can stand against the honest, 'I did it, and I am sorry it has vexed you'? Who is then the highest, the judge or the culprit? The latter may say in his heart, 'A Higher than thou hath forgiven me, and I care not what thou, His subject, may do.' Oh! be open in all your ways. It is a girdle around your loins, strengthening you in all your wayfarings.

"Do not ignore the Third Person's presence. Where there are two gathered together for whatever purpose, there is He also, ready to help to the uttermost, and more than willing. Let people tell lies of you, He will blunt the shafts. . . .

"Do you want to be loved, respected, and trusted? Then ignore the likes and dislikes of man in regard to your actions; do to them as you would have them do to you, leave their love for God, taking Him only; you will find that, as you do so, men will like you; they may despise some things in you, but they will lean on you and trust you, and He will give you the spirit of comforting them. But try to please men and ignore God, you will fail miserably and get nothing but disappointment.

"The ninety-first Psalm is a mountain of strength to all believers, as is also the twentieth verse of the thirty-first Psalm.

"If a man speaks well of me, divide it by millions and then it will be millions of times too favourable. If a man speaks evil of me, multiply it by millions and it will be millions of times too favourable. Man is disguised, as far as his neighbour is concerned; this disguise is his outward goodness. . . . Nothing evil was ever said of any man which was not true; his worst enemies could not say a thousandth part of the evil that is in him."¹

And this fearless righteousness was the girdle of his loins. From the Soudan in 1876 he wrote :

"The more one acts from principle and not from feelings, the straighter is our course. No one can be perfectly honest, but the nearer one can be so the stronger one is, and the wish is accepted as the deed. To deceive when you have knowledge of the fact is a lie to all intents and purposes. (Footnote—'The willful suppression of truth is a lie.'—Latin adage.)"²

And in 1877 :

"I thank God He has given me strength to avoid all tricks; to tell them (the slave-dealers) that I would no longer allow their goings on, and to speak to them truthfully."³

And as he sought nothing but the truth, he feared nothing from the opposition or criticism of others. He would say, as his letter about lies indicates, that no one could abuse him too much. To claim to be

¹ "Letters to His Sister," pp. 23-26.

² *Ibid.*, p. 98.

³ *Ibid.*, p. 112.

superior to others he called hypocrisy. So he was regardless of what others thought about him. "Do not think I am ill-tempered," he wrote after the Taiping war, "but I do not care one jot about my promotions or what people may say." He hated conventional society, and felt sure as we have seen that if Christ came back to it He would be "altogether outré."

"You may think I am cantankerous," he wrote to his sister from the Soudan in 1875; "so I am, but it is on principle. I will not cater to this world's appetites, nor be drawn into its coteries and squabbles. What have we in common? They think men deserve credit for this or that. I do not think so. Am I to agree with them?"¹

God deserves all the credit for what men do.

"I do not believe," he said, "in the foresight of Napoleon or the Duke. People made out the talent and foresight after the thing was done. God gives the thought, man carries it out, for the thought is given so strongly as to force him to act thus."² When he resigned the secretaryship to Lord Ripon, he wrote, "I am glad to have cast off every anchor I had to attach me to this world." From Mauritius, "How I hate society! How society hates me!" And after the unpleasant experience in South Africa:

"Whether men praise you, it does not make you better, or whether they blame you, it does not make you worse. God judges by motives, men by actions (Thomas à Kempis). When I went to the Cape, I prayed for glory to

¹ "Letters to His Sister," p. 88f.

² *Ibid.*, p. 128.

God and the welfare of the people, so I am glad *I* got no glory out of it.”¹

The world had no dominion over such a man,—“the man whom other men fear and yet trust,”—those are the words about him written in the copy of Hake’s “Story of Chinese Gordon” which I have read, by an old man who was one of the best students of character I ever knew.

Of a piece with this freedom and independence of mind, and springing from the same source, was Gordon’s modesty. To the illustrations already given of it, I would add several. From Mauritius he wrote in 1882 :

“I have to thank God for several precious working truths—‘God forbid that I should glory’ is one. It is connected with idols in one’s heart. One hugs a thought of some sort or another about politics or such like. God shoots an arrow at it and it becomes dust. Again, the only way to fight Anak is to keep in union with God in Christ—when one goes out or in, when one writes or receives a letter, meets or speaks to any one. This is what is meant by the jealousy of God ; He will be partners with us entirely in all we do or think.

“He also has been gracious enough to let me see the benefits derived from earthly snubs ; they are reminders of Him, like the iron belt or hair shirt of the old monks ; one is so very forgetful that one needs these thorns, though they are no longer thorns when they give this benefit, but are healthful lancet stabs.

“Another truth God gave me is that the Holy Ghost

¹ “ Letters to His Sister,” p. 210.

must participate in our Scripture reading ; that we must in mind keep in union with Christ. Thank God also, I can now pray and wish that every one in the world were holier than I am,—higher than I in the future world.”¹

“When he went to Gravesend, he used to take his place in the gallery of the parish church among the poor. Nobody in the town knew anything about his history, and he was allowed to keep to this place in the gallery, until by and by it began to leak out that he was no other than the leader of the ‘Ever-victorious Army’ in China. Then the church wardens approached the stranger, and graciously asked him to come down and occupy a place in the luxurious seats in the area appointed for the grandees. Gordon thanked them, but declined, preferring to keep the seat in which he had so long sat unnoticed and unknown.”²

I have related the story of his tearing out page after page from the biography which has been written of him. One who saw much of him at that time and who relates that incident, adds :

“I tried to get at the bottom of this feeling of his, telling him he might be justly proud of these things ; but was answered that no man has a right to be proud of anything, inasmuch as he has no *native* good in him—he has received it all ; and he maintained that there was deep cause for intense humiliation on the part of every one, that all wearing of medals, adorning the body, or any form of self-glorification, was quite out of place. Also, he said, he had no right to possess anything, having once given himself to God.

¹ “Letters to His Sister,” p. 191.

² “Gordon Anecdotes,” p. 106.

"He had a great number of medals, for which he cared nothing. There was a gold one, however, given to him by the Empress of China, with a special inscription engraved upon it, for which he had a great liking. But it suddenly disappeared; no one knew where or how. Years afterwards it was found out, by a curious accident, that Gordon had erased the inscription, and sent the medal anonymously to Canon Miller for the relief of the sufferers from the cotton famine at Manchester."¹

The disinterestedness of soul, the devotion to duty for its own sake alone was the dominant characteristic of the man. It greatly impressed the world. To him it was only a matter of course that men should do their duty and die for it without concern. Salary was no consideration whatever to him. He would often refuse what was offered. When he was called to China in 1881 he telegraphed, "As for conditions, Gordon indifferent." He did not require that his calls should stipulate the wages. He gave away his money constantly to relieve human need. After the capture of Soochow during the Taiping Rebellion, the Chinese government ordered that he be given a donation of 10,000 taels and a decoration of the first class. Gordon was still indignant at the treachery of Li Hung Chang in murdering the Taiping leaders after having promised to preserve their lives, and when the treasure-bearers entered his presence, with bowls of bullion on their heads, he flogged them from the chamber with his "magic wand." When the Rebellion was over, he stipulated for ample rewards for his officers and men, but refused to take anything

¹ Hake, "The Story of Chinese Gordon," p. 226f.

himself. His salary during the war he had spent in comforts for his army and in the relief of the Chinese. His conduct was a revelation to the Chinese. The day before Bruce was leaving for England, Prince Kung, the Regent of China, came to him and said : "You will be astonished to see me again, but I felt I could not allow you to leave without coming to see you about Gordon. We do not know what to do. He will not receive money from us."

This was the motive and spirit in which Gordon gave his service. He went out into the world as the missionary goes, not to get but to give, content with a bare living and eager only to do work for the good of men, to fight lies, to establish truth, to erect righteousness. "If you take the case of this man," said Mr. Gladstone after his death, "pursue him into privacy, investigate his heart and his mind, and you will find that he proposed to himself not any ideal of wealth and power, or even fame, but to do good was the object he proposed to himself in his whole life, and on that one object it was his desire to spend his existence."

Before he left for the Soudan in 1873 he wrote :

"I have been more or less acted on by sharks, who want to go with me for money. I have told them that, if it is in my power to employ them, they must belong to the A class—*i. e.*, those who come for the occupation and interest it may give them, and who are content if they are fairly reimbursed their expenses; not the B class, who go for the salary only and who want to make a good thing of it.

"My object is to show the Khedive and his people that

gold and silver idols are not worshipped by all the world. They are very powerful gods, but not so powerful as our God ; so, I refuse a large sum ; you—and I am responsible alone—will not be angry at my doing so. From whom does all the money come ? From poor miserable creatures who are ground down to produce it. Of course, these ideas are outrageous. ‘ Pillage the Egyptians ! ’ is still the cry.”¹

He hated the pillaging of the weak, and all selfish exploitation of men. He held to the old-fashioned notions of honour and sincerity and unselfishness, and he sought simply to ameliorate the lot of the people whom God gave him opportunity to help. He sought to do this, not as a matter of political principle, but as a moral duty and privilege, and he sought to do it, not by political expedients only but by the glorious power of a Christlike life. How different the influence of Christianity in Asia and Africa and South America would have been if all our representatives from the East had been such men. The trouble is that our movement upon the non-Christian world is a contradiction in itself. We preach a pure religion and we too often show the non-Christian nations an immoral life, commercial greed, and an unjust political code. A great part of the influence of Eastern diplomacy and commerce, judged by Gordon’s standard, has been sheer treason alike to Christianity and to the interests of the less privileged nations. Chinese Gordon is a standing illustration of the missionary character of the true representative of a Christian nation wherever he goes.

¹ “ Letters to His Sister,” p. 69.

The people to whom Gordon went always discovered what he was. He invariably entered with deepest sympathy into their lives. Thus he writes from the Soudan in 1877 : "I confess to being somewhat tired of the length of these negotiations, etc., etc. ; but it is better to be tired and worn than that one poor black skin should have a bullet-hole in it." ¹

"I feel strongly," he wrote years later from Jaffa in 1883, "that the grace God gave me to pray for my enemies in the Soudan led to my success." He made men love him with a great love. During the Taiping Rebellion he drew about him captured Taipings, so that at the last his personal body-guard was made up of his former enemies. He had a simply magic gift of inspiring confidence, even in the hearts of savages. Mr. Power, the London *Times* correspondent, wrote of his influence in Khartoum :

"Gordon is a most lovable character—quiet, mild, gentle and strong ; he is so humble, too. The way he pats you on the shoulder when he says, 'Look here, dear fellow, now what do you advise ?' would make you love him. When he goes out-of-doors, there are always crowds of Arab men and women at the gate to kiss his feet, and twice to-day the furious women, wishing to lift his feet to kiss them, threw him over. Numbers of women flock here every day to ask him to touch their children, to cure them ; they call him the 'father and the saviour of the Soudan.' He has found me badly up in Thomas à Kempis, which he reads every day, and has given me an 'Imitation of Christ.' He is, indeed, I believe, the greatest and best man of this century."

¹ "Letters to His Sister," p. 112f.

Again he writes :

"I like Gordon more and more every day ; he has a most lovable manner and disposition, and is so kind to me. He is glad if you would show the smallest desire to help him in his great trouble. How one man could have dared to attempt his task I wonder. One day of his work and bother would kill any other man, yet he is so cheerful at breakfast, lunch, and dinner ; but I know he suffers fearfully from low spirits. I hear him walking up and down his room all night (it is next to mine). It is only his great piety carries him through."

He handled men with the most daring freedom. He would trust when others would imprison. He would march among hosts of armed foes and manage them as he would. He did not do it by duplicity or diplomacy but by straightforwardness and truth. The boy Capsune, whom Gordon rescued from slavery one day, said that he was "quite sure Gordon Pasha could see quite well in the dark, because he had the light inside him." "Have nothing to do with the Turk," he once said to General Butler. "I know him well. He is hopeless." But he would not have been hopeless in Gordon's hands. His faith and love would never have abandoned any people.

To dwell further on Gordon's energy, vigour, love of hard tasks, ceaseless activity, patience in duty and restless impatience until duty was done, his thought of detail and his wrath at mere routine and its petty mechanicalism, the thoroughgoing honesty of his work, would be but to draw out the great warm living qualities which I am sure we have already felt in the man. I have only in bringing this study to a

close to deal a little more directly with his missionary relations.

But one great missionary lesson of his life and character has been already drawn,—the lesson, namely, of the right character and the right purpose in the secular representation of what we call Christian civilization. What a worthless thing it is to send men, who know nothing about the Christianity from which civilization springs and which is its very guarantee, out to the non-Christian world as its ambassadors! But a more direct lesson for us is the lesson of Christian character, of relationship to Christ, of joyful devotion to duty, of contempt for hardship and sacrifice and death as parts of the game, “as a matter of course,” of sympathy and understanding, as all essential to the religious missionary. We cannot all have Chinese Gordon’s personality, but every Christian man should study his life and resort to its secret springs. Would that we had more—thank God we have so many—men of the same high type, men of the old chivalry, prepared and tuned to our own day’s duty who still go forth with the knightly Spirit, reverencing conscience as king, to war against evil and to give help to all that calls them and to serve the Lord Christ.

But Gordon sustained also some interesting direct relations to the organized missionary enterprise. It was his work on the upper Nile that led, with Stanley’s letter regarding Uganda, to the establishment of the Church Missionary Society Mission in Eastern Equatorial Africa, now become perhaps the most remarkable foreign mission in the world. When the second party was sent out it went, on Gordon’s offer of as-

sistance, via Khartoum. When he heard of the murder of the first missionaries he wrote to the Church Missionary Society: "I will engage to send up safe any persons you may wish to send, to secure you free passage for letters, etc., and to do this free of cost within my government. . . . *Don't send 'lukewarms.'*" And when the party came he sent it on by his steamer and at his personal expense right up to the frontier of Uganda.

He had strong notions as to the kind of men needed in Africa for mission work, and he had characteristic opinions as to this work in the region of which he knew. He had the greatest admiration for Livingstone as a missionary. He rejoiced at all the missionary effort made for all these countries, but his ideals were as high for it as they were for himself. Thus he wrote to a party of missionaries on their way to the interior of Africa:

"I want you to like my people, not to look upon them as utterly evil. Mr. Wright has, I dare say, told you my views about missionaries. They must hate father, mother and their own life also. You are sure to succeed if you will entirely trust Him. Shut your eyes to Stanley, to Egyptian government, to all things, and nothing will go wrong, and you must succeed, though it may not be as you would think the best way. You have counted the cost and embarked in this work for His sake, and, though inferior far, for our nation's sake. You must go through with it. Are you missionaries? So am I. The letter must be one which he who runs can read—*the life.*"¹

He did not care greatly for sky-scraping missionary

¹ "Gordon Anecdotes," p. 152.

enterprises. He was very outspoken in his comments and criticisms on what he considered wrong in the spirit in which missionary work was entered upon.

There is one letter to his sister written from Mas-sowah in 1878, in which he writes freely about mission work in North Africa :

“What you ask requires me to be plain-spoken. There is not the least doubt that there is an immense virgin field for *an apostle* in these countries among the black tribes. They are virgin to my belief, and the apostle would have nothing to contend with in the fanaticism of the Arabs. But where will you find an apostle? I will explain what I mean by that term. He must be a man who has died entirely to the world ; who has no ties of any sort ; who longs for death when it may please God to take him ; who can bear the intense dullness of these countries ; who seeks for few letters ; and who can bear the thought of dying deserted. Now, there are few, very, very few men who can accept this post. But no half-measures will do. . . .

“A man must give up everything, understand *everything, everything*, to do anything for Christ here. No half nor three-quarter measures will do. And yet, what a field ! . . .”

Such men Gordon wanted for himself, such he thought missions ought to have. In 1877 he had written on his road to Shaka :

“Find me the man and I will take him as my help, who utterly despises money, name, glory, honour ; one who never wishes to see his home again ; one who looks to God as the source of good and the controller of evil ; one who has a healthy body and energetic spirit, and who looks on

death as a release from misery ; and if you cannot find him then leave me alone. To carry myself is enough for me—I want no other baggage.”¹

What would not a score of such men accomplish in any cause which might win them ?

I have already said that Gordon was no impeccable, faultlessly regular man. He was like the winds of spring. They blow where they list and you cannot tell whence they come or whither they go. No, not just so, for you know he came from God and went to God. Here he saw many things and lived them. He felt and lived the enigmas of which life is full and he did not suppress the paradoxes, the inconsistencies.

But he wrought for the truth as he saw it, and his consolation in the midst of opposition and failure and at last in death was that “everything is for the best.” That was ever his philosophy. God was unfolding His will. Let man be still. In life and death this satisfied him.

“Man must die,” he wrote from the Holy Land, “to the things of the world before he can produce any fruit ; *this is certain*. What is death to the things of the world ? It is to be counted an idiot, an idealist, an impossible sort of person, a theorist, an indiscreet person, an (apparent) condoner of evil, an enthusiast, a mean-spirited person, etc., etc. It is *not* prayer-meetings, or churchgoing, or parish-visiting.

“I speak of myself. In my spiritual nature, I despise the world, its praise or blame. I know of nothing to be

¹ Butler, “Gordon,” p. 136.

admired in my body or its actions from my birth to this day. The world's praises are satires on me ; its blame is just, though not from right motives. In my bodily nature I scheme and work as if everything depended on my sending this or that telegram, or my ordering this or that ; but, thank God, my spiritual nature rules, and I can, when exposed to rebuffs, fall back on that spiritual nature and be comforted." ¹

So while he lived he rested on the living God whose home above all worlds was yet within his own heart, and now that he has died, it is as he said, just as though the grain had escaped the sheath, the sword were from its scabbard drawn. The servant who had ruled well here has gone on, and there in the boundless countries of God has received "a greater government to administer."

Three monuments have been erected to his memory. One is the statue in Trafalgar Square of the solitary figure looking out for the relief that did not come. The second is the great tablet in St. Paul's with its noble inscription :

*Major General Charles George Gordon, C. B.
who at all times and everywhere gave his strength to
the weak, his substance to the poor, his sympathy to
the suffering, his heart to God*

Born at Woolwich 28 Jan., 1833

Slain at Khartoum 26 Jan., 1885

*He saved an empire by his warlike genius, he ruled
vast provinces with justice, wisdom and power ; and
lastly, obedient to his sovereign's command, he died
in the heroic attempt to save men, women and chil-
dren from imminent and deadly peril. Greater
love hath no man than this, that a man lay down
his life for his friends.*

¹ " Letters to His Sister," p. 274.

And the third and noblest of all is the statue of Gordon on a dromedary in the public square at Khartoum. The man and the dromedary are not facing the river by which he might at any time have escaped, nor the palace where he ruled and fell, but they are gazing out over the desert, the great desert with its hopeless tribes, whose mute cry for help he alone heard and in whose sob he caught the accents of the voice of God. This, at the end, as through all his days, had been the secret of his power.

“ For this man was not great
 By wealth or kingly state
 By bright sword or knowledge of earth's wonder,
 But more than all his race
 He saw life face to face,
 And heard the still small Voice above its thunder.”

I pray that in our study of him and of these other leaders in the great movement which is slowly working out the purpose of God in the world, some one here also may have been enabled to look life full in the face and to hear whispering above it the heavenly call to the wider ministry of man and the deeper service of God.

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We notice that v contains only c and the known quantity h . Hence, subtracting the squares of these equations in order to eliminate u , we find

$$\pm \sqrt{(l^2 - h^2)} = c \left(e^{\frac{h}{2c}} - e^{-\frac{h}{2c}} \right) \dots \dots \dots (D).$$

This agrees with the equation given by Poisson in his *Traité de Mécanique*.

The value of c has to be found from this equation. It gives two real finite values of c , one positive and the other negative but numerically equal. A negative value for c would make y negative and would therefore correspond to a catenary with its concavity downwards. It is therefore clear that the positive value of c is to be taken.

To analyse the equation (D), we let $c = 1/\gamma$, and arrange the terms of the equation in the form

$$z = e^{m\gamma} - e^{-m\gamma} - a\gamma = 0 \dots \dots \dots (E),$$

so that a and m are both positive. We have $a^2 = l^2 - h^2$, and $2m = h$. Since the length l of the string must be longer than the straight line joining the points of suspension, it is clear that a must be greater than $2m$. By differentiation,

$$\frac{dz}{d\gamma} = m(e^{m\gamma} + e^{-m\gamma}) - a.$$

Thus $dz/d\gamma$ is negative when $\gamma = 0$, so that, as γ increases from zero, z is at first zero, then becomes negative and finally becomes positive for large values of γ . There is therefore some one value of γ , say $\gamma = i$, at which $z = 0$. If there could be another, say $\gamma = i'$, then $dz/d\gamma$ must vanish twice, once between $\gamma = 0$ and $\gamma = i$, and again between $\gamma = i$ and $\gamma = i'$. We shall now show that this is impossible. By differentiating twice we have

$$\frac{d^2z}{d\gamma^2} = m^2(e^{m\gamma} - e^{-m\gamma});$$

thus $d^2z/d\gamma^2$ is positive when γ is greater than zero. Hence $dz/d\gamma$ continually increases with γ from its initial value $2m - a$ when $\gamma = 0$. It therefore cannot vanish twice when γ is positive. It appears from this reasoning that the equation gives only one positive value of c .

The solitary positive value of c having been found from (D), we can form a simple equation to find u by adding one of the equations (C) to the other. In this way we find one real value of x . The value of y is then found from the first of the equations (A). Thus it appears that, *when a uniform string is suspended from two fixed points of support, there is only one position of equilibrium.*

The equation (D) can be solved by approximation when h/c is so small that we can expand the exponentials and retain only the first powers of h/c which do not disappear of themselves. This occurs when c is large, i.e. when the string is nearly tight. In such cases, however, it will be found more convenient to resume the problem from the beginning rather than to quote the equations (D) or (E).

448. Ex. 1. A uniform string of length l is suspended from two points A and B in the same horizontal line, whose distance apart is h . If h and l are nearly equal, find the parameter of the catenary.

Referring to the figure of Art. 443, we see that $s = \frac{1}{2}l$, $x = \frac{1}{2}h$. Hence using one of the equations (5) of that article, we have
$$l = c \left(e^{\frac{h}{2c}} - e^{-\frac{h}{2c}} \right).$$

Whatever the given values of h and l may be, the value of c must be found from this equation. When h and l are nearly equal, we know by Art. 446, Ex. 2, that h/c

is small. Hence, expanding the exponentials and retaining only the lowest powers of h/c which do not disappear, we have
$$c^2 = \frac{h^3}{24(l-h)}.$$

Since the string considered in this problem is nearly horizontal, the tension of every element is nearly the same. If the string be slightly extensible, so that the extension of any element is some function of the tension, the stretched string will still be homogeneous. The form will therefore be a catenary, and its parameter will be given by the same formula, provided l represents its stretched length.

In order to use this formula, the length l of the string and the distance h between A and B must be measured. But measurements cannot be made without error. To use any formula correctly it is necessary to estimate the effects of such errors. Taking the logarithmic differential we have
$$\frac{2\delta c}{c} = \frac{3\delta h}{h} - \frac{\delta l}{l-h}.$$

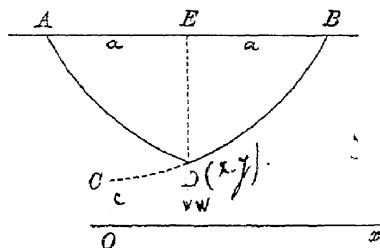
Here δh and δl are the errors of h and l due to measurement. We see that the error in c might be a large proportion of c if either h or $l-h$ were small. In our case $l-h$ is small. Hence to find c we must so make our measurements that the error of $l-h$ is small compared with the small quantity $l-h$, while the length h need be measured only so truly that its error is within the same fraction of the larger quantity h . Thus greater care must be taken in measuring $l-h$ than h .

Suppose, for example, that $h=30$ feet and $l=31$ feet, with possible errors of measurement either way of only one thousandth part of the thing measured. The value of c given by the formula is 33.5 feet, but its possible error is as much as one thirtieth part of itself.

Ex. 2. A uniform measuring chain of length l is tightly stretched over a river, the middle point just touching the surface of the water, while each of the extremities has an elevation k above the surface. Show that the difference between the length of the measuring chain and the breadth of the river is nearly $\frac{8}{3} \frac{k^2}{l}$.

Ex. 3. A heavy string of length $2l$ is suspended from two fixed points A, B in the same horizontal line at a distance apart equal to $2a$. A ring of weight W can slide freely on the string, and is in equilibrium at the lowest point. Find the parameter of the catenary and the position of the weight.

Let D be the position of the heavy ring, then BD and AD are equal portions of a catenary. Produce BD to its vertex C , and let Ox, OC be the directrix and axis of the catenary DB . Let x be the abscissa of D . Then since l is the difference of the arcs



$$CB, CD, \text{ we have } l = \frac{c}{2} \left(e^{\frac{x+a}{c}} - e^{-\frac{x+a}{c}} \right) - \frac{c}{2} \left(e^{\frac{x}{c}} - e^{-\frac{x}{c}} \right) \dots \dots \dots (1).$$

Also, since the weight of the ring is supported by the two vertical tensions of the string,

$$W = 2w \frac{c}{2} \left(e^{\frac{x}{c}} - e^{-\frac{x}{c}} \right) \left[T \cos \theta = cw \right] \dots \dots \dots (2).$$

The equations (1) and (2) determine x and c . Thence the ordinates of D and B may be found, and therefore the depth of D below AB .

If the weight of the ring is much greater than the weight of the string, each string is nearly tight. Thus a/c is small, but x/c is not necessarily small, for the vertex C may be at a considerable distance from D . If we expand the terms containing the exponent a/c and eliminate those containing x/c , we find

$$c = Wa/2w\sqrt{l^2 - a^2} \text{ nearly.}$$

The contrary holds if the weight of the ring is much smaller than the weight of the string. If W were zero the two catenaries BD and DA would be continuous, and the vertex would be at D . Hence when W is very small, the vertex will be near D and therefore x/a will be small. But a/c is not necessarily small. Expanding the terms with small exponentials, we find from (2) that $x = W/2w$. Then

$$(1) \text{ gives } l = \frac{c}{2} \left(e^{\frac{a}{c}} - e^{-\frac{a}{c}} \right) + \frac{W}{2w} \left\{ \frac{1}{2} \left(e^{\frac{a}{c}} + e^{-\frac{a}{c}} \right) - 1 \right\}.$$

If the weight W were absent this equation would reduce to the one already discussed above. If γ be the change produced in the value of c there found by adding the weight W , we find, by writing $c + \gamma$ for c in the first term on the right-hand side, that $\left(l - \frac{ak}{c} \right) \gamma + \frac{W}{2w} (k - c) = 0$, where k is the ordinate of B before the addition of W .

If the weight W had been attached to any point D of the string not its middle point, AD , BD would still form catenaries, whose positions could be found in a similar manner. We may notice that, however different the two strings may appear to be, the catenaries have equal parameters. For consider the equilibrium of the weight W ; we see, by resolving horizontally that the wc of each catenary must be the same.

If the string be passed through a fine smooth ring fixed in space through which it could slide freely, the two strings on each side must have their tensions equal. Hence the two catenaries have the same directrix. The parameters are not necessarily equal, for the difference between the horizontal tensions of the two catenaries is equal to the horizontal pressure on the ring, which need not be zero.

Ex. 4. A heavy string of length l is suspended from two points A , A' in the same horizontal line, and passes through a smooth ring D fixed in space. If DN be a perpendicular from D on AA' and $NA = h$, $NA' = h'$, $DN = k$, prove that the parameters c , c' may be obtained from

$$4c^2 = l^2 \left\{ \cosh \frac{h'}{2c'} \operatorname{cosech} \left(\frac{h}{2c} + \frac{h'}{2c'} \right) \right\}^2 - k^2 \left(\operatorname{cosech} \frac{h}{2c} \right)^2,$$

and a similar equation with the accented and unaccented letters interchanged.

Ex. 5. A portion AC of a uniform heavy chain rests extended in the form of a straight line on a rough horizontal plane, while the other portion CB hangs in the form of a catenary from a given point B above the plane. The whole chain is on the point of motion towards the vertical through B . If l be the length of the whole chain and h be the altitude of B above the plane, show that the parameter c of the catenary is equal to $\mu(l + \mu h) - \mu\sqrt{\{(u^2 + 1)h^2 + 2\mu hl\}}$.

Ex. 6. A heavy string hangs over two small smooth fixed pegs. The two ends of the string are free, and the central portion hangs in a catenary. Show that the free ends are on the directrix of the catenary. If the two pegs are on the same level and distant $2a$ apart, show that equilibrium is impossible unless the length of the string is equal to or greater than $2ae$. [Coll. Exam.]

Ex. 7. A heavy uniform chain is suspended from two fixed points A and B in the same horizontal line, and the tangent at A makes an angle 45° with the horizon.

Prove that the depth of the lowest point of the chain below AB is to the length of the chain as $\sqrt{2} - 1 : 2$

✓ Ex. 8. A uniform heavy chain is fastened at its extremities to two rings of equal weight, which slide on smooth rods intersecting in a vertical plane, and inclined at the same angle α to the vertical: find the condition that the tension at the lowest point may be equal to half the weight of the chain; and, in that case, show that the vertical distance of the rings from the point of intersection of the rods is $l \cot \alpha \log(\sqrt{2} + 1)$, where $2l$ is the length of the chain. [Math. Tripos, 1856.]

Ex. 9. A heavy string of uniform density and thickness is suspended from two given points in the same horizontal plane. A weight, an n th that of the string, is attached to its lowest point; show that, if θ, ϕ be the inclinations to the vertical of the tangents at the highest and lowest points of the string, $\tan \phi = (1 + n) \tan \theta$.

[Math. Tripos, 1858.]

✓ Ex. 10. If α, β be the angles which a string of length l makes with the vertical at the points of support, show that the height of one point above the other is

$$l \cos \frac{1}{2}(\alpha + \beta) / \cos \frac{1}{2}(\alpha - \beta). \quad [\text{Pet. Coll., 1855.}]$$

✓ Ex. 11. A heavy endless string passes over two small smooth fixed pegs in the same horizontal line, and a small smooth ring without weight binds together the upper and lower portions of the string: prove that the ratio of the cosines of the angles which the portions of the string at either peg make with the horizon, is equal to that of the tangents of the angles which the portions of the string at the ring make with the vertical. [Math. Tripos, 1872.]

✓ Ex. 12. A and B are two smooth pegs in the same horizontal line, and C is a third smooth peg vertically below the middle point of AB ; an endless string hangs upon them forming three catenaries AB, BC , and CA : if the lowest point of the catenary AB coincides with C , prove that the pegs AB divide the whole string into two parts in the ratio of $2w + w'$ to $2w - w'$, where w and w' are the vertical components of the pressures on A and C respectively. [Math. Tripos, 1870.]

Ex. 13. An endless uniform chain is hung over two small smooth pegs in the same horizontal line. Show that, when it is in a position of equilibrium, the ratio of the distance between the vertices of the two catenaries to half the length of the chain is the tangent of half the angle of inclination of the portions near the pegs.

[Math. Tripos, 1855.]

Ex. 14. A heavy uniform string of length $4l$ passes through two small smooth rings resting on a fixed horizontal bar. Prove that, if one of the rings be kept stationary, the other being held at any other point of the bar, the locus of the position of equilibrium of that end of the string which is the further from the stationary ring may be represented by the equation $x = 2\sqrt{(ly)} \log \frac{l}{y}$. [Coll. Ex.]

Ex 15. A heavy uniform string is suspended from two points A and B in the same horizontal line, and to any point P of the string a heavy particle is attached. Prove that the two portions of the string are parts of equal Catenaries.

Prove also that the portion of the tangent at A intercepted between the verticals through P and the centre of gravity of the string is divided by the tangent at B in a ratio independent of the position of P .

If θ, ϕ be the angles the tangents at P make with the horizon, α and β those made by the tangents at A and B , show that $\frac{\tan \theta + \tan \phi}{\tan \alpha + \tan \beta}$ is constant for all positions of P .

[St John's Coll.]

Ex. 16. A heavy uniform string hangs over two smooth pegs in the same horizontal line. If the length of each portion which hangs freely be equal to the length between the pegs, prove that the whole length of the string is to the distance between the pegs as $\sqrt{3}$ to $\log \sqrt{3}$. Compare also the pressures on each peg with the weight of the string.

Ex. 17. A uniform endless string of length l is placed symmetrically over a smooth cube which is fixed with one diagonal vertical. Prove that the string will slip over the cube unless the side of the cube is greater than $\frac{1}{3}l\sqrt{2} \log(1+\sqrt{2})$.

[Emm. Coll., 1891.]

Ex. 18. An endless inextensible string hangs in two festoons over two small pegs in the same horizontal line. Prove that, if θ be the inclination to the vertical of one branch of the string at its highest point, the inclination of the other branch at the same point must be either θ or ϕ , where ϕ has only one value and is a function of θ only. If $\cot \frac{1}{2}\theta = e^{\sec \theta}$, then $\phi = \theta$.

[Coll. Ex.]

Ex. 19. Four smooth pegs are placed in a vertical plane so as to form a square, the diagonals being one vertical and one horizontal. Round the pegs an endless chain is passed so as to pass over the three upper and under the lower one. If the directions of the strings make with the vertical angles equal to α at the upper peg, β and γ at each of the middle and δ at the lower peg, prove the following relations:

$$\sin \beta \log \cot \frac{1}{2}\alpha \tan \frac{1}{2}\beta = \sin \gamma \log \cot \frac{1}{2}\gamma \tan \frac{1}{2}\delta,$$

$$\sin \beta \sin \delta + \sin \alpha \sin \gamma = 2 \sin \alpha \sin \delta.$$

[Claus Coll.]

Ex. 20. A bar of length $2a$ has its ends fastened to those of a heavy string of length $2l$, by which it is hung symmetrically over a peg. The weight of the bar is n times, and the horizontal tension $\frac{1}{2}m$ times the weight of the string. Show that

$$m^2 + n^2 = \left\{ (n+1) \operatorname{cosech} \frac{a}{ml} - n \coth \frac{a}{ml} \right\}^2. \quad [\text{Coll. Ex., 1889.}]$$

Ex. 21. One end of a heavy chain is attached to the extremity of a fixed rod, the other end is fastened to a small smooth ring which slides on the rod: prove that in the position of equilibrium $\log \left\{ \cot \frac{1}{2}\theta \cot \left(\frac{1}{2}\pi - \frac{1}{2}\psi \right) \right\} = \cot \theta (\sec \psi - \operatorname{cosec} \theta)$, θ being the inclination of the rod to the horizon, and ψ that of the chain at its highest point.

[Coll. Ex.]

Ex. 22. A string of length πa is fastened to two points at a distance apart equal to $2a$, and is repelled by a force perpendicular to the line joining the points and varying inversely as the square of the distance from it. Show that the form of the string is a semi-circle.

[Coll. Ex., 1882.]

Ex. 23. A chain, of length $2l$ and weight $2W$, hangs with one end A attached to a fixed point in a smooth horizontal wire, and the other end B attached to a smooth ring which slides along the wire. Initially A and B are together. Show that the work done in drawing the ring along the wire till the chain at A is inclined at an angle of 45° to the vertical is $Wl(1 - \sqrt{2} + \log \sqrt{1+\sqrt{2}})$.

[Coll. Ex., 1883.]

Ex. 24. Determine if the catenary is the only curve such that, if AB be any arc whose centre of gravity is G , and AT , BT tangents at A and B , then GT is always parallel to a fixed line in space.

Ex. 25. A uniform heavy chain of length $2a$ is suspended from two points in the same horizontal line; if one of these points be moveable, find the equation of the locus of the vertex of the catenary formed by the string; and show that the area cut off from this locus by a horizontal line through the fixed point is $\frac{1}{4}a^2(\pi^2 - 4)$.

[Math. Tripos, 1867.]

449. Stability of equilibrium. Some problems on the equilibrium of heavy strings may be conveniently solved by using the principle that the depth of the centre of gravity below some fixed straight line is a maximum or minimum, Art. 218. If the curve of the string be varied from its form as a catenary, the use of this principle will require the calculus of variations. But if we restrict the arbitrary displacements to be such that the string retains its form as a catenary, though the parameter c may be varied, the problem may be solved by the ordinary processes of the differential calculus.

This method presents some advantages when we desire to know whether the equilibrium is stable or not. We know, by Art. 218, that *the equilibrium will be stable or unstable according as the depth of the centre of gravity below some fixed horizontal plane is a true maximum or minimum.*

Ex 1 A string of length $2l$ hangs over two smooth pegs which are in the same horizontal plane and at a distance $2a$ apart. The two ends of the string are free, and its central portion hangs in a catenary. Show that equilibrium is impossible unless l be at least equal to ae , and that, if $l > ae$, the catenary in the position of stable equilibrium for symmetrical displacements will be defined by that root of $ce^{\frac{a}{c}} = l$ which is greater than a . [Math. Tripos, 1878.]

Let $2s$ be the length of the string between the pegs. Taking the horizontal line joining the pegs for the axis of x , we easily find (Art. 399) that the depth \bar{y} of the centre of gravity of the catenary and the two parts hanging over the pegs is given by

$$2l\bar{y} = sy - ca + (l - s)^2.$$

Substituting for y and s their values in terms of c , we find

$$2l \frac{d\bar{y}}{dc} = \left(c - \frac{l}{\rho}\right) \frac{\rho^2(c-a) - (c+a)}{c},$$

where ρ stands for $e^{\frac{a}{c}}$. It is easy to see that the second factor on the right-hand side is negative for all positive values of c . Equating $d\bar{y}/dc$ to zero, we find that the possible positions of equilibrium are given by $l = c\rho$. To find the least value of l given by this equation we put $dl/dc = 0$; this gives $c = a$, so that l must be equal to or greater than ae .

For any value of l greater than ae there are two possible values of c , one greater and the other less than a . To determine which of these two catenaries is stable, we examine the sign of the second differential coefficient, Art. 220. We easily find,

$$\text{when } l = c\rho, \quad 2l \frac{d^2\bar{y}}{dc^2} = (c-a) \frac{\rho^2(c-a) - (c+a)}{c^2}.$$

In order that the equilibrium may be stable, this expression must be negative. This requires that c should be greater than a .

Ex. 2. A heavy string of given length has one extremity attached to a fixed point A , and hangs over a small smooth peg B on the same level with A , the other extremity of the string being free. Show that, if the length of the string exceed a certain value, there are two positions of equilibrium, and that the one in which the catenary has the greater parameter is stable.

450. Heterogeneous chain. A heavy heterogeneous chain is suspended from two given points A and B . Find the equation to the catenary.

This problem may be solved in a manner similar to that used in Art. 443 for a homogeneous chain. Since the equations (1) and (2) of that article are obtained by simple resolutions, they will be true with some slight modifications when the string is not uniform. In our case the weight of the string measured from the lowest point is $\int w ds$ between the limits $s = 0, s = s$, Art. 442. We have therefore by the same resolutions

$$T \cos \psi = T_0 \dots (1), \quad T \sin \psi = \int w ds \dots (2).$$

Dividing one of these by the other as before, we find

$$\int w ds = T_0 \tan \psi, \quad \therefore w = \frac{T_0}{\rho \cos^3 \psi} \dots (3),$$

substituting for ρ and $\tan \psi$, their Cartesian values

$$w = T_0 \frac{d^2 y}{dx^2} \frac{dx}{ds} = T_0 \left\{ 1 + \left(\frac{dy}{dx} \right)^2 \right\}^{-\frac{1}{2}} \frac{d^2 y}{dx^2} \dots (4).$$

Conversely, when the law of density is known, say $w = f(s)$, the equation (3) gives a relation between s and dy/dx which we may write in the form $dy/dx = f_1(s)$. We easily deduce from this

$$x = \int \{1 + (f_1(s))^2\}^{-\frac{1}{2}} ds, \quad y = \int \{1 + (f_1(s))^2\}^{-\frac{1}{2}} f_1(s) ds,$$

whence x and y can be expressed in terms of an auxiliary variable which has a geometrical meaning.

Ex. 1. Prove that the tension at any point P of the heterogeneous catenary is equal to the weight of a uniform chain whose length is the projection of the radius of curvature on the vertical and whose density is the same as that of the catenary at P .

Ex. 2. A straight line BR is drawn through any fixed point B in the axis of y parallel to the normal at P to the curve, cutting the axis of x in R . Prove that (1) the tension at P is (T_0/c) times the length BR and (2) the weight of the arc OP , measured from the lowest point O , is (T_0/c) times the length OR , where $OB = c$ and T_0 is the horizontal tension; Art. 35.

451. Cycloidal chain. A heterogeneous chain hangs in the form of a cycloid under the action of gravity: find the law of density.

In a cycloid we have $\rho = 4a \cos \psi$ and $s = 4a \sin \psi$, where a is the radius of the rolling circle. Substituting, we find

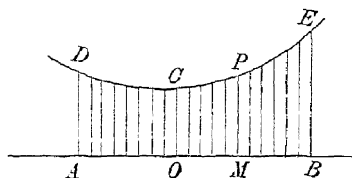
$$w = \frac{T_0}{4a} \sec^3 \psi = \frac{16a^3 T_0}{(16a^2 - s^2)^{\frac{3}{2}}}.$$

It appears from this result that all the lower part of the chain is of nearly uniform density; thus the density at a point whose distance from the vertex measured along the arc is equal to the radius of the rolling circle is about ten ninths of the density at the vertex. The density increases rapidly higher up the chain and is infinite at the cusp. If then the chain when suspended from two points in the same horizontal line is not very curved, the chain may be regarded as nearly uniform.

The chief interest connected with this chain is that, when slightly disturbed from its position of equilibrium, it makes small oscillations whose periods and amplitudes can be investigated.

Ex. Drawing the usual figure for a cycloid, let O be the lowest point of the curve, B the middle point of the line joining the cusps. Let the normal at any point P of the curve intersect the line joining the cusps in M , and let BR be drawn through B parallel to MP to intersect the horizontal through O in R . Prove that the centre of gravity of the arc OP is the intersection of BR with the vertical through M . We find $\bar{x} = 2a\psi$, $\bar{y} = 2a\psi \cot \psi$, if B is the origin.

452. Parabolic chain. A heavy chain AOB is suspended from another chain DCE by vertical strings, which are so numerous that every element of AOB is attached to the corresponding element of DCE . If the weights of DCE and of the vertical strings are inconsiderable compared with that of AOB , find the form of the chain DCE that the chain AOB may be horizontal in the position of equilibrium.



The tensions at O , M of the chain AOB being equal and horizontal, the weight of the length OM is supported by the tensions at C and P of the chain DCE . Thus DCE may be regarded as a heterogeneous heavy chain, such that the weight of any length PC is $m\pi$. Resolving horizontally and vertically for this portion of the chain, we have

$$T \cos \psi = T_0, \quad T \sin \psi = m\pi.$$

Dividing one of these by the other,

$$m\pi = T_0 \tan \psi = T_0 dy/dx, \quad \therefore \frac{1}{2} m\pi^2 = T_0 (y - c).$$

The form of the chain DCE is therefore a parabola.

One point of interest connected with this result is that the chain AOB might be replaced by a uniform heavy bar to represent the roadway of a bridge. The tensions of the chains due to the weight of the bridge would not then tend to break or bend the roadway. It is only necessary that the roadway should be strong enough to bear without bending the additional weights due to carriages. But this would not be true if the light chain DCE were not in the form of a parabola.

The results are more complicated if the weight of the chain DCE is taken into account, and if the chains of support, instead of being vertical, are arranged in some other way.

This problem was first discussed by Nicolas Fuss, *Nova Acta Petropolitanae*, Tom. 12, 1794. It was proposed to erect a bridge across the Neva suspended by vertical chains from four chains stretched across the river. He decided that the chains of his day could not support the necessary tension without breaking.

Ex. 1. Prove that in the parabolic catenary the tension at any point P is $(T_0/2a)$ times the length of the normal between P and the axis of the parabola, where $2a$ is the semi-latus rectum. Prove also that the line density w at P is T_0 divided by the length of the normal.

Ex. 2. Prove that the weight of the chain OP measured from the lowest point O of the curve is $(T_0/2a)$ times the distance of P from the axis of the parabola; and deduce $T_0 = 2am$.

Ex. 3. The centre of gravity G of an arc bounded by any chord lies in the diameter bisecting the chord, and $PG = \frac{1}{2}PN$ where the diameter cuts the parabola in P and the chord in N .

Ex. 4. Referring to the figure, we notice that, since the tensions at C and P support the weight of the roadway OM , the tangents at C and P must intersect in a point vertically over the centre of gravity of OM . Thence deduce that the curve CP is a parabola.

Ex. 5. If the weight of any element ds of the string $DCPE$ is represented by $w(ds + ndx)$, show that the catenary is given by $x = \int \frac{cdz}{n + \sqrt{(1+z^2)}}$, where z is the tangent of the inclination of the tangent to the horizon, and c is a constant. [Fuss.]

Ex. 6. Prove that the form of the curve of the chain of a suspension bridge when the weight of the rods is taken into account, but the weight of the rest of the bridge neglected, is the orthogonal projection of a catenary, the rods being supposed vertical and equidistant. [Math. Tripos, 1880.]

453. The Catenary of equal strength. A heavy chain, suspended from two fixed points, is such that the area of its section is proportional to the tension. Find the form of the chain.

If wds be the weight of an element ds , the conditions of the question require that $T = cw$, where c is some constant. The equations (1) and (2) of Art. 450 now become

$$T \cos \psi = T_0, \quad T \sin \psi = \frac{1}{c} \int T ds.$$

Substituting in the second equation the value of T given by the first, we have $c \tan \psi = \int \sec \psi ds$. Differentiating, we find $c \sec^2 \psi = \sec \psi ds/d\psi$ and $\therefore \rho \cos \psi = c$.

This result also easily follows from the intrinsic equation of equilibrium (2) given in Art. 454. We have $T ds/\rho = wds \cos \psi$. But when the string is equally strong throughout $T = cw$, hence $\rho \cos \psi = c$. The projection of the radius of curvature on the vertical is therefore constant and equal to c .

To deduce the Cartesian equation we substitute for ρ and $\cos \psi$,

$$\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{-1} \frac{d^2y}{dx^2} = \frac{1}{c}, \quad \therefore \tan^{-1} \frac{dy}{dx} = \frac{x}{c} + A.$$

If the origin be taken at the lowest point, the constant A is zero. We then find

$$y = c \log \sec \frac{x}{c}.$$

Tracing this curve, we see that the ordinate y increases from zero as x increases from zero positively or negatively, and that there are two vertical asymptotes given by $x = \pm \frac{1}{2}\pi c$. When x lies between $\frac{1}{2}\pi c$ and $\frac{3}{2}\pi c$, the ordinate is imaginary; when x lies between $\frac{3}{2}\pi c$ and $\frac{5}{2}\pi c$, the curve is the same as that between $x = \pm \frac{1}{2}\pi c$. For greater values of x , the ordinate is again imaginary and so on. The curve therefore consists of an infinite number of branches all equal and similar to that between $x = \pm \frac{1}{2}\pi c$. This is therefore the only part of the curve which it is necessary to consider. Since the ordinates of the bridge must be finite, the values of x are restricted to lie between $\pm \frac{1}{2}\pi c$. The span therefore cannot be as great as πc .

Let O be the lowest point of the curve, C the centre of curvature at any point P , and PH a perpendicular on the vertical through C . Then $CH = c$. The sides of the triangle PCH are perpendicular and proportional to the forces which act on the arc OP , viz. the tension at P , the weight of OP and the horizontal tension T_0 at O . It follows that (1) the tension at P is (T_0/c) times the length of the radius of

curvature and (2) the weight of the arc OP is (T_0/c) times the projection of the radius of curvature on the horizontal.

This curve was called the catenary of equal strength by Davies Gilbert, who invented it on the occasion of the erection of the suspension bridge across the Menai Straits. See *Phil. Trans.* 1826, part iii, page 202. In the first volume of *Liouville's Journal*, 1836, there is a note by G. Coriolis on the "chainette" of equal resistance. Coriolis does not appear to have been aware that this form of chain had already been discussed several years before.

Ex 1. Prove (1) $x = c\psi$, (2) $s = c \log \tan \frac{1}{2}(\pi + 2\psi)$ Use $\rho = \frac{ds}{d\psi}$; $\cos \psi = \frac{dx}{ds}$

Ex 2. Prove that the depth of the centre of gravity of any arc below the intersection of the normals at its extremities is constant and equal to c . Prove also that its abscissa is equal to that of the intersection of the tangents at the same points.

Ex 3. The distance between the points of support of a catenary of uniform strength is a , and the length of the chain is l . Show that the parameter c must be found from $\tanh \frac{l}{4c} = \tan \frac{a}{4c}$. Show also that this equation gives a positive value of c greater than a/π . Use $s = c \log \tan \frac{1}{2}(\pi + 2\psi)$.

Ex 4. Show that the horizontal projection of the span is in every case less than π times the greatest length of uniform chain of the same material that can be hung by one end. Assume the strength of any part of the chain to be proportional to the mass per unit of length. [Kelvin, *Math. Tripos*, 1874.]

If L be the length of uniform chain spoken of, the tension at the point of support is its weight, i.e. wL . Again, the tension at any point of the heterogeneous chain is cw , hence c must be less than L . Hence the span must be less than πL .

454. String under any Forces. To form the general intrinsic equations of equilibrium of a string under the action of any forces. Let A be any fixed point of reference on the string, $AP = s$, $AQ = s + ds$. Let T be the tension at P ; then, since T is a function of s , $T + dT$ is the tension at Q .*

Let the impressed forces on the element PQ be resolved along the tangent, radius of curvature, and binormal at P . Thus Fds is the force on ds resolved along the tangent in the direction in which s is measured; Gds is the force on ds resolved along the radius of curvature ρ in the direction in which ρ is measured, i.e. inwards, Hds is the force on ds resolved perpendicular to the plane of the curve at P , and estimated positive in either direction of the binormal. These three directions are called the principal directions or principal axes of the curve at P .

Let $d\psi$ be the angle between the tangents at P and Q . Hence also the angle $PCQ = d\psi$. The element ds is in equilibrium under

* It should be noticed that, if s were measured from B towards A , so that $BQ = s$, then T would be the tension at Q , $T + dT$ that at P .

the forces T , $T + dT$ acting along the tangents at P , Q and the forces Fds , Gds , Hds . Resolving along the tangent at P ,

$$(T + dT) \cos d\psi - T + Fds = 0,$$

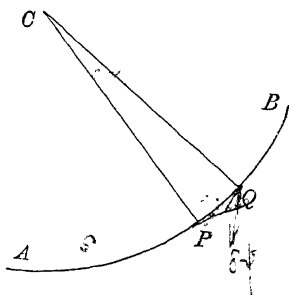
which reduces to

$$dT + Fds = 0 \dots \dots \dots (1).$$

Resolving along the radius of curvature at P , we have

$$(T + dT) \sin d\psi + Gds = 0,$$

$$\therefore T \frac{ds}{\rho} + Gds = 0 \dots \dots \dots (2).$$



We have now to resolve perpendicular to the osculating plane at P of the curve. Since two consecutive tangents to a curve lie in the osculating plane, the tensions have no component perpendicular to this plane. We have therefore

$$Hds = 0 \dots \dots \dots (3).$$

The three equations (1), (2), (3) are the general intrinsic equations of equilibrium.

The density of the string is supposed to be included in the expressions Fds , Gds , Hds for the forces on the element. The equations of equilibrium therefore apply, whether the string is uniform, or whether its density varies from point to point.

From these equations we infer that the tensions T and $T + dT$, acting at the extremities of any element, are equivalent to two other forces, viz. dT and $T \frac{ds}{\rho}$, acting respectively along the tangent to, and the radius of curvature of, the curve at either extremity of the element. In problems on strings it is often convenient to replace the tensions by these two forces. The advantage of this change is that the direction cosines of the tangent and of the radius of curvature are known by the differential calculus. When therefore we form the equations of statics, we can easily resolve these two forces and the given impressed forces in any directions we may find convenient.

Ex. Show that the form of the string is such that at every point the resultant of the applied forces lies in the osculating plane, and makes with the principal normal to the string an angle $\tan^{-1} \frac{d \log T}{d\psi}$.

455. To form the general Cartesian equations of equilibrium of a string*.

Let ds be the length of any element PQ of the string. Let the forces on this element when resolved parallel to the positive directions of the axes be Xds , Yds , Zds . The element is in equilibrium under the action of the tensions at P and Q and these three impressed forces.

Let us resolve all these parallel to the axis of x . The resolved tension at P is $T \frac{dx}{ds}$, and pulls the element PQ towards the left hand. At Q , s has become $s + ds$, the horizontal tension at Q is therefore

$$\left(T \frac{dx}{ds}\right) + \frac{d}{ds} \left(T \frac{dx}{ds}\right) ds,$$

and this pulls the element PQ towards the right-hand side. Taking both these and the force Xds , we have

$$\frac{d}{ds} \left(T \frac{dx}{ds}\right) ds + Xds = 0.$$

Treating the other components in the same way, we find

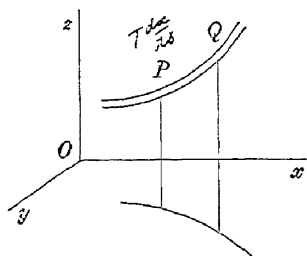
$$\left. \begin{aligned} \frac{d}{ds} \left(T \frac{dx}{ds}\right) + X &= 0 \\ \frac{d}{ds} \left(T \frac{dy}{ds}\right) + Y &= 0 \\ \frac{d}{ds} \left(T \frac{dz}{ds}\right) + Z &= 0 \end{aligned} \right\}.$$

456. Ex. 1. Show that the polar equations of equilibrium of a string in one plane under forces Pds , Qds , acting along and perpendicular to the radius vector, are

$$\frac{d}{ds} (T \cos \phi) - \frac{T}{r} \sin^2 \phi + P = 0, \quad \frac{d}{ds} (T \sin \phi) + \frac{T}{r} \sin \phi \cos \phi + Q = 0,$$

where $\cos \phi = dr/ds$ and $\sin \phi = r d\theta/ds$. Thence deduce the equations of equilibrium of a string in space of three dimensions, referred to cylindrical coordinates.

* The equations of equilibrium of a string under the action of any forces in two dimensions were given in a Cartesian form by Nicolas Fuss, *Nova Acta Petropolitana*, 1796. He gives two solutions, one by moments, and another by considering the tension. In this second solution, after resolving parallel to the axes, he deduces algebraically equations equivalent to those obtained by resolving along the tangent and normal. He goes on to apply his equations to the chainette and other similar problems.



Ex. 2. A string is in equilibrium in the form of a helix, and the tension is constant throughout the string. Show that the force on any element tends directly from the axis of the helix.

Ex. 3. The extremities of a string of given length are attached to two given points, and each element ds of the string is acted on by a repulsive force tending directly from the axis of z and equal to $2\mu r ds$. If $(r\theta z)$ be the cylindrical coordinates of any point, prove that

$$T = A - \mu r^2, \quad \left(\frac{dr}{dz}\right)^2 = C \left(1 - \frac{\mu}{A} r^2\right)^2 - \frac{B^2}{r^2} - 1.$$

Show how the five arbitrary constants are determined. Explain how a helix is, in certain cases, the solution.

✓ Ex. 4. A heavy chain is suspended from two points, and hangs partly immersed in a fluid. Show that the curvatures of the portions just inside and just outside the surface of the fluid are as $D - D'$ to D , where D and D' are the densities of the chain and fluid. [St John's Coll.]

~ The weights of the elements just above and just below the surface of the fluid are proportional to $D ds$ and $(D - D') ds$. If T be the tension, the resolved parts of these weights along the normal must be $T ds/\rho$ and $T ds/\rho'$. Hence $D/(D - D') = \rho'/\rho$.

Ex. 5. A heavy string is suspended from two fixed points A and B , and the density is such that the form of the string is an equiangular spiral. Show that the density at any point P is inversely proportional to $r \cos^2 \psi$, where r is the distance of P from the pole, and ψ is the angle which the tangent at P makes with the horizon.

[Trin. Coll., 1881.]

Ex. 6. A heavy string, which is not uniform, is suspended from two fixed points. Prove that the catenary formed of a given uniform string which touches at any point the curve in which the string hangs and has the same tension at that point will be of invariable dimensions.

• 457. **Constrained Strings.** *A string rests on a curve of any form in one plane, and is acted on by forces at its extremities. It is required to find the conditions of equilibrium and the tension at any point.*

There are four cases of this proposition which are of considerable importance; we shall consider these in order.

Let us first suppose that the weight of the string is so slight that it may be neglected compared with the forces applied at the two extremities of the string. Let us also suppose that the curve is perfectly smooth. The forces on an element ds are merely the tensions at its ends and the reaction or pressure of the curve. Let $R ds$ be this pressure, then R is the pressure per unit of length of the string. For the sake of brevity this is usually expressed by saying that R is the pressure at the element. It is usual to estimate the pressure of the curve on the string as positive when it acts in the direction opposite to that in which the radius of curvature is measured.

Resolving along the tangent and normal to the string, we have

by Art. 454, $\therefore dT=0, \quad T \frac{ds}{\rho} - Rds=0.$

We infer from these equations that, *when a light string rests on a smooth curve, the tension is constant, and the pressure at any point varies as the curvature.*

458. This theorem has a wider range than would perhaps appear at first sight. Since the curve may be of any form, the result includes the case of a string in equilibrium under any forces which are at every point normal to the curve. Supposing the normal forces given, the form of the curve can be found from the result just proved, viz. that at every point the curvature is proportional to the normal force.

As an example we may consider Bernoulli's problem; *to find the form of a rectangular sail*, two opposite sides of which are fixed so as to be parallel to each other and perpendicular to the direction of the wind. The weight of the sail is neglected compared with the pressure produced by the wind. Let us enquire what is the curve formed by a plane section of the sail drawn perpendicular to the fixed sides.

Two answers may be given to this question according as the wind after acting on the sail immediately finds an issue, or remains to press on the sail like a gas in equilibrium. On the former hypothesis we assume as the law of resistance, that the pressure of the wind on any element of the sail acts along the normal to the element and is proportional to the square of the resolved velocity of the wind. We have therefore $R=w \cos^2 \psi$, where ψ is the angle the normal to the section of the sail makes with the direction of the wind, and w is a constant. This gives $c/\rho = \cos^2 \psi$. By Art. 444 we infer that the curve is a catenary, whose axis is in the direction of the wind, and whose directrix is vertical.

If the air presses on the sail like a gas in equilibrium, the pressure on each side of the sail is equal in all directions by the laws of hydrostatics, but the pressure is greater on one side than on the other. We have therefore R equal to this constant difference, hence also ρ is constant, and the required curve is a circle.

Ex. 1. A "square sail" of a ship is fastened to the mast by two yard-arms, and in such that when filled with wind every section by a horizontal plane is a straight line parallel to the yards. Show that, assuming the ordinary law of resistance, it will have the greatest effect in propelling the ship when $3 \sin(\alpha - 2\phi) - \sin \alpha = 0$, where α is the angle between the direction from which the wind comes and the ship's keel, and ϕ is the angle between the yard and the ship's keel. [Caius Coll.]

Ex. 2. A light string has one end fixed at the vertex of a smooth cycloid; prove that as the string, while taut, is wound on the curve, the line of action of the resultant pressure on the cycloid envelopes another cycloid of double parameter.

[Coll. Ex., 1890.]

[The resultant pressure of the curve on an arc of the string balances the tensions at the extremities of the arc. It therefore passes through the intersection of the tangents at those extremities and bisects the angle between them.]

459. Heavy smooth string. Let us next suppose that the weight of the string cannot be neglected. Let wds be the weight

of the element ds . Let ψ be the angle the tangent PK at P makes with the horizontal.

The element PQ is in equilibrium under the action of wds along the ordinate PN , Rds along the normal PG , and the tensions at P and Q . Resolving along the tangent and normal at P , we have

$$dT - wds \sin \psi = 0 \quad \dots\dots(1),$$

$$T \frac{ds}{\rho} - wds \cos \psi - Rds = 0 \quad \dots\dots(2).$$

Since $\sin \psi = dy/ds$, the first equation gives by integration

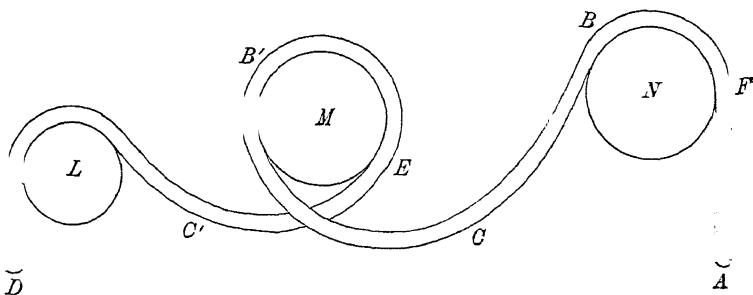
$$T = wy + C \quad \dots\dots\dots(3).$$

Hence, if T_1, T_2 be the tensions at two points whose ordinates are y_1, y_2 ,

$$T_2 - T_1 = w(y_2 - y_1).$$

This important result may be stated thus, *If a heavy string rest on a smooth curve, the difference of the tensions at any two points is equal to the weight of a string whose length is the vertical distance between the points.*

460. It may be remarked that this result has been obtained solely by resolving along the tangent to the string, and is altogether independent of the truth of the second equation. If then the whole length of the string does not lie on the curve, but if



part of it be free and stretch across to and over some other curve, the theorem is still true. Thus if the string $ABCD$ stretch round the smooth curves L, M, N , as indicated in the figure, the tension at any point B or C exceeds that at A by the weight of a string whose length is the vertical distance of B or C above A .

Since the tensions at A and D are zero, it follows that the free extremities of a heavy chain are in the same horizontal line.

In the same way the tension is a maximum at the highest point. Also no point of the string, such as C or C' , can be beneath the horizontal line joining the free extremities.

To determine the pressure at any point P (see fig. of Art. 459) we write the equation (2) in the form

$$R\rho = T - w\rho \cos \psi,$$

where the pressure R of the curve on the string, when positive, acts outwards, i.e. in the direction opposite to that in which the radius of curvature ρ is measured, Art. 457. If T_1 be the tension at any fixed point A , and z the altitude of any point P above A , we have by (3) $T = T_1 + wz$. It therefore follows that

$$R\rho = T_1 + w(z - \rho \cos \psi).$$

If we measure a length $PS = \rho$ along the normal at P outwards, the point S may be called the *anti-centre*. It is clear that $z - \rho \cos \psi$ is the altitude of S above A . Hence, if a heavy string rest on a smooth curve, the value of $R\rho$ at any point P exceeds the tension at A by the weight of a string whose length is the altitude of the anti-centre of P above A .

If the extremity A be free, as in the figure of this article, then $R\rho$ at any point B is equal to w multiplied by the altitude of the anti-centre of B above A . If part of the string is free, as at C and C' , the pressure R is zero. Hence the anti-centres of curvature all lie in the straight line joining the free extremities A and D . *This is the common directrix of all the catenaries.*

In these equations Rds is the pressure outwards of the curve on the string. It is clear that, if R were negative and the string on the convex side, the string would leave the curve and equilibrium could not exist. At any such point as B , the anti-centre is above B and R is clearly positive. But at such a point as E the anti-centre is below E , and if it were also below the straight line AD , the pressure at E would be *negative*. If the string rest on the concave side of the curve, these conditions are reversed. In general, it is necessary for equilibrium that $R\rho$ should be positive or negative according as the string is on the convex or concave side of the curve.

Summing up the results arrived at in this article, we see that a horizontal straight line can be drawn such that the tension at each point P of the string is wy , where y is the altitude of P above the straight line. *This straight line may be called the statical*

directrix of the string. No part of the string can be below the statical directrix, and the free ends, if there are any, must lie on it. If R be the outward pressure of the curve on the string, $R\rho$ is equal to wy' , where y' is the altitude of the anti-centre of P above the directrix. It is therefore necessary that at every point of the string the anti-centre should be above or below the directrix according as the string is on the convex or concave side of the curve.

Ex. 1. Show that the locus of the anti-centre of a circle is another circle.

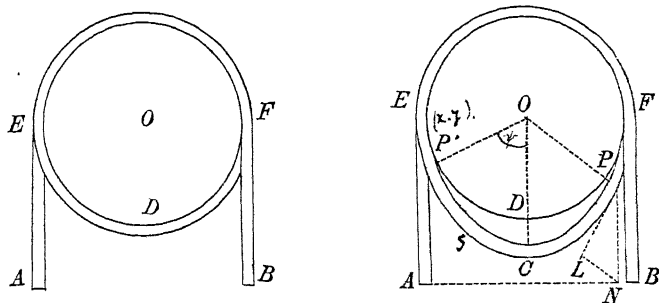
Ex. 2. Show that the coordinates of the anti-centre at any point P of an ellipse referred to its axes are given by $ax = 2a^2 \cos \phi - c^2 \cos^3 \phi$ $by = 2b^2 \sin \phi + c^2 \sin^3 \phi$, where $c^2 = a^2 - b^2$, and ϕ is the eccentric angle of P .

Ex. 3. If S be the anti-centre at any point P of a curve, show that the normal to the locus of S makes with PS an angle θ given by $\tan \theta = \frac{1}{2} d\rho/ds$.

461. It should be noticed that at the points where the string leaves the constraining curve, both the curvature of the string and the pressure R may change abruptly. Thus in the figure of Art. 460 at a point a little below F the radius of curvature of the string is infinite and R is zero. At a point a little above F the curvature of the string is the same as that of the body N , and the pressure R is equal to T/ρ . At such a point as E the abrupt change if any in the value of the product $R\rho$ (in accordance with the rule of Art. 460) is equal to the weight of a string whose length is the vertical distance between the anti-centres on each side of the point.

When the external forces which act on the string are such that their magnitudes per unit of length are finite, an abrupt change of tension cannot occur. If the tensions on each side of any point could differ by a finite quantity, an infinitesimal length of string containing the point would be in equilibrium under the influence of two unequal forces acting in opposite directions. In the same way there can be no abrupt change in the direction of the tangent *except at a point where the tension is zero*, for if the tangents on each side of any point made a finite angle with each other, the element of string at that point would be in equilibrium under the action of two finite tensions not opposed to each other.

462. Ex. 1. A heavy string (length $2l$) passes completely round a smooth horizontal cylinder (radius a) with the two ends hanging freely down on each side. The parts of the string on the upper semi-circumference are close together, so that the whole string may be regarded as lying in a vertical plane perpendicular to the



axis of the cylinder. Find the position of rest and the least length of string consistent with equilibrium.

First, let us suppose that the string is in contact with the circle along the lower semi-circumference as well as the upper. Then a length $l - \frac{3}{2}\pi a$ hangs vertically on each side. Let D be the lowest point of the circle, the anti-centre of D is at a depth $2a$ below the centre O of the circle. Hence, unless $l - \frac{3}{2}\pi a > 2a$, the string cannot rest in contact with the circle.

Secondly, let us suppose that a portion of the string hangs freely in the form of a catenary. Let P' be one of the points of contact of the catenary with the circle. Let P be any point on the catenary, drawn in the figure merely to show the triangle PLN , Art. 444. Let the angle $P'OD = \psi$, so that ψ is the inclination of the tangent at P' to the horizon. Let x, y be the coordinates of P' , $s = CP'$. By examining the triangle PLN , we see that $y = c \sec \psi$, $s = c \tan \psi$. Since $x = a \sin \psi$, we have by (5) of Art. 443

$$\sec \psi + \tan \psi = e^{\frac{a \sin \psi}{c}} \dots \dots \dots (1).$$

As already explained, the free extremities A, B of the string are on a level with the directrix, Art. 460. Hence $BF = y + a \cos \psi$, also the arc $FE = \pi a$, $EP' = (\frac{1}{2}\pi - \psi)a$, and $P'C = s$. The sum of these four quantities is l ,

$$c (\sec \psi + \tan \psi) + a \cos \psi - a\psi + \frac{3}{2}\pi a = l \dots \dots \dots (2).$$

Putting $v = \frac{1}{2} \log \frac{1 + \sin \psi}{1 - \sin \psi}$, we find from (1) and (2)

$$c = \frac{a \sin \psi}{v} \quad \frac{l}{a} = \sqrt{\frac{1 + \sin \psi}{1 - \sin \psi}} \left(\frac{\sin \psi}{v} + 1 - \sin \psi \right) - \psi + \frac{3}{2}\pi.$$

The second of these equations gives the length of the string corresponding to any given position of equilibrium.

To find the least value of l consistent with equilibrium, we equate to zero the differential coefficient of l . As this leads to some rather long reductions, the results only are here stated. Noticing that $dv/d\psi = \sec \psi$, we find

$$\frac{1}{a} \frac{dl}{d\psi} = \frac{(1-v)(v \cos^2 \psi - \sin \psi)}{v^2 (1 - \sin \psi)} = 0.$$

By expanding v in powers of $\sin \psi$, we may show that $(v \cos^2 \psi - \sin \psi)$ is negative and does not vanish for any value of $\sin \psi$ between zero and unity. Equating to zero the factor $(1-v)$, we find that $\sin \psi = (e^2 - 1)/(e^2 + 1)$. As $dl/d\psi$ changes sign from $-$ to $+$ as $\sin \psi$ increases, we see that l is a minimum. Effecting the numerical calculations, we have $\psi = 86^\circ$, and $l - \frac{3}{2}\pi a = (e - \psi)a$, which reduces to $(1.85)a$.

For any given value of l , greater than this minimum, there are two positions of equilibrium. In one a portion of the string hangs freely in the form of a catenary; in the other the string fits closely to the cylinder or hangs free according as the given value of $l - \frac{3}{2}\pi a$ is greater or less than $2a$.

Ex. 2. A uniform chain, having its ends fastened together, is hung round the circumference of a vertical circle. If a be the radius of the circle, $2a\gamma$ the arc which the string touches, and l the whole length, prove

$$(l - 2a\gamma) \{ \log(-\cos \gamma) - \log(1 + \sin \gamma) \} = 2a \sin^2 \gamma \sec \gamma. \quad [\text{May Exam.}]$$

Ex. 3. A uniform inextensible string of given length hangs freely from two fixed points. It is then enclosed in a fine fixed tube which touches no part of the string, and is cut through at a point where the tangent makes an angle γ with the horizon. Prove that at a point where the tangent makes an angle ψ with the horizon the ratio of the pressure on the tube to the weight of the string per unit of length becomes $\cos^2 \psi \sec \gamma$.

[Math. Tripos, 1886.]

463. Rough curve, light string. *To consider the case in which the weight of the string is inconsiderable, but the curve is rough.* Referring to the figure of Art. 459, we shall suppose the extremities A and B to be acted on by unequal forces F, F' . Our object is to find the conditions of limiting equilibrium; let us then suppose the string is on the point of motion in the direction AB . The friction on every element PQ is equal to μRds , where μ is the coefficient of friction. This force acts in the direction opposite to motion, viz. from B to A .

Introducing this force into the equations obtained in Art. 459 by resolving the forces along the tangent and normal, and omitting the terms containing the weight of the element, we have

$$dT - \mu Rds = 0 \dots (1), \quad T \frac{ds}{\rho} - Rds = 0 \dots (2).$$

Eliminating R , we find,
$$\frac{dT}{T} = \mu \frac{ds}{\rho} = \mu d\psi;$$

$$\therefore \log T = \mu \psi + A, \quad \therefore T = Be^{\mu \psi},$$

where A and B are undetermined constants. If T_1, T_2 be the tensions at two points at which the tangents make angles ψ_1, ψ_2 with the axis of x , this equation gives

$$T_2 = T_1 e^{\mu(\psi_2 - \psi_1)} \dots (3).$$

It will be found useful to put the result in the form of a rule. *If a light string rest on a rough curve in a state bordering on motion, the ratio of the tensions at any two points is equal to e to the power of μ times the angle between the tangents or between the normals at those points.*

The sign to be given to μ in this equation depends on the direction in which the friction acts. In using the rule, however, no difficulty arises from this ambiguity; for (1) it is evident that that tension is the greater of the two which is opposed to the friction, and (2) it must be the ratio of the greater tension to the lesser (not the lesser to the greater) which is equal to the exponential with the positive index.

To determine the angle between the tangents; let a straight line, starting from a position coincident with one tangent, roll on the string until it coincides with the other tangent; the angle turned round by this moving tangent is the angle required.

The pressure at any point is given by (2), and we see that *$R\rho$ at any point is equal to the tension at that point.*

464. If the forces F, F' which act at the extremities A, B are given, and if the length l of the string is also given, we may

find the limiting positions of equilibrium in the following manner. Put the equation to the curve in the form $\psi = f(s)$. Let s be the required arc-coordinate of A , then $s + l$ is that of B . The ψ 's of A and B are therefore $f(s)$ and $f(s + l)$. Hence, by taking the logarithms of equation (3),

$$\log F_2 - \log F_1 = \mu \{f(s + l) - f(s)\}.$$

From this equation s must be found. The other limiting position may be found by writing $-\mu$ for μ .

465. It should be noticed that the equation (3) of Art. 463 is independent of the size of the curve. Suppose a *heavy string to pass through a small rough ring or over a small peg*, and to be in a state bordering on motion; the weight of the portion of string on the pulley may sometimes be neglected compared with the tensions of the string on either side. If the strings on either side make a finite angle with each other, the pressures and therefore the frictions will not be small, and cannot be neglected. We infer that, *when a heavy tight string passes through a small rough ring, the ratio of the tensions on each side is given by the same rule as that for a light string.*

466. Ex. 1. A rope is wound twice round a rough post, and the extremities are acted on by forces F , F' . Find the ratio of F to F' when the rope is on the point of slipping. [Here the angle between the tangents is 4π , hence the ratio of the greater force to the other is $e^{4\pi\mu}$]

Ex. 2. A circle has its plane vertical, and is pressed against a vertical wall by a string fixed to a point in the wall above the circle. The string sustains a weight P , the coefficient of friction between the string and circle is μ , and the wall is perfectly rough. When the circle is on the point of sliding, prove that, if W be the weight of the circle and θ the angle between the string and the wall, $P(1 + \cos \theta) e^{\mu\theta} = W + 2P$.
[Coll. Exam.]

Ex. 3. A light string is placed over a rough vertical circle, and a uniform heavy rod, whose length is equal to the diameter of the circle, has one end attached to each end of the string, and rests in a horizontal position. Find within what points on the rod a given mass may be placed, without disturbing the equilibrium of the system, and show that the given mass may be placed anywhere on the rod, provided the ratio of its weight to that of the rod does not exceed $\frac{1}{2}(e^{\mu\pi} - 1)$, where μ is the coefficient of friction between the string and the circle. [Coll. Exam., 1880.]

Ex. 4. A string, whose weight is neglected, passes over a rough fixed horizontal cylinder and is attached to a weight W ; P is the weight which will just raise W , and P' the weight which will just sustain W ; show that, if R , R' are the corresponding resultant pressures of the string on the cylinder, $P \cdot P' : R^2 : R'^2$. [Math. T., 1880.]

Ex. 5. A band without weight passes tightly round the circumference of two unequal rough wheels. One wheel is fixed while the other is made to turn slowly round its centre. Show that the band will slip first on the smaller wheel.

Ex. 6. On the top of a rough fixed sphere (radius c) is placed a heavy particle, to which are tied two equally heavy particles by light strings each of length $c\theta$; show that, when the latter particles are as near together as possible, the planes of the strings make with one another an angle ϕ , where $2 \sin(\theta - \lambda) \cos \frac{\phi}{2} = \sin \lambda \cdot e^{\theta \tan \lambda}$, and λ is the angle of friction between the particles and the sphere, and between the strings and the sphere. [Coll. Exam., 1887.]

Ex. 7. A uniform heavy string of length $2l$ passes through two given small fixed rings A, B in the same horizontal line. Supposing the string to be on the point of slipping inwards at both A and B , find the position of equilibrium.

If $2s$ be the portion of the string between the pegs, y the ordinate of the catenary at either peg, the tensions at the two sides of either ring are proportional to y and $l-s$. Referring to the triangle PLN in the figure of Art. 443, we see that the angle through which the string has been turned is the supplement of the least angle whose sine is c/y . Hence we have by (3) $\log \frac{y}{l-s} = \left(\pi - \sin^{-1} \frac{c}{y} \right) \mu$. Also if $2a$ be the known distance between the rings, we have $x=a$. Substituting for y and s their values in terms of x or a given in Art. 443, we have an equation to find c . Hence y and s may be found.

Ex. 8. A, B, C are three rough points in a vertical plane; P, Q, R are the greatest weights which can be severally supported by a weight W when connected with it by strings passing over A, B, C , over A, B , and over B, C respectively. Show that the coefficient of friction at B is $\frac{1}{\pi} \log \frac{QR}{PW}$. [Math. Tripos, 1851.]

Let α, β, γ be the angles through which the string is bent at ABC , their sum is π . By Art. 463 $\log P/W, \log Q/W, \log R/W$ are respectively equal to $\mu\alpha + \mu'\beta + \mu''\gamma, \mu\alpha + \mu'(\beta + \gamma), \mu'(a + \beta) + \mu''\gamma$. The result follows by substitution. It is supposed that B lies between the verticals through A and C .

Ex. 9. A string, whose length is l , is hung over two rough pegs at a distance a apart in a horizontal line. If one free end of the string is as much as possible lower than the other, the inclination to the vertical of the tangent to the string at either peg is given by the equation $\frac{l}{a} \sin \theta \cdot \log \cot \frac{\theta}{2} = \cos \theta + \cosh \mu(\pi - \theta)$. [St John's Coll., 1881.]

Ex. 10. An endless uniform heavy chain is passed round two rough pegs in the same horizontal line, being partly supported by a smooth peg situated midway in the line between the other pegs, so that the chain hangs in three festoons. If α, β are the angles which the tangents at one of the rough pegs make with the vertical, and μ is the coefficient of friction, prove that the limiting values of α and β are given by the equation $e^{\pm \mu(\pi - \alpha + \beta)} = 2 \frac{\sin \alpha \log \cot \frac{1}{2} \alpha}{\sin \beta \log \cot \frac{1}{2} \beta}$. [Math. Tripos, 1879.]

467. Rough curve, heavy string. We shall now consider the general case in which both the weight of the string and the roughness of the curve are taken account of.

Referring to the figure of Art. 459, and introducing both the weight and the roughness into the equations (1) and (2), we have

$$dT - wds \sin \psi - \mu Rds = 0 \dots\dots\dots (1),$$

$$\frac{Tds}{\rho} - wds \cos \psi - Rds = 0 \dots\dots\dots (2).$$

In applying these equations to other forms of the string we must remember that the friction is μ times the pressure taken positively. Thus as the string is heavy it might lie on the concave side of the curve. We must then change the sign of R in the second equation, but not in the first.

We shall presently have occasion to write $\rho = ds/d\psi$. If the figure is not so drawn that s and ψ increase together, we shall have $\rho = -ds/d\psi$. To solve these equations, we eliminate R ,

$$\therefore \frac{dT}{d\psi} - \mu T = w\rho (\sin \psi - \mu \cos \psi) \dots \dots \dots (3).$$


This is one of the standard forms in the theory of differential equations. According to rule we multiply by $e^{-\mu\psi}$ and integrate ;

$$\therefore Te^{-\mu\psi} = \int w\rho (\sin \psi - \mu \cos \psi) e^{-\mu\psi} d\psi + C \dots \dots (4)$$

We cannot effect this integration until the form of the curve is given. By using the rules of the differential calculus we first express ρ as a function of ψ . Then substituting and integrating, we find

$$Te^{-\mu\psi} = f(\psi) + C \dots \dots \dots (5).$$

The value of T having been found by this equation, R follows from either (1) or (2). *It should be noticed that we have not assumed that the string is necessarily uniform.*


 The pressure at any point is given by the equation

$$R\rho = T - w\rho \cos \psi.$$

It may be noticed that this is the same as the corresponding equation for a heavy string on a smooth curve, Art. 460.

If the string is not on the point of motion, we replace the term $-\mu Rds$ in (1) by $-Fds$, where F is the friction per unit of length.

Ex. If the string is uniform and of finite length, and if the extremities are acted on by forces P_1, P_2 , prove that the whole friction called into play is $\int Fds = P_2 - P_1 - wz$, where $z = y_2 - y_1$, so that z is the vertical distance between the extremities of the string.


 **468** It appears from the last article that the determination of the circumstances of the equilibrium of a heavy string on a rough curve depends on the integral

$$I = \int w\rho e^{-\mu\psi} (\sin \psi - \mu \cos \psi) d\psi.$$

This integral can be found in several cases.

If the curve is a circle and the string homogeneous, we have $\rho = a$. We easily find

$$I = \frac{wa}{\mu^2 + 1} \{ (\mu^2 - 1) \cos \psi - 2\mu \sin \psi \} e^{-\mu\psi}.$$

 If the curve is an equiangular spiral and the string homogeneous, we have $r = ae^{\theta \cot \alpha}$. Since $\rho \sin \alpha = r$ and $\psi = \theta + \alpha$, the integral may be obtained from the last by writing $\mu - \cot \alpha$ for μ , and $ae^{-\alpha \cot \alpha} \operatorname{cosec} \alpha$ for a .

If the curve is a cycloid with its base inclined to the horizon at any angle, we have $\rho = 4a \cos(\psi - \alpha)$, where a is the radius of the generating circle. More generally, if the curve is such that $w\rho$ can be expanded in a series of *positive integral powers* of $\sin \psi$ and $\cos \psi$, we can express $w\rho(\sin \psi - \mu \cos \psi)$ in a series of sines and cosines of multiple angles. In this case the integral can be found by a method similar to that used for the circle.

If the curve is a catenary we have $\rho \cos^2 \psi = c$ and $I = wc \sec \psi e^{-\mu \psi}$. More generally, if the curve is such that $\rho = a \cos^n \psi$, where n is a *positive or negative integer*, we may find I by a formula of reduction. We easily see that

$$\begin{aligned} & \{\mu^2 + (n+1)^2\} I_n - (n-1)(n+2) I_{n-2} \\ &= wc (\cos \psi)^{n-1} e^{-\mu \psi} \{n+2 - \mu(n+2) \sin \psi \cos \psi - (n+1-\mu^2) \cos^2 \psi\}. \end{aligned}$$

469. Ex. 1. A heavy string occupies a quadrant of the upper half of a rough vertical circle in a state bordering on motion. Prove that the radius through the lower extremity makes an angle α with the vertical given by $\tan(\alpha - 2\epsilon) = e^{-\frac{1}{2}\mu\pi}$ where $\mu = \tan \epsilon$.

Ex. 2. A heavy string, resting on a rough vertical circle with one extremity at the highest point, is on the point of motion. If the length of the string is equal to a quadrant, prove that $\frac{1}{2}\pi \tan \epsilon = \log \tan 2\epsilon$. [Coll. Ex., 1881.]

Ex. 3. A single moveable pulley, of weight W , is just supported by a power P , which is applied at one end of a cord which goes under the pulley and is then fastened to a fixed point; show that, if ϕ be the angle subtended at the centre by the part of the string in contact with the pulley, ϕ is given by the equation

$$P(1 - 2e^{\mu\phi} \cos \phi + e^{2\mu\phi})^{\frac{1}{2}} = W. \quad [\text{Coll. Ex., 1882.}]$$

Ex. 4. If a heavy string be laid on a rough catenary, with its vertex upwards and its axis vertical, so that one extremity is at the vertex, the string will just rest if its length be equal to the parameter of the catenary, provided the coefficient of friction be $(2 \log 2)/\pi$. [Coll. Ex., 1885.]

Ex. 5. A heavy string AB is placed on the concave side of a rough cycloidal curve whose base is inclined at an angle α to the horizon, with one extremity A at the lowest point and the other B at the vertex. Prove that the string will be in a state bordering on motion if $\frac{\tan \epsilon - 2 \tan \alpha}{\tan \epsilon + (1 - 3 \cos^2 \epsilon) \tan \alpha} = e^{a \tan \epsilon}$, where $\tan \epsilon$ is the coefficient of friction.

Ex. 6. A heavy string rests on a rough cycloid with its base horizontal and plane vertical. The normals at the extremities of the string make with the vertical angles each equal to α , which is also the angle of friction between string and cycloid. If, when the cycloid is tilted about one end till the base makes an angle α with the horizontal, the string is on the point of motion, show that

$$3 - 2 \sec^2 \alpha = e^{-2\alpha \tan \alpha}.$$

[It is assumed that no part of the string hangs freely.] [Coll. Ex., 1883.]

Ex. 7. A heavy uniform flexible string rests on a smooth complete cycloid, the axis of which is vertical and vertex upwards, the whole length of the string exactly coinciding with the whole arc of the cycloid; prove that the pressure at any point of the cycloid varies inversely as the curvature. [Math. Tripos, 1865.]

Ex. 8. A heavy string AB is laid on a rough convex curve in a vertical plane, and the friction at every point acts in the same direction along the curve. Show

that it will rest if the inclination of the chord AB to the horizon be less than $\tan^{-1} \mu$, where μ is the coefficient of friction. [June Ex., 1878.]

470. The following proposition will be found to include a number of problems which lead to known integrals.

Let the form be known in which a heterogeneous unconstrained string, supported at each end, rests in equilibrium in one plane under the action of any forces. Let this known curve be $y=f(x)$. Let us now suppose this string to be placed in the same position on a rough curve fixed in space whose equation is also $y=f(x)$. If the extremities of the string be acted on by forces such that the string is on the point of slipping, then

$$(T + G\rho) e^{-\mu\psi} = C, \quad R\rho e^{-\mu\psi} = C \quad (1),$$

where C is constant throughout the length of the string. Here, as in Art. 454, Gds is the resolved normal force inwards on the element ds . The standard case is the same as that taken in Art. 467. The string is just slipping in that direction along the curve in which the ψ of any point of the string increases. Also the pressure R of the curve on the string, when positive, acts outwards. If either of these assumptions is reversed, the sign of μ must be changed. In order that the string may not leave the curve, the sign of C should be such that R acts from the curve towards that side on which the string lies.

To prove these results, we refer to equations (1) and (2) Art. 454. Introducing the pressure R into these equations, we have

$$dT + Fds - \mu Rds = 0, \quad \frac{Tds}{\rho} + Gds - Rds = 0 \quad (2).$$

Eliminating R , as in Art. 467 $Te^{-\mu\psi} = -\int (F - \mu G) \rho e^{-\mu\psi} d\psi + C \quad (3).$

When the string is hanging freely, $R=0$; by eliminating T between the equations (2) we find that $F\rho = \frac{d}{d\psi}(G\rho)$ is true along the curve. When the string is constrained to lie on a curve which possesses this property, we can substitute this value of $F\rho$ in the equation (3). We then find $Te^{-\mu\psi} = -e^{-\mu\psi} G\rho + C$. The first result to be proved follows immediately, the second is obtained by substituting this value of T in the second of equations (2).

471 Ex. 1. A uniform heavy string AB is placed on the upper side of a rough curve whose form is a catenary with its directrix horizontal. If the lower extremity is at the vertex, find the least force F which, acting at the upper extremity, will just move the string.

At the upper end of the string we have $T=F$, $G = -g \cos \psi$, at the lower $T=0$, $G = -g$, $\psi=0$. Hence by Art. 470 $(F - g\rho \cos \psi) e^{\pm\mu\psi} = -gc$, $\therefore F = g(y - ce^{\mp\mu\psi})$. The upper sign of μ gives the larger value of F , i.e. the force which will just move the string upwards, the lower sign gives the force which will just sustain the string. Instead of quoting equation (1), the reader should deduce this result from the equations of equilibrium.

Ex. 2. A uniform string AB rests on the circumference of a rough circle under the action of a central force tending to a point O situated at the opposite extremity of the diameter through A . If the force of attraction vary as the inverse cube of the distance, prove that the force F acting at A necessary to prevent the string from slipping is $F = k(\sec^3 \beta e^{-2\mu\beta} - 1)$, where β is the angle AOB , $\frac{2k}{a}$ the force at A , and a is the diameter.

472. Endless and other strings. When a heavy inextensible string rests in equilibrium in contact with a smooth curve without singularities in a vertical plane, the pressure and tension can be found as in Art. 459, with one undetermined constant. This constant is usually found by equating to zero the tension at the free extremity. If, however, the string is either endless or has both its extremities attached to the curve and is tightened at pleasure, there is nothing to determine the constant.

Let us suppose the string to be in contact along the under side of the curve. Let the string be gradually loosed until its length exceeds the length of the arc in contact by an infinitely small quantity. The string is then just on the point of leaving the curve at some unknown point Q , and is then said to *just fit* the curve. If the length of the string were still further increased a finite portion of the string would be off the curve and hang in the form of a catenary. In the same way if the portion of the string under consideration rest with its weight supported on the upper and concave side of the curve, we may conceive the string to be gradually tightened until it separates from the curve at some point Q . If still further tightened or shortened a finite part of the string would hang in the form of a catenary, while the remainder would still rest on the curve.

To determine the position of the point Q we notice that the pressure of the curve on the string measured towards that side on which the string lies must be positive at every point of the curve and zero at Q . The pressure thus measured is therefore a minimum at Q .

Referring to Art. 460, the outward pressure R is given by

$$R\rho = T_0 + w(y - \rho \cos \psi) \dots \dots \dots (1).$$

Differentiating, and remembering that both R and dR/ds are zero at Q , we find

$$0 = \frac{dy}{ds} - \cos \psi \frac{d\rho}{ds} + \rho \sin \psi \frac{d\psi}{ds},$$

except when ρ is infinite at the point thus determined. Since $dy/ds = \sin \psi$ and

$$\rho = ds/d\psi, \text{ this gives at once} \quad 2 \tan \psi = \frac{d\rho}{ds} \dots \dots \dots (2).$$

This equation determines the points at which $R\rho$ is a maximum, a minimum, or stationary. When both R and dR/ds are zero, we have

$$\rho \frac{d^2 R}{ds^2} = \frac{d^2 R\rho}{ds^2} = \cos \psi \left(\frac{2}{\rho} - \frac{d^2 \rho}{ds^2} \right) + \sin \psi \frac{1}{\rho} \frac{d\rho}{ds}.$$

The sign of this expression determines whether R is a maximum or a minimum. When the length of the string is finite, some of these maxima or minima may be excluded as being beyond the given limits. But we must then also take into consideration the extremities of the string, for it is manifest that the pressure at either end may be less than that at any point between the limits of the string. *The required point Q is that one of all these points at which the pressure measured towards the string is least.* The undetermined constant T_0 is then found by making the pressure zero at this point.

If the string leave the curve at the lowest point we have $d\rho/ds = 0$, i.e. the radius of curvature ρ must be either a maximum, a minimum, or stationary at that point. Since $R\rho$ must be a minimum or a maximum according as the string is outside or inside, it is also necessary that $d^2 R\rho/ds^2$ should be positive in the first case and negative in the second.

We may express these conditions in a geometrical form. Consider a portion of the string on the under and convex side of a curve, and let it be gradually loosened

until it leaves the curve. Let Q be the point whose anti-centre is lowest, and let the constant T_0 be determined by making the statical directrix pass through that anti-centre, Art. 460. If R represent the outward pressure on the string, $R\rho$ is then positive at every point of the string and equal to zero at Q . The string therefore leaves the curve at Q .

Next, let the string rest on the upper and concave side of a curve. If gradually tightened it will leave the curve at the point Q whose anti-centre is highest. For, choosing the constant T_0 so that the statical directrix passes through the anti-centre, and assuming that the whole string is still above the directrix (Art. 460), the value of $R\rho$ is negative at every point of the string and equal to zero at Q .

473. Ex. 1. A heavy string *just fits* round a vertical circle: show that the tension at the highest point is three times that at the lowest.

Let T_0, T_1 be the tensions at the lowest and highest points, and let a be the radius. Then $T_1 - T_0 = 2wa$. Since ρ is constant the only solution of (2) is $\psi = 0$, and this makes the outward pressure R a minimum. The pressure is therefore zero at the lowest point. The weight, viz. $w ds$, of the lowest element is therefore supported by the tensions at each end, i.e. $w ds = T_0 ds/a$. These equations give $T_0 = wa$, and $T_1 = 3wa$.

We may obtain the result more simply by using the geometrical rule given in the last article. The locus of the anti-centre is obviously another circle of radius $2a$ and concentric with the given circle. Taking the tangent at its lowest point for the statical directrix, the altitudes of the highest and lowest points of the given circle are as 3 : 1, Art. 460. The tensions at these points are therefore also in the same ratio. We see also that if the string be slightly loosened, it will begin to leave the curve at the lowest point.

Ex. 2. A heavy string (length $2l$) rests on the inner or concave side of a segment of a smooth sphere (radius a , angle 2β) and hangs down symmetrically over the smooth rim which is in a horizontal plane. Find the conditions of equilibrium.

Since every point of the string must be above the statical directrix, it will be seen on drawing a figure that $l > a(\beta + 1 - \cos \beta)$. Since the string rests on the concave side, the outward pressure R must be negative and therefore every point of the anti-centric curve must be below the statical directrix, hence $l < a(\beta + \cos \beta)$. These two conditions require that β should be less than $\frac{1}{2}\pi$. If the second inequality be reversed the string will leave the spherical segment at the highest point.

Ex. 3. A heavy string is attached to two points of the arc of a catenary with its axis vertical, and rests against its under surface. If the string is gradually loosed, show that it will leave the curve at every point at the same instant.

Ex. 4. A heavy string has one end fastened to the lowest point of the arc of a cycloid with the axis vertical and the vertex at the lowest point. The string envelopes the arc outside up to the cusp, and passing over a small smooth pulley has the other end hanging freely. Prove that the least length of the string hanging down which is consistent with equilibrium is equal to six times the radius of the generating circle. Find also in this case the resultant pressure on the cycloid.

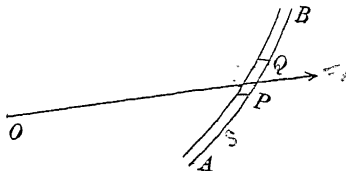
[Queens' Coll.]

Ex. 5. A heavy string just fits the under surface of a cycloidal arc, the extremities of the string being attached to the cusps. Show that the pressure is zero at the point Q given by the negative root of the equation $3 \sin(2\phi + \alpha) = -\sin \alpha$, where ϕ is the inclination of the normal at Q to the axis of the cycloid, and α is the inclination of the axis to the vertical. Find also the tension at the vertex.

Ex. 6. A heavy string surrounds an oval curve, and is so much longer than the perimeter that a finite portion hangs in the form of a catenary. If the string is gradually shortened until the arc of the catenary is evanescent, show (1) that the curve and the catenary have four consecutive points coincident, and (2) that the evanescent arc is situated at a point of the curve determined by $2 \tan \psi = dp/ds$.

Ex. 7. A string is bound tightly round a smooth ellipse, and is acted on by a central repulsive force in the focus varying directly as the square of the distance. Find the law of variation of the tension, and prove that, if the string be slightly loosened, it will leave the curve at the points at a distance from the focus equal to $7/4$ times the semi-major axis, provided the eccentricity be greater than $3/4$. If the eccentricity be less than $3/4$, where will it leave the curve? [Coll. Ex., 1887.]

474. Central forces. *A string of given length is attached to two fixed points, and is under the action of a central force. Find the relation between the form of the curve and the law of force. Let the arc be measured from any fixed point A on the string in the direction AB , and let $s = AP$. Let O be the centre of force, and let Fds be the force on the element ds estimated positive when acting in the positive direction of the radius vector, i.e. when the force is repulsive.*



The element PQ is in equilibrium under the action of the tensions T and $T + dT$ and the central force Fds . Resolving along the tangent at P , we have

$$dT + Fds \cos \phi = 0,$$

where ϕ is the radial angle, i.e. the angle OPA . Since $\cos \phi = dr/ds$,

this reduces to

$$\frac{dT}{dr} + F = 0 \dots \dots \dots (1).$$

We might obtain a second equation by resolving the same forces along the normal at P , but the result is more easily found by taking the moment of the forces which act on the finite portion of string AP . This portion is in equilibrium under the action of the tensions T_0 , T and the central force tending from O on each element. Taking moments about O , these latter disappear; we have therefore

$$Tp = A \dots \dots \dots (2),$$

where p is the perpendicular from O on the tangent at P , and A is the moment about O of the tension T_0 .

Let the tangents at any two points A , B of the curve meet in C . Then the arc AB is in equilibrium under the action of the tensions at A and B and the resultant

R of the central forces on all the elements. This resultant force must therefore act along the straight line joining the centre of force O to the intersection C of the tangents at A and B . Also if OY , OZ are the perpendiculars from O on the tangents at A and B , we see by compounding the tensions that $R = A \cdot \frac{YZ}{OY \cdot OZ}$.

As the point P moves from A to B , the foot of the perpendicular on the tangent at P traces out the pedal curve. This curve, when sketched, exhibits to the eye the magnitude of the tension at all points of the catenary.

475. Two cases have now to be considered.

First. Suppose the form of the string to be given, and let the force be required. By known theorems in the differential calculus we can express the equation to the curve in the form $p = \psi(r)$. The equations (1) and (2) then give

$$T = \frac{A}{\psi(r)}, \quad F = \frac{A\psi'(r)}{\psi(r)^2} \dots\dots\dots(3).$$

The constant A remains indeterminate, for it is evident that the equilibrium would not be affected if the magnitude of the central force were increased in any given ratio. The tension at any point of the string and the pressures on the fixed points of suspension would be increased in the same ratio.

Secondly. Suppose that the force is given, and that the form of the curve is required. Eliminating T between (1) and (2), we find

$$\frac{A}{p} = B - \int F dr \dots\dots\dots(4).$$

This differential equation has now to be solved. Put $u = 1/r$ and $\int F dr = f(u)$; we find by a theorem in the differential calculus

$$A^2 \left\{ u^2 + \left(\frac{du}{d\theta} \right)^2 \right\} = (B - fu)^2 \dots\dots\dots(5).$$

Separating the variables, we have

$$\int \frac{\pm A du}{\{(B - fu)^2 - A^2 u^2\}^{\frac{1}{2}}} = \theta + C \dots\dots\dots(6).$$

When this integration has been effected the polar equation to the curve has been found.

There are three undetermined constants, viz. A , B , C , in this equation. To discover their values we have given the polar coordinates $(u_0\theta_0)$, $(u_1\theta_1)$ of the points of suspension. After integrating (6) we substitute in turn for $(u\theta)$ these two terminal values, and thus obtain two equations connecting the three con-

stants. We have also given the length of the string. To use this datum we must find the length of the arc. We easily find

$$(ds)^2 = (dr)^2 + (r d\theta)^2 = \frac{1}{u^2} \{ (du)^2 + (u d\theta)^2 \}.$$

Substituting from (5), we have

$$s = \int \frac{(B - fu) du}{u^2 \{ (B - fu)^2 - A^2 u^2 \}^{\frac{1}{2}}} \dots\dots\dots (7).$$

Taking this between the given limits of u , and equating the result to the given length of the string, we have a third equation to find the three constants.

The equation (6) agrees with that given by John Bernoulli, *Opera Omnia, Tomus Quartus*, p. 238. He applies the equation to the case in which the force varies inversely as the n th power of the distance, and briefly discusses the curves when $n=0$ and $n=2$.

476. Ex. 1. A string is in equilibrium under the action of a central force. If F be the force at any point per unit of length, prove that the tension at that point $= F\chi$, where χ is the semi-chord of curvature through the centre of force.

Show also that $F = A \frac{r}{p^2}$, where A is a constant.

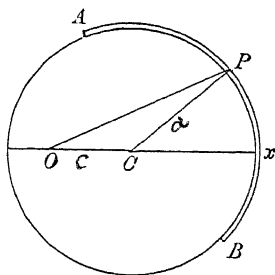
Ex. 2. A uniform string is in equilibrium in the form of an arc of a circle under the influence of a centre of force situated at any point O . Find the law of force. Let C be the centre, $OC = c$, $CP = a$. Then $2ap = r^2 + a^2 - c^2$,

$$\therefore F = -A \frac{d}{dr} \frac{1}{p} = 4aA \frac{r}{(r^2 + a^2 - c^2)^2}.$$

If the centre of force is situated at any point of the arc not occupied by the string the law of force is the inverse cube of the distance.

Since $Tp = A$, A is positive, hence F is positive, i.e. the force must be repulsive. If the centre of force is outside the circle, p is negative for that part of the arc nearest O which is cut off by the polar line of O . If the string occupy this part of the arc, A is negative and the force F must be attractive.

We have taken r or u as the independent variable. If the centre of force be at the centre of the circle, this would be an impossible supposition. This case therefore requires a separate investigation. It is however clear that the string will be in equilibrium whatever the law of force may be, provided it is repulsive.



Ex. 3. A uniform string is in equilibrium in the form of the curve $r^n = a^n \cos n\theta$ under a central force F in the origin: prove that $F = \mu u^{n+2}$.

Ex. 4. A string of infinite length has one extremity attached to a fixed point A , and passing through a small smooth fixed ring at B stretches to infinity in a straight line, the whole being under the influence of a central repulsive force $= \mu u^n$, where

$n > 1$. Show that the form of the string between A and B is $r^{n-2} = b^{n-2} \cos(n-2)\theta$. If $n=2$ the curve is an equiangular spiral.

Ex 5. A closed string surrounds a centre of force $=\mu u^n$, where $n > 1$ and < 2 . Show that, as the length of the string is indefinitely increased so that one apse becomes infinitely distant from the centre of force, the equilibrium form of the string tends to become $r^{n-2} = b^{n-2} \cos(n-2)\theta$. If $n = \frac{3}{2}$ the form of the curve is a parabola.

Ex 6. A uniform string of length $2l$ is attached to two fixed points A, B at equal distances from a centre O of repulsive force $=\mu u^2$. If $OA = OB = b$ and the angle $AOB = 2\beta$, prove that the equation to the string is

$$\frac{M}{r} = 1 + \frac{\cos(\theta \sin \alpha)}{\cos \alpha},$$

where the real and imaginary values of M and α are determined from the equations

$$\frac{M}{b} = 1 + \frac{\cos(\beta \sin \alpha)}{\cos \alpha} \quad \sin \alpha = \pm \frac{b}{l} \sin(\beta \sin \alpha).$$

The equations (1) and (2) of Art. 474 become here $\frac{dT}{Adu} = \mu du$, $Tp = A$.

Proceeding as explained in Art. 475, we find $\pm \int \frac{1}{\{(B + \mu u)^2 - A^2 u^2\}^{\frac{1}{2}}} = \theta + C$.

This integral is one of the standards in the integral calculus, and assumes different forms according as $A^2 - \mu^2$ is positive, negative or zero. Taking the first assumption, we have after a slight reduction

$$\frac{A^2 - \mu^2}{B} u = \mu \pm A \cos \left(1 - \frac{\mu^2}{A^2} \right)^{\frac{1}{2}} (\theta + C).$$

The formula really includes all cases, for when $A^2 - \mu^2$ is negative we may write for the sine of the imaginary angle on the right-hand side its exponential value.

Proceeding to find the arc in the manner already explained, we easily arrive at

$$B\theta = \pm \{(Br + \mu)^2 - A^2\}^{\frac{1}{2}} + D,$$

where the radical must have opposite signs on opposite sides of an apse.

The conditions of the question require that the string should be symmetrical about the straight line determined by $\theta = 0$. We have therefore $C = 0$ and $D = 0$.

Putting $A = \mu \sec \alpha$, the equation to the curve reduces to $\frac{\mu \tan^2 \alpha}{B} \frac{1}{r} = 1 \pm \frac{\cos(\theta \sin \alpha)}{\cos \alpha}$.

We also have $B^2 l^2 = (Bb + \mu)^2 - \mu^2 \sec^2 \alpha$.

Eliminating B between these equations, we find $l \sin \alpha = \pm b \sin(\beta \sin \alpha)$. We now put M for the coefficient of $1/r$ and include the double sign in the value of α . Since $r = b$ when $\theta = \pm \beta$ the three results given above have been obtained.

Ex 7. A string is in equilibrium in the form of a closed curve about a centre of repulsive force $=\mu u^2$. Show that the form of the curve is a circle.

Referring to the last example, we notice that, since r is unaltered when θ is increased by 2π , r must be a trigonometrical function of θ . Hence $\sin \alpha = 1$ or 0 . Putting $M \cos \alpha = M'$, the first makes $M'/r = \cos \theta$, which is not a closed curve, the second gives $M = r$, which is a circle.

Ex 8. If the curve be a parabola, and the centre of force at the focus, and if the equilibrium be maintained by fixing two points of the string, find the law of force, and prove that the tension at any point P is $2fr$, where $r = SP$ and f is the force at P per unit of length.

[St John's Coll., 1883.]

* Ex 9. An infinite string passes through two small smooth rings, and is acted on by a force tending from a given fixed point and varying inversely as the cube of the distance from that point. Show that the part of the string between the rings assumes the form of an arc of a circle.

[Coll. Ex., 1884.]

Ex. 10. If a string, the particles of which repel each other with a force varying as the distance, be in equilibrium when fastened to two fixed points, prove that the tension at any point varies as the square root of the radius of curvature.

[Math. Tripos, 1860.]

Ex. 11. Show that the *catenary of equal strength* for a central force which varies as the inverse distance is $r^n \cos n\theta = a^n$, where $1-n$ is the ratio of the line density to the tension. Show also that this system of curves includes the circle, the rectangular hyperbola, the lemniscate, and when n is zero the equiangular spiral.

[O. Bonnet, *Liouville's J.*, 1844.]

Ex. 12. A string is placed on a smooth plane curve under the action of a central force F , tending to a point in the same plane; prove that, if the curve be such that a particle could freely describe it under the action of that force, the pressure of the string on the curve referred to a unit of length will be equal to $\frac{F \sin \phi}{2} + \frac{c}{\rho}$, where ϕ is the angle which the radius vector from the centre of force makes with the tangent, ρ is the radius of curvature, and c is an arbitrary constant.

If the curve be an equiangular spiral with the centre of force in the pole, and if one end of the string rest freely on the spiral at a distance a from the pole, then the pressure is equal to $\frac{\mu \sin \phi}{2r} \left(\frac{1}{r^2} + \frac{1}{a^2} \right)$.

[Math. Tripos, 1860.]

Ex. 13. A free uniform string, in equilibrium under the action of a repulsive central force F , has a form such that a particle could freely describe it under a central force F' tending to the same centre. Show that $F = kpF'$, where k is a constant. If v be the velocity of the particle and T the tension of the string, show also that $T = kpv^2$. See Art. 476, Ex. 1.

Ex. 14. It is known that a particle can describe a rectangular hyperbola about a repulsive central force which varies as the distance and tends from the centre of the curve. Thence show that a string can be in equilibrium in the form of a rectangular hyperbola under an attractive central force which is constant in magnitude and tends to the centre of the curve. Show also that the tension varies as the distance from the centre.

For a comparison of the free equilibrium of a uniform string with the free motion of a particle under the action of a central force, see a paper by Prof. Townsend in the *Quarterly Journal of Mathematics*, vol. XIII., 1873.

477. When there are two centres of force the equations of equilibrium are best found by resolving along the tangent and normal. Let r, r' be the distances of any point P of the string from the centres of force; F, F' the central forces, which are to be regarded as functions of r, r' respectively. Let p, p' be the perpendiculars from the centres of force on the tangent at P . We then have

$$dT + Fdr + F'dr' = 0 \dots (1), \quad \frac{T}{\rho} - F \frac{p}{r} - F' \frac{p'}{r'} = 0 \dots (2).$$

The first equation gives

$$T = B - \int Fdr - \int F'dr' \dots \dots \dots (3).$$

We may suppose the lower limits of these integrals to correspond to any given point P_0 on the string. If this be done B will be the tension at P_0 . Substituting the value of T thus obtained from (1) and (2) and remembering that $\rho = rdr/dp$,

$$\frac{d}{dp} (p \int Fdr) + \frac{d}{dp'} (p' \int F'dr') = B \dots \dots \dots (4);$$

on the other hand, if we find T from (2) and substitute in (1), we find after reduction

$$\frac{1}{p} \frac{d}{ds} \left(\frac{F p^2 \rho}{r} \right) + \frac{1}{p'} \frac{d}{ds'} \left(\frac{F' p'^2 \rho}{r'} \right) = 0 \quad \dots \quad (5).$$

Thus of the four elements, viz. (1) the force F , (2) the force F' , (3) the tension T , (4) the equation to the curve, if any two are given, sufficient equations have now been found to discover the other two.

Ex. 1. A string can be in equilibrium in the form of a given curve under the action of each of two different centres of force. Show that it is in equilibrium under the joint action of both centres of force, and that the tension at any point is equal to the sum of the tensions due to the forces acting separately.

Ex. 2. Prove that a uniform string will be in equilibrium in the form of the curve $r^2 = 2a^2 \cos 2\theta$ under the action of equal centres of repulsive force situated at the points, $(a, 0)$, $(-a, 0)$, the force of each per unit of length at a distance R being μ/R . Prove also that the tension at all points will be the same and equal to $\frac{1}{2}\mu$.

[Coll. Ex., 1891.]

478. String on a surface. *A string rests on a smooth surface under the action of any forces. To find the position of equilibrium.*

Let the equation to the surface be $f(x, y, z) = 0$. Let Rds be the outward pressure of the surface on the string. Let (l, m, n) be the direction cosines of the inward direction of the normal. By known theorems in solid geometry, l, m, n are proportional to the partial differential coefficients of $f(x, y, z)$ with regard to x, y, z respectively.

If the equations are required to be in Cartesian coordinates, we deduce them at once from those given in Art. 455 by including R among the impressed forces. We thus have

$$\left. \begin{aligned} \frac{d}{ds} \left(T \frac{dx}{ds} \right) + X - Rl &= 0 \\ \frac{d}{ds} \left(T \frac{dy}{ds} \right) + Y - Rm &= 0 \\ \frac{d}{ds} \left(T \frac{dz}{ds} \right) + Z - Rn &= 0 \end{aligned} \right\}.$$

We have here one more unknown quantity, viz. R , than we had in Art. 455, but we have also one more equation, viz. the given equation to the surface.

479 *Let us next find the intrinsic equations to the string.* Let PQ be any element of the string, PA a tangent at P . Let APB be a tangent plane to the surface, PB being at right angles to PA . Let PN be the normal to the surface. Let PC be the radius of

curvature of the string, then PC lies in the plane BPN . Let χ be the angle CPN , then χ is also the angle the osculating plane CPA of the string makes with the normal PN to the surface.

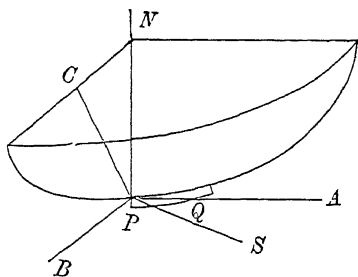
The element PQ is in equilibrium under the action of (1) the forces Xds , Yds , Zds acting parallel to the axes of coordinates, which are not drawn in the figure, (2) the reaction Rds along NP , (3) the tensions at P and Q , which have been proved in Art. 454 to be equivalent to dT along PQ and Tds/ρ along PC .

Resolving these forces along the tangent PA , we have

$$dT + Xds \frac{dx}{ds} + Yds \frac{dy}{ds} + Zds \frac{dz}{ds} = 0,$$

$$\therefore T + \int (Xdx + Ydy + Zdz) = A \dots\dots\dots(1).$$

The forces are said to be *conservative*, when their components X , Y , Z are respectively partial differential coefficients with regard to x , y , z , of some function W which may be called the work function, Art. 209. Assuming this to be the case, the integral in (1) is equal to the work of the forces. It follows from this equation that *the tension of the string plus the work of the forces is the same at all points of the string*. Taking the integral between limits for any two points P , P' of the string, we see that *the difference of the tensions at two points P , P' is independent of the length or form of the string joining those points and is equal to the difference of the works at the points P' , P taken in reverse order*.



We shall suppose that, while ρ is measured inwards along PC , the pressure R of the surface on the string is measured outwards along NP , Art. 457. We shall also suppose that (l, m, n) are the direction cosines of the normal PN measured inwards. With this understanding we now resolve the forces along the normal PN to the surface; we find

$$\frac{Tds}{\rho} \cos \chi + Xds l + Yds m + Zds n - Rds = 0.$$

By a theorem in solid geometry, if ρ' be the radius of curvature of the section of the surface made by the plane NPA , i.e. by

a plane containing the normal to the surface and the tangent to the string, then $\rho' \cos \chi = \rho$. We therefore have

$$\frac{T}{\rho} + Xl + Ym + Zn = R \dots\dots\dots (2).$$

It follows from this equation that *the resultant pressure on the surface is equal to the normal pressure due to the tension plus the pressure due to the resolved part of the forces*. The tension at any point P having been found by (1), the pressure on the surface follows by (2), provided we know the direction of the tangent PA to the string. This last is necessary in order to find the value of ρ' .

Lastly, let us resolve the forces along the tangent PB to the surface. Let λ, μ, ν be the direction cosines of PB . Since PB is at right angles to both PN and PA , these direction cosines may be found from the two equations

$$\lambda f_x + \mu f_y + \nu f_z = 0, \quad \lambda \frac{dx}{ds} + \mu \frac{dy}{ds} + \nu \frac{dz}{ds} = 0.$$

We then have by the resolution

$$\frac{T}{\rho} \sin \chi + X\lambda + Y\mu + Z\nu = 0 \dots\dots\dots (3).$$

Ex. An endless string lies along a central circular section of a smooth ellipsoid, prove that $b^4 F^2 = T^2 (b^2 - p^2)$, where F is the force per unit of length which acting transversely to the string in the tangent plane is required to keep the string in its place, p is the perpendicular from the centre on the tangent plane and b is the mean semi-axis.
[Trin. Coll., 1890.]

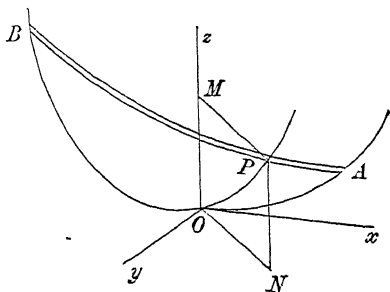
480. Geodesics. If any portion of the string is not acted on by external forces, we have for that portion $X = 0, Y = 0, Z = 0$. The equation (1) then shows that *the tension of the string is constant*. The equation (2) shows that *the pressure at any point is proportional to the curvature of the surface along the string*. The equation (3) (assuming the string not to be a straight line) shows that $\chi = 0$, i.e. at every point *the osculating plane of the curve contains the normal to the surface*. Such a curve is called a *geodesic*, in solid geometry.

Conversely, if the string rest on the surface in the form of a geodesic under the action of forces, we see by (3) that they must be such that *at every point of the string their resolved part perpendicular to the osculating plane of the string is zero*.

Returning to the general case in which the string is under the action of forces, we notice that $\sin \chi / \rho$ is the resolved curvature of the string in the tangent plane at P to the surface. When the resolved curvature vanishes and changes sign as P

moves along the string the concavity changes from one side of the string to the other. Such a point may be regarded as a point of geodesic inflexion. It follows from the equation (3) that a string stretched on a surface can have a point of geodesic inflexion only when the force transverse to the string and tangential to the surface is zero.

481. A string on a surface of revolution. When the surface on which the string rests is one of revolution, we can replace the rather complicated equation (3) of Art. 479 by a much simpler one obtained by taking moments about the axis of figure. If also the resultant force on each element is either parallel to or intersects the axis of figure, there is a further simplification. This includes the useful case in which the only force on the string is its weight, and the axis of figure of the surface is vertical.



Let the axis of figure be the axis of z , and let (r, θ, ϕ) be the polar coordinates and (r', ϕ, z) the cylindrical coordinates of any point on the string, so that in the figure $r' = ON$, $z = PN$, and $\phi =$ the angle NOx . Then from the equation to the surface we have $z = f(r')$. Let the forces on the element ds be Pds , Qds , Zds when resolved respectively parallel to r' , $r'd\phi$, and z .

We shall now take moments about the axis of figure. The moment of R is clearly zero. To find the moment of T , we resolve it perpendicular to the axis and multiply the result by the arm r' . In this way we find that the moment is $Tr' \sin \psi$, where ψ is the angle the tangent to the string makes with the tangent to the generating curve of the surface, i.e. ψ is the curvilinear angle OPA . The equation of moments is therefore

$$d(Tr' \sin \psi) + Qr'ds = 0 \dots\dots\dots(4).$$

We also have by resolving along the tangent as in Art. 479,

$$dT + Pdr' + Qr'd\phi + Zdz = 0 \dots\dots\dots(5).$$

We have also the geometrical equation expressing $\sin \psi$ in terms of the differentials of the coordinates of P . Let the generating curve OP turn round Oz through an angle $d\phi$ and then intersect the string in P' and a plane drawn through MP parallel

to xy in Q . Then $PQ = PP' \sin \psi$, i.e. $r'd\phi = ds \cdot \sin \psi$. We therefore have

$$(r'd\phi)^2 = \{(dr')^2 + (r'd\phi)^2 + (dz)^2\} \sin^2 \psi \dots\dots\dots (6).$$

Eliminating T and $\sin \psi$ between (4), (5) and (6) we have an equation from which the form of the string can be deduced.

If the only force acting on the string is gravity, and if the axis is vertical, the equations take the simple forms

$$Tr' \sin \psi = wB, \quad T = w(z + A) \quad . \quad (7).$$

Eliminating T and $\sin \psi$, by help of (6), we have

$$(z + A)^2 r'^2 = B^2 \left\{ 1 + \left(\frac{dr'}{r'd\phi} \right)^2 + \left(\frac{dz}{r'd\phi} \right)^2 \right\} \quad . \quad (8)$$

Substituting for z from the equation of the surface, viz. $z = f(r')$, this becomes the polar differential equation of the projection of the string on a horizontal plane. The outward normal pressure of the surface on the string may be deduced from equation (2) of Art 479.

482. Heavy string on a sphere. Using polar coordinates referred to the centre O as origin, the fundamental equations take the simple forms

$$T \sin \theta \sin \psi = wB', \quad T = w(a \cos \theta + A), \\ (\sin \theta d\phi)^2 = \{(\sin \theta d\phi)^2 + (d\theta)^2\} \sin^2 \psi, \quad Ra = w(2a \cos \theta + A),$$

where ψ is the angle the string makes with the meridian arc drawn through the summit and $B = aB'$. These give as the differential equation* of the string

$$\left(\frac{d\theta}{d\phi} \right)^2 + \sin^2 \theta = \sin^4 \theta \left(\frac{a \cos \theta + A}{B'} \right)^2.$$

The tension at any point $P = wz$ where z is the altitude of P above a fixed horizontal plane called the directrix plane, and every point of the string must be above this plane. The plane is situated at a depth A below the centre of the sphere. At each point P let the normal OP be produced to cut in some point S a concentric sphere whose radius is twice that of the given sphere. The point S is the anti-centre of P , and the outward pressure on the string is wz'/a where z' is the altitude of S above the directrix plane. As already explained every anti-centre must be above or below the directrix plane according as the string lies on the convex or concave side of the sphere, Art 460

The values of the constants A, B depend on the conditions at the ends of the string. We see that $B' = 0$, (1) if either end is free, for then T vanishes at that end, (2) if the string pass through the summit of the sphere, for then $\sin \theta$ vanishes, (3) if a meridian can be drawn from the summit to touch the sphere, for $\sin \psi = 0$ at the point of contact. In all these cases, $\sin \psi$ vanishes throughout the string, i.e. the string lies in a vertical plane.

If the string form a closed curve, the three quantities $T, \sin \theta, \sin \psi$ cannot

* The reduction of the integral giving ϕ in terms of θ to elliptic functions is given by Clebsch in *Crelle's J.*, vol. 57. A model was exhibited at the Royal Society, June 1895, by Greenhill and Dewar of an algebraical spherical catenary. By a proper choice of the constants the projection of the chain on a horizontal plane became a closed algebraical curve of the tenth degree; see also *Nature*, Jan. 10, 1895.

vanish or change sign at any point of the string. The highest and lowest points of the string are therefore given by $\psi = \frac{1}{2}\pi$, hence at these points

$$T \sin \theta = wB', \quad T = w(a \cos \theta + A), \quad \therefore \sin \theta (a \cos \theta + A) = B'.$$

These equations yield only two available values of $\cos \theta$; for tracing the two curves whose common abscissa is $\xi = \cos \theta$ and whose ordinates are the *reciprocals* of the two values of T , we have an ellipse and a rectangular hyperbola, which, since T must be positive, give only two intersections. Let $\theta = \alpha$, $\theta = \beta$ be the meridian distances of the highest and lowest points of the string, both being positive. Then

$$-\frac{2A}{a} = \frac{\sin 2\alpha - \sin 2\beta}{\sin \alpha - \sin \beta}, \quad -\frac{B'}{a} = \sin \alpha \sin \beta \frac{\cos \alpha - \cos \beta}{\sin \alpha - \sin \beta}.$$

It follows that the directrix plane passes through the centre of the sphere when α and β are complementary. In general the tensions, and therefore the depths of the directrix plane below the highest and lowest points, are inversely as the distances of those points of the string from the vertical diameter.

It has been proved in Art. 480, that the string can have a point of geodesic inflexion when the transverse tangential force is zero. This requires that the meridian drawn from the summit should touch the string, and this, we have already seen, cannot occur. It follows that *the string must be concave throughout its length on the same side.*

If the form of the string is a circle its plane must be either horizontal or vertical, and in the latter case it must pass through the centre of the sphere. To prove this we give the string a virtual displacement without changing its form, it is easy to see that the altitude of the centre of gravity can be a max-min only in the cases mentioned. In both cases the altitude is a maximum and the equilibrium is therefore unstable. Art. 218. In the same way it may be shown that *any position of equilibrium of a heavy free string on a smooth sphere is unstable.*

Ex. 1. A heavy uniform chain, attached to two fixed points on a smooth sphere, is drawn up just so tight that the lowest point just touches the sphere. Prove that the pressure at any point is proportional to the vertical height of the point above the lowest point of the string. [Coll. Ex., 1892.]

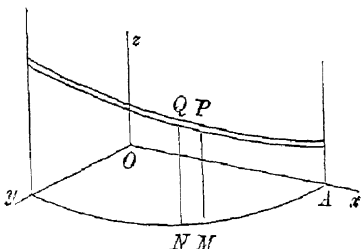
Ex. 2. A string rests on a smooth sphere, cutting all the sections through a fixed diameter at a constant angle. Show that it would so rest if acted on by a force varying inversely as the square of the distance from the given diameter, and that the tension varies inversely as that distance. [Coll. Exam., 1884.]

Ex. 3. A string can rest under gravity on a sphere in a smooth undulating groove lying between two small circles whose angular distances from the highest point of the sphere are complementary, without pressing on the sides of the groove. If ψ is the acute angle at which the string cuts the vertical meridian prove that the points at which ψ is a minimum occur at angular distances $\frac{1}{2}\pi$ from the highest point and find the value of ψ at these points. [Math. T., 1889.]

483. String on a Cylindrical Surface. Ex. 1. A heavy string is in equilibrium on a cylindrical surface whose generators are vertical, the extremities of the string being attached to two fixed points on the surface. Find the circumstances of the equilibrium.

Let $PQ = ds$ be any element, w its weight. Let the axis of z be parallel to the generators, and let z be measured in the direction opposite to gravity. Resolving

along a tangent to the string, we have as in (1) Art. 479, $T - wz = d$. Resolving vertically, we have by Art. 478, $\frac{d}{ds} \left(T \frac{dz}{ds} \right) - w = 0$. These are the same as the equations to determine the equilibrium of a heavy string in a vertical plane. The constants, also, of integration are determined by the same conditions in each case. We see therefore that *if the cylinder is developed on a vertical plane, the equilibrium of the string is not disturbed*. The circumstances of the equilibrium may therefore be deduced from the ordinary properties of a catenary



To find the pressure on the cylinder, we either resolve along the normal at P to the surface, or quote the general result found in Art. 479. We thus find $R = T/\rho'$, also $\frac{1}{\rho'} = \frac{\cos^2 \psi}{\rho_1} + \frac{\sin^2 \psi}{\infty} = \frac{\cos^2 \psi}{\rho_1}$, by Euler's theorem on curvature, where ρ_1 is the radius of curvature at M of the section AMN of the cylinder made by a horizontal plane, and ψ is the angle the tangent at P to the string makes with the horizontal plane.

Ex. 2. If a string be suspended symmetrically by two tacks upon a vertical cylinder, and if z_1, z_2, z_3 be the distances above the lowest point of the catenary at which the string crosses itself, then $z_1 z_2 z_3 = (z_{n+1} - z_n)^2$. [Math. Tripos, 1859.]

Ex. 3. If an endless chain be placed round a rough circular cylinder, and pulled at a point in it parallel to the axis, prove that, if the chain be on the point of slipping, the curve formed by it on the cylinder when developed will be a parabola; and find the length of the chain when this takes place. [Math. Tripos.]

Ex. 4. A heavy uniform string rests on the surface of a smooth right circular cylinder, whose radius is a and whose axis is horizontal. If (a, θ, z) be the cylindrical coordinates of a point on the string, θ being measured from the vertical, prove that $T = w(b + a \cos \theta)$, $z = \int \frac{acd\theta}{\{(b + a \cos \theta)^2 - c^2\}^{\frac{1}{2}}}$, where b and c are two constants.

It is clear that the tension resolved parallel to z is constant, i.e. $T dz/ds = wc$. Combining this result with the value of T found in Art. 483, Ex. 1, we obtain the second result in the question.

Ex. 5. The extremities of a heavy string are attached to two small rings which can slide freely on a rod which is placed along the highest generator of a right circular horizontal cylinder, and are held apart by two forces each equal to wa . The lowest point of the string just reaches to a level with the axis of the cylinder. If D be the distance between the rings and L the length of the string, prove that

$$\frac{D}{4a} = \int \frac{d\psi}{\sqrt{(3 + \sin^2 \psi)}}, \quad \frac{L}{8a} = \int \frac{d\psi}{\sqrt{(3 + \sin^2 \psi)}} \frac{1}{1 + \sin^2 \psi},$$

the limits of integration being 0 to $\frac{1}{2}\pi$.

These follow from the results in the last question. The conditions of the question give $a = b = c$. The integrals are reduced by putting $\tan \frac{1}{2}\theta = \sin \psi$.

Ex. 6. A uniform string rests on a horizontal circular cylinder of radius a with its ends fastened to the highest generator and its lowest point at a depth a below it; prove that the curvature at the lowest point is $1/a$, and that the inclination of the

string at any point to the axis is $\sec^{-1}(1+z/a)$, where z is the height of the point above the axis, supposing the string cuts the highest generator at an angle of 60° .

[June Exam.]

Ex. 7. A heavy uniform string has its two ends fastened to points in the highest generator of a smooth horizontal cylinder of radius a , and is of such a length that its lowest point just touches the cylinder. Prove that, if the cylinder be developed, the origin being at one of the fixed points, the curve on which the string lay is given by $c^2 \left(\frac{dy}{dx} \right)^2 = a^2 \cos^2 \frac{y}{a} + 2ac \cos \frac{y}{a}$. [Math. T., 1883.]

484. String on a right cone. Ex. 1. A string has its extremities attached to two fixed points on the surface of a right cone, and is in equilibrium under the action of a centre of repulsive force F at the vertex. Show that the equilibrium is not disturbed by developing the cone and string on a plane passing through the centre of force.

Let the vertex O be the origin, (r', θ', z) the cylindrical coordinates of any point P on the string. Let $OP=r$. Taking moments about the axis and resolving along the tangent, we have as in Art. 481,

$$Tr' \sin \psi = B, \quad T + \int F dr = C \dots\dots\dots(1).$$

We may imagine the cone divided along a generator and together with the string on its surface unwrapped on a plane. Let (r, θ) be the polar coordinates of the position of P in this plane. Let p be the perpendicular from O on the tangent to the unwrapped string, then $p = r \sin \psi$. The equations (1) become

$$Tp = B', \quad T + \int F dr = C \dots\dots\dots(2).$$

These are the equations of equilibrium of a string in one plane under the action of a central force, and the constants of integration are determined by the same conditions in each case. We may therefore transfer the results obtained in Art. 474 to the string on the cone. In transferring these results we notice that the point (r, θ) on the plane corresponds to $(r' \theta' z)$ on the cone, where $r' = r \sin \alpha$, $\theta' \sin \alpha = \theta$, $z = r \cos \alpha$.

The pressure R is given by $R = \frac{T}{\rho} = \frac{\sin \phi}{r^2} \cdot \frac{B \cos \alpha}{\sin^2 \alpha}$, since $\frac{1}{\rho} = \frac{\cos^2 \phi}{\infty} + \frac{\sin^2 \phi}{r' \sec \alpha}$ by Euler's theorem on curvature. Art. 479.

Ex. 2. The two extremities of a string, whose length is $2l$, are attached to the same point A on the surface of a right cone. The equation to the projection of the string on a plane perpendicular to the axis is $\pi r' = l \cos(\theta' \sin \alpha)$, the point A being given by $\theta' = \pi$. Show that the string will rest in equilibrium under the influence of a centre of force in the vertex varying inversely as the cube of the distance.

Ex. 3. A heavy uniform string has its ends fastened to two points on the surface of a right circular cone whose axis is vertical and vertex upwards, the string lying on the surface of the cone. Prove that, if the cone be developed into a plane, the curve on which the string lay is given by $p(a+br)=1$, the origin being the vertex, p the perpendicular on the tangent, and a, b constants.

[Coll. Ex., 1890.]

485. String on a rough surface. A string rests on a rough surface under the action of any forces, and every element borders on motion; to find the conditions of equilibrium.

The required conditions may be deduced from the equations for a smooth surface by introducing the limiting friction. The pressure of the surface on the element ds being Rds , the limiting friction will be μRds . This friction acts in some direction PS lying in the tangent plane to the surface. See figure of Art. 479. Let ψ be the angle SPA . Resolving along the principal axes at any point of the string exactly as in Art. 479, we have

$$\left. \begin{aligned} dT + Xdx + Ydy + Zdz + \mu Rds \cos \psi &= 0 \\ \frac{T}{\rho'} + Xl + Ym + Zn - R &= 0 \\ \frac{T}{\rho'} \tan \chi + X\lambda + Y\mu + Z\nu + \mu R \sin \psi &= 0 \end{aligned} \right\}.$$

These three equations express the conditions of equilibrium.

486. The simplest case is that in which the applied forces can be neglected compared with the tension. We then have, putting zero for X, Y, Z ,

$$\left. \begin{aligned} \frac{dT}{ds} + \mu R \cos \psi &= 0 \\ \frac{T}{\rho'} &= R \\ \frac{T}{\rho'} \tan \chi + \mu R \sin \psi &= 0 \end{aligned} \right\}.$$

It easily follows from these equations that $\tan \chi + \mu \sin \psi = 0$. This requires that $\tan \chi$ should be less than μ , thus equilibrium is impossible if the string be placed on the surface so that its osculating plane at any point makes an angle with the normal greater than $\tan^{-1} \mu$. Eliminating ψ and R from these equations,

$$\frac{dT}{ds} + \frac{T}{\rho'} (\mu^2 - \tan^2 \chi)^{\frac{1}{2}} = 0,$$

$$\therefore \log T = C - \int \frac{ds}{\rho'} (\mu^2 - \tan^2 \chi)^{\frac{1}{2}}.$$

Thus, when the string is laid on the surface in a given form and is bordering on motion, the tension at any point can be found.

It also follows from the equations of Art. 486 that, if $\chi = 0$, then $\psi = 0$. If therefore the string is placed along a geodesic line on the surface, the friction must act along a tangent to the string. Putting $\psi = 0$, we have from the two first equations

$$\log T = C - \mu \int \frac{ds}{\rho'}.$$

Since along a geodesic $\rho' = \rho$, we may deduce from this equation the following extension of the theorem in Art. 463. *If a light string rest on a rough surface in a state bordering on motion, and the form of the string be a geodesic, then (1) the friction at any point acts along the tangent to the string, and (2) the ratio of the tensions at any two points is equal to e to the power of $\pm \mu$ times the sum of the infinitesimal angles turned through by a tangent which moves from one point to the other.*

The conditions of equilibrium of a string on a rough surface are given in Jellett's *Theory of Friction*. He deduces from these the equations obtained in Art. 486.

487. Ex. 1. A fine string of inconsiderable weight is wound round a right circular cylinder in the form of a helix, and is acted on by two forces F, F' at its extremities. Show that, when the string borders on motion, $\log \frac{F'}{F} = \pm \mu \frac{\cos^2 \alpha}{a} s$, where s is the length of the string in contact with the cylinder, α the angle of the helix and a the radius of the cylinder.

Since the helix is a geodesic, this result follows from the equations of Art. 486 by writing for $1/\rho'$ its value $\cos^2 \alpha/a$ given by Euler's theorem on curvature.

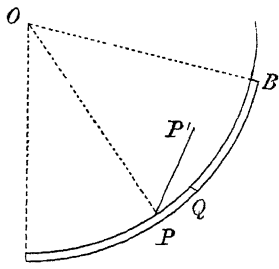
Ex. 2. A heavy string AB , initially without tension, rests on a rough horizontal plane in the form of a circular arc. Find the least force F which, applied along a tangent at one extremity B , will just move the string.

Let O be the centre of the arc, let the angle $\angle OP = \theta$, the arc $AP = s$. Let the element PQ of the string begin to move in some direction PP' , where $P'PQ = \psi$; then by the nature of friction the angle ψ must be less than a right angle. The friction at P therefore acts in the opposite direction, viz. $P'P$, and is equal to $\mu v ds$. The equations of equilibrium are

$$\left. \begin{aligned} dT - \mu v ds \cos \psi &= 0 \\ T d\theta - \mu v ds \sin \psi &= 0 \end{aligned} \right\} \dots\dots\dots (1).$$

Substituting in the first equation the value of T given by the second, we have, since $ds = a d\theta$, $d\psi = d\theta$, and therefore $\psi = \theta + C \dots\dots\dots (2)$.

We have by substituting in (1) $T = \mu v a \sin(\theta + C)$. A



If every element of the string border on motion, the equations (1) hold throughout the length. Since T must be zero when $\theta = 0$, we find that $C = 0$. Hence, if aa be the given length of the string AB , the force required to just move it is given by $F = \mu v a \sin \alpha$. It is evident that this result does not hold if the length of the string exceed a quadrant, for then ψ at the elements near B would be greater than a right angle.

Supposing the arc AB to be greater than a quadrant, let the force F acting at B increase gradually from zero. When $F = \mu v a \sin \alpha$, where $\alpha < \frac{1}{2}\pi$, it follows from what precedes that a finite arc EB , terminating at B and subtending at O an angle $\angle EOB$ equal to α , is bordering on motion, and that the tension at E is zero. When $F = \mu v a$ the resolved part of the tension at B along the normal is $\mu v a d\theta$, and is just balanced by the friction. When F increases beyond the value $\mu v a$, the whole friction is insufficient to balance the normal force.

Summing up, the force required to move the string is $F = \mu aw \sin \alpha$ if the length is less than a quadrant. If the length exceed a quadrant, the force is μaw , and the string begins to move at the extremity at which the force is applied. See Art. 190.

Ex. 3. If a weightless string stretched by two weights lie in one plane across a rough sphere of radius a , show that the distance of the plane from the centre cannot exceed $a \sin \epsilon$, where ϵ is the angle of friction. [St John's Coll., 1889]

438. Virtual Work. The equations of equilibrium of a string may be deduced from the principle of virtual work by taking each element separately, and following the general method indicated in Art. 203. In fact the left-hand side of the x equation given in Art. 455, after multiplication by $ds \cdot dx$, is the virtual moment resulting from a displacement dx . This method requires that the tensions at the ends of the element should be included as part of the impressed forces. The principle may also be expressed as a max-min condition (Art. 212) in a form which includes only the given external forces. As an example of this let us consider the following problem

A heterogeneous string of given length l , fixed at its extremities A, B , is in equilibrium in one plane in a field of force whose potential is V . It is required to find the form of the string

Supposing $m = f(s)$ to be the line density at a point whose arc distance from A is s , the work function for the whole string is $\int V m ds$, the limits being 0 to l . We shall take the arc s as the independent variable and regard x, y as two functions of s connected by the equation

$$\left(\frac{dx}{ds}\right)^2 + \left(\frac{dy}{ds}\right)^2 = 1 \quad \dots \dots \dots (1).$$

Following Lagrange's rule we remove the restriction (1) and make

$$u = \int \left\{ Vm + \lambda \left(\left(\frac{dx}{ds}\right)^2 + \left(\frac{dy}{ds}\right)^2 - 1 \right) \right\} ds \quad \dots \dots \dots (2),$$

a max-min for all variations of x and y , the quantity λ being an arbitrary function of s , afterwards chosen to make the resulting values of x, y satisfy the condition (1)*.

As the limits are fixed, there is no obvious advantage in varying all the coordinates. We shall therefore take the variation of u on the supposition that x, y are variable and s constant. We have

$$\delta u = \int \left\{ m \left(\frac{dV}{dx} \delta x + \frac{dV}{dy} \delta y \right) + 2\lambda \left(\frac{dx}{ds} \frac{\delta x}{ds} + \frac{dy}{ds} \frac{\delta y}{ds} \right) \right\} ds.$$

Integrating the third and fourth terms by parts and remembering that $\delta x, \delta y$ vanish at the fixed ends of the string, we find

$$\delta u = \int \left\{ \left(m \frac{dV}{dx} - 2 \frac{d}{ds} \left(\lambda \frac{dx}{ds} \right) \right) \delta x + \left(m \frac{dV}{dy} - 2 \frac{d}{ds} \left(\lambda \frac{dy}{ds} \right) \right) \delta y \right\} ds.$$

At a max-min, this must be zero for all values of $\delta x, \delta y$, hence

$$m \frac{dV}{dx} - 2 \frac{d}{ds} \left(\lambda \frac{dx}{ds} \right) = 0, \quad m \frac{dV}{dy} - 2 \frac{d}{ds} \left(\lambda \frac{dy}{ds} \right) = 0 \quad \dots \dots (3).$$

Restoring the condition (1) we have now three equations from which x, y , and λ

* We regard s as the abscissa, x, y as the two ordinates of an unknown curve, which is to be found by making u a max-min for all variations of x, y . The rules of the calculus of variations then enable us to write down the equations to find the curve. The equation of this curve contains λ and is made to satisfy (1) by a proper choice of this quantity. Then since (2) is a max-min for all variations of x, y , it follows that $\int V m dx$ is a max-min for those variations of x, y which satisfy the condition (1).

may be determined as functions of s . It is evident that these agree with the equations already found in Art. 455, with -2λ written for T .

We may also deduce the value of λ by multiplying the equations (3) respectively by dx/ds and dy/ds and adding. We then find

$$m \frac{dV}{ds} = \frac{1}{\lambda} \frac{d}{ds} \lambda^2 \left\{ \left(\frac{dx}{ds} \right)^2 + \left(\frac{dy}{ds} \right)^2 \right\} = 2 \frac{d\lambda}{ds},$$

which agrees with the equation to determine the tension in Art. 479.

If the string is in three dimensions and constrained to rest on a smooth surface, we make $\int V m ds$ a max-min subject to the two conditions

$$x'^2 + y'^2 + z'^2 - 1 = 0, \quad F(x, y, z) = 0 \dots \dots \dots (I),$$

where accents denote differentiations with regard to s . Following the same method as before we make

$$u = \{ V m + \lambda (x'^2 + y'^2 + z'^2 - 1) + \mu F(x, y, z) \} ds$$

a max-min. Varying only x, y, z and integrating by parts exactly as before, we find on equating the coefficients of $\delta x, \delta y, \delta z$ to zero

$$m \frac{dV}{dx} - 2 \frac{d}{ds} \left(\lambda \frac{dx}{ds} \right) + \mu \frac{dF}{dx} = 0, \quad \&c. = 0, \quad \&c. = 0 \dots \dots \dots (II),$$

the two latter equations being obtained from the first by writing y and z respectively for x . These three equations joined to the conditions (I) determine x, y, z, λ, μ in terms of s . These agree with the equations obtained in Art. 478, when -2λ and $-\mu(F_x^2 + F_y^2 + F_z^2)^{\frac{1}{2}}$ are written for T and R .

489. Elastic Strings. The theory of elastic strings depends on a theorem which is usually called *Hooke's law*. This may be briefly enunciated in the following manner. Let an extensible string uniform in the direction of its length have a natural length l_1 . Let this string be stretched by the application of two forces at its extremities, and let these forces be each equal to T . Let the stretched length of the string be l . Then it is found by experiment that the extension $l - l_1$ bears to the force T a ratio which is constant for the same string.

If the natural or unstretched length of the string were doubled so as to be $2l_1$, the force T being the same as before, it is clear that each of the lengths l_1 would be stretched exactly as before to a length l . The extension of this string of double length will therefore be twice that of the single string. More generally, we infer that the extension must be proportional to the natural length when the stretching force is the same.

Joining these two results together, we see that

$$l - l_1 = l_1 \frac{T}{E},$$

where E is some constant, which is independent to the natural length of the string and of the force by which it is stretched.

It is clear that, if two similar and equal strings are placed side by side, they will together require twice the force to produce the same extension that each string alone would require. It follows that the force required to produce a given extension is proportional to the area of the section of the unstretched string. The coefficient E is therefore proportional to the area of the section of the string when unstretched. The value of E when referred to a sectional area equal to the unit of area is called *Young's modulus*.

To find the meaning of the constant E , let us suppose that the string can be stretched to twice its natural length without violating Hooke's law. We then have $l = 2l_1$, and therefore $E = T$. Thus E is a force, it is the force which would theoretically stretch the string to twice its natural length.

490. This law governs the extension of other substances besides elastic strings. It applies also to the compression and elongation of elastic rods. It is the basis of the mathematical theory of elastic solids. But at present we are not concerned with its application except to strings, wires, and such like bodies.

The law is true only when the extension does not exceed certain limits, called the limits of elasticity. When the stretching is too great the body either breaks or receives such a permanent change of structure that it does not return to its original length when the stretching force is removed. In all that follows, we shall suppose this limit not to be passed.

The reader will find tables of the values of Young's modulus and the limits of elasticity for various substances given in the article *Elasticity*, written by Sir W. Thomson (Lord Kelvin), for the *Encyclopædia Britannica*.

491. Ex. 1. A uniform rod AB , suspended by two equal vertical elastic strings, rests in a horizontal line; a fly alights on the rod at C , its middle point, and the rod is thereupon depressed a distance h ; if the fly walk along the rod, then when he arrives at P , the depression of P below its original level is $2h (AP^2 + BP^2)/AB^2$, and the depression of Q , any other point of the rod, is $2h (AP \cdot AQ + BP \cdot BQ)/AB^2$.

[St John's Coll., 1887.]

Ex. 2. A heavy lamina is supported by three slightly extensible threads, whose unstretched lengths are equal, tied to three points forming a triangle ABC . Show that when it assumes its position of equilibrium the plane of the lamina will meet what would be its position in case the threads were inelastic in the line whose areal equation is $xx_0/E + yy_0/F + zz_0/G = 0$, where E, F, G are the moduli, and x_0, y_0, z_0 the areal coordinates of the centre of gravity of the lamina referred to the triangle ABC .

[St John's Coll., 1885.]

492. A uniform heavy elastic string is suspended by one extremity and has a weight W attached to the other extremity. Find the position of equilibrium and the tension at any point.

Let OA_1 be the unstretched string, P_1Q_1 any element of its length. Let OA be the stretched string, PQ the corresponding position of P_1Q_1 . Let w be the weight of a unit of length of unstretched string, $l_1 = OA_1$, $x_1 = OP_1$; $l = OA$, $x = OP$. The tension T at P clearly supports the weight of PA and W . Hence

$$T = w(l_1 - x_1) + W \dots \dots \dots (1).$$

If PA were equally stretched throughout we could apply Hooke's law to the finite length PA . But as this is not the case we must apply the law to an elementary length PQ . We have therefore

$$dx - dx_1 = dx_1 \epsilon T \dots \dots \dots (2),$$

where ϵ has been written for the reciprocal of E .

Eliminating T ,
$$\frac{dx}{dx_1} = 1 + \epsilon \{w(l_1 - x_1) + W\}.$$

Integrating,
$$x = x_1 + \epsilon \{w(l_1 x_1 - \frac{1}{2} x_1^2) + W x_1\} + C.$$

The constant C introduced in the integration is clearly zero, since x_1 and x must vanish together. Putting $x_1 = l_1$, we find

$$l - l_1 = \frac{1}{2} \epsilon \cdot w l_1^2 + \epsilon W l_1.$$

If the string had no weight, the extension due to W would be $\epsilon W l_1$. If there were no weight W at the lower end, the extension would be $\frac{1}{2} \epsilon w l_1^2$. Hence the extension due to the weight of the string is equal to that due to half its weight attached to the lowest point. We also see that the extension due to the weight of the string and the attached weight is the sum of the extensions due to each of these treated separately.

Ex. 1. A heavy elastic string OA placed on a rough inclined plane along the line of greatest slope is attached by one extremity O to a fixed point, and has a weight W fastened to the other extremity A . Find the greatest length of the stretched string consistent with equilibrium.

When the string is as much stretched as possible, the friction on every element acts down the plane and has its limiting value. Let α be the inclination of the plane to the horizon. Let μ, μ' be the coefficients of friction between the plane and the string and between the plane and the weight respectively. If $f = \sin \alpha + \mu \cos \alpha$, then $f w$ replaces w in Art. 492. We therefore find for the whole elongation $l' - l = \frac{1}{2} \epsilon f w l_1^2 + \epsilon f' W l$, where f' is what f becomes when μ' is written for μ .

Ex. 2. A heavy elastic string AA' is placed on a rough inclined plane along the

line of greatest slope. Supposing the inclination of the plane to be less than $\tan^{-1}\mu$, find the greatest length to which the string could be stretched consistent with equilibrium. Compare also the stretching of the different elements of the string.

The frictions near the lower end A of the string will act down the plane, while those near the upper end A' will act up the plane. There is some point O separating the string into two portions OA , OA' in which the frictions act in opposite directions. Each of these portions may be treated separately by the method used in the last example. An additional equation, necessary to find the unstretched length z of OA , is obtained by equating the tensions at O due to the two portions. The results are

$$z = \frac{l_1}{2} \left(1 - \frac{\tan \alpha}{\mu} \right), \quad l - l_1 = \frac{1}{2} \epsilon \mu w \cos \alpha l_1^2 \left(1 - \frac{\tan^2 \alpha}{\mu^2} \right).$$

Ex. 3. A series of elastic strings of unstretched lengths l_1, l_2, l_3 are fastened together in order, and suspended from a point, l_1 being the lowest. Show that the total extension is

$$\frac{1}{2} (\epsilon_1 w_1 l_1^2 + \epsilon_2 w_2 l_2^2 + \dots) + w_1 l_1 (\epsilon_2 l_2 + \epsilon_3 l_3 + \dots) + w_2 l_2 (\epsilon_3 l_3 + \dots) + \dots,$$

where w_1, w_2, \dots are the weights per unit of length of unstretched string, $\epsilon_1, \epsilon_2, \dots$ the reciprocals of the moduli of elasticity. [Coll. Exam., 1888.]

493. Work of an elastic string. If the length of a light elastic string be altered by the action of an external force, *the work done by the tension is the product of the compression of the string and the arithmetic mean of the initial and final tensions.*

In the standard case let the length be increased from a to a' , then $a - a'$ is the shortening or compression of the string. As before, let l_1 be the unstretched or natural length.

By referring to Art. 197, we see that the work required is

$$- \int T dl = - \int E \frac{l - l_1}{l_1} dl = - E \frac{(a' - l_1)^2 - (a - l_1)^2}{2l_1},$$

the limits of the integral being from $l = a$ to $l = a'$. This result may be put into the form $\frac{1}{2} (T_1 + T_2) (a - a')$, where T_1 and T_2 represent the values of T when a and a' are written for l . The rule follows immediately. See the author's *Rigid Dynamics* 1877.

This rule is of considerable use in dynamics where the length of the string may undergo many changes in the course of the motion. It is important to notice that the rule holds even if the string becomes slack in the interval, provided it is tight in the initial and final states. If the string is slack in either terminal state, we may still use the same rule provided we suppose the string to have its natural or unstretched length in that terminal state.

Ex. 1. Show that the depth below the point of suspension O of the centre of gravity of the elastic string considered in Art. 492 is $\frac{1}{2} l_1 + \epsilon l_1 (\frac{1}{3} S + \frac{1}{2} W)$, where S is the weight of the string. Show also that the work done by gravity as the string and weight are moved from the unstretched position OA_1 to the stretched position OA , is $\epsilon l_1 (\frac{1}{3} S^2 + SW + W^2)$ where $\epsilon = 1/E$.

Ex. 2. Let one end of an elastic string be fixed to the rim of a wheel sufficiently rough to prevent sliding, and let the other be attached to a mass resting on the

ground, so that when the string (of length a) is just taut it shall be vertical. Show that the work which must be spent in turning the wheel so as just to lift the mass off the ground is $Mga + Ea \log E/(E + Mg)$, where E is the tension which would double the length of the string, neglecting the weight of the string. [Math. Tripos.]

Ex. 3. A disc of radius r is connected by n parallel equal elastic strings, of natural length l_1 , to an equal fixed disc; the wrench necessary to maintain the discs at a distance x apart with the moveable one turned through an angle θ about the common axis, consists of a force X and a couple L given by

$$X = nEx \left(\frac{1}{l_1} - \frac{1}{\xi} \right), \quad L = 2nEr^2 \sin \theta \left(\frac{1}{l_1} - \frac{1}{\xi} \right),$$

where $\xi^2 = x^2 + 4r^2 \sin^2 \frac{1}{2} \theta$.

[Coll. Exam., 1885.]

One disc being moved to a distance x from the other and turned round through an angle θ , we first show that the length of each string is changed from l_1 to ξ . Using the rule above, the work function is $W = n \cdot \frac{1}{2} T (\xi - l_1) = nE (\xi - l_1)^2 / 2l_1$.

By Art. 208 we have $Xdx + Ld\theta = \frac{dW}{dx} dx + \frac{dW}{d\theta} d\theta$.

Effecting the differentiations $X = dW/dx$, $L = dW/d\theta$, we obtain the results given.

494. Heavy elastic string on a smooth curve. Ex. 1. A heavy elastic string is stretched over a smooth curve in a vertical plane: show that the difference between the values of $T + T^2/2E$ at any two points of the string is equal to the weight of a portion of the string whose unstretched length is the vertical distance between the points. It follows from this theorem that any two points at which the tensions are equal are on the same level.

If ds_1 is the unstretched length of any element ds of the string, we have by Hooke's law $ds_1 = dsE/(T + E)$. If then w is the weight per unit of *unstretched length*, the weight of any element ds of the stretched string is equal to $w'ds$, where $w' = wE/(T + E)$. Let us now form the equations of equilibrium, using the same figure and reasoning as in Art. 459, where a similar problem is discussed for an inextensible string. We evidently arrive at the same equations (1) and (2) with w' written for w . Substituting for w' and integrating, we find that (1) leads to the result given above.

Ex. 2. A heavy elastic string is stretched on a smooth curve in a vertical plane:

show that $T + \frac{T^2}{2E} = wy$, $R\rho - \frac{T^2}{2E} = wy'$,

where T is the tension at any point P , R the outward pressure of the curve on the string per unit of length of unstretched string, w the weight of a unit of length of unstretched string, and y, y' the altitudes of P and its anti-centre above a fixed horizontal line called the *statical directrix* of the string, Art. 460. Show also that no part of the string can be below the directrix, and that the free ends, if there are any, must lie on it.

Ex. 3. A heavy elastic string rests in equilibrium on a smooth cycloid with its cusps upwards. If one extremity is attached to a point on the curve while the free extremity is at the vertex, prove that the stretched length of any unstretched arc s_1 measured from the vertex is given by $\gamma s = \sinh \gamma s_1$, where $4aE\gamma^2 = w$, and a is the radius of the generating circle.

Ex. 4. An elastic string rests on a smooth curve whose plane is vertical with its ends hanging freely. Show that the natural length σ may be found from the

equation $\left(\frac{d\sigma}{ds}\right)^2 = \frac{b}{2y+b}$, where y is the vertical height above the free extremities, and b the natural length of a portion of the string whose weight is the coefficient of elasticity. If the natural length of each vertical portion be l , and if $(l+b)^2 = 2ab$, and if the curve be a circle of radius a , prove that the natural length of the portion in contact with the curve is $2\sqrt{(ab) \log(\sqrt{2}+1)}$. [June Exam, 1877.]

Ex 5. An elastic string, uniform when unstretched, lies at rest in a smooth circular tube under the action of an attracting force (μr) tending to a centre on the circumference of the tube diametrically opposite to the middle point of the string. If the string when in equilibrium just occupies a semicircle, prove that the greatest tension is $\{\lambda(\lambda + 2\mu a^2)\}^{\frac{1}{2}} - \lambda$, where λ is the modulus of elasticity, a the radius of the tube, ρ the mass of a unit of length of the unstretched string.

[Trinity Coll., 1878]

Ex. 6. An infinite elastic string, whose weight per unit of length when unstretched is m , and which requires a tension ma to stretch any part of it to double its length (when on a smooth table), is placed on a rough table (coefficient μ) in a straight line perpendicular to its edge. The string just reaches the edge, which is smooth. A weight $\frac{1}{2}ma\mu$ is attached to the end and let hang over the edge. If the weight takes up its position of rest quietly, so that no part of the string re-contracts after having been once stretched, show that the distance of the weight below the edge of the table is $\frac{3}{2}a\mu(3\mu+4)$, and that beyond a distance $\frac{3}{2}a(\mu+2)$ from the edge of the table the string is unstretched.

[Trinity Coll.]

495. Light elastic string on a rough curve. Ex. 1. An elastic string is stretched over a rough curve so that all the elements border on motion. If no external forces act on the string except the tensions F, F' at its extremities, then $\frac{F'}{F} = e^{\pm\mu\psi}$, where ψ is the angle between the normals to the curve at its extremities.

This follows by the same reasoning as in Art. 463.

Ex. 2. An elastic string (modulus λ) is stretched round a rough circular arc so that every element of it is just on the point of slipping, if T, T' are the tensions at its extremities, the ratio of the stretched to the unstretched length is

$$\log \frac{T'}{T} \cdot \log \frac{T'(T+\lambda)}{T(T'+\lambda)}. \quad [\text{St John's Coll., 1884.}]$$

Ex. 3. An endless cord, such as a cord of a window blind, is just long enough to pass over two very small fixed pulleys, the parts of the cord between the pulleys being parallel. The cord is twisted, the amount of twisting or torsion being different in the two parts, and the portions in contact with the pulleys being unable to untwist. If the pulleys be made to turn slowly through a complete revolution of the string, show that the quotient of the difference by the sum of the torsions is decreased in the ratio $e^4 : 1$.

[Math. Tripos, 1853.]

Ex. 4. An elastic band, whose unstretched length $= 2a$, is placed round four rough pegs A, B, C, D , which constitute the angular points of a square of side $= a$. If it be taken hold of at a point P between A and B , and pulled in the direction AB , show that it will begin to slip round both A and B at the same time if $AP = a/(e^{\frac{1}{2}\mu\pi} + 1)$.

[May Exam.]

Ex. 5. An endless slightly extensible strap is stretched over two equal pulleys: prove that the maximum couple which the strap can exert on either pulley is $\frac{2a(c+\pi a)}{c \coth \frac{1}{2}\mu\pi + 2a/\mu} T$, where a is the radius of either pulley, c the distance of their

centres, μ the coefficient of friction, and T the tension with which the strap is put on. [Math. Tripos, 1879.]

Ex. 6. A rough circular cylinder (radius a) is placed with its axis horizontal, and a string, whose natural length is l , is fastened to a point Q on the highest generator of the cylinder and to an external point P at a distance l from Q , PQ being horizontal and perpendicular to the axis of the cylinder; the cylinder is then slowly turned upon its fixed axis in the direction away from P ; show that the string will slip continually along the whole of the length in contact with the cylinder until S (the natural length of the part wound up) $= a/\mu$, when all slipping will cease, and that up to this stage the relation between S and θ (the angle turned through by the cylinder) is $le^{\mu\phi} = (l - a\phi)e^{\mu\theta} + a\phi$, where $S = a\phi$. [Coll. Exam., 1880.]

496. Elastic string, any forces. *To form the equations of equilibrium of an elastic string under the action of any forces.*

Let ds_1 be the unstretched length of any element ds of the string. Then by Hooke's law $ds = ds_1(T + E)/E$. The forces on the element, due to the attraction of other bodies, will be proportional to the unstretched length. Let then the resolved parts of these forces along the principal axes of the string be Fds_1 , Gds_1 , Hds_1 , as in Art. 454. The equations of equilibrium (1), (2), and (3) of that article are obtained by equating to zero the resolved parts of the forces along the principal axes of the curve; these equations will therefore apply to the elastic string if we replace Fds , Gds , Hds , by Fds_1 , Gds_1 , Hds_1 . The equations of equilibrium for the elastic string may therefore be derived from those for an inelastic string by treating the forces as

$$Fds \frac{E}{T + E}, \quad Gds \frac{E}{T + E}, \quad Hds \frac{E}{T + E},$$

i.e. reducing all the impressed forces in the ratio $E : T + E$.

497. Suppose, for example, that the string rests on any smooth surface. The resolution along the tangent to the string (as in Art. 479) gives

$$\left(1 + \frac{T}{E}\right) dT + Xdx + Ydy + Zdz = 0. \quad \therefore T + \frac{T^2}{2E} + \int (Xdx + Ydy + Zdz) = C.$$

It follows that $T + T^2/2E$ + the work function of the forces is constant along the whole length of the string, Art. 479.

Ex. When gravity is the only force acting, show that the equations of equilibrium of an elastic string corresponding to (1), (2), (3) of Art. 479 may be written in the simple forms

$$T + \frac{T^2}{2E} = wz, \quad R\rho' - \frac{T^2}{2E} = wz', \quad \left(wz + \frac{T^2}{2E}\right) \tan \chi = w\rho' \sin \theta,$$

where T is the tension at any point P , R the outward pressure of the surface on the string per unit of unstretched length, χ the angle the radius of curvature of the string makes with the normal to the surface, z and z' the altitudes of P and the

anti-centre S above a certain horizontal plane, θ the angle the vertical makes with the plane containing the normal to the surface and the tangent to the string, and w the weight of a unit of unstretched length. If PS be a length measured outwards along the normal to the surface equal to the radius of curvature of a normal section of the surface drawn through the tangent at P to the string, S is the anti-centre of P .

If the surface is one of revolution with its axis vertical, we replace the third equation by $Tr' \sin \psi = B$, where r' is the distance of P from the axis of the surface, ψ the angle the tangent to the string makes with the meridian and B is a constant. See Art. 481.

498. To take another example, suppose that the elastic string is under the action of a central force. Taking moments about the centre of force, and resolving along the tangent to the string, we find, after integration,

$$T^2 = A, \quad T + \frac{T^2}{2E} + \int F dr = C.$$

These equations may be treated in a manner somewhat similar to that adopted for inelastic strings.

499 Ex. 1. An elastic string rests in equilibrium in the form of an arc of a circle under the influence of a centre of force at any unoccupied point of the circle. Show that the law of force is $F = \frac{\mu}{r^3} \left(1 + \frac{\mu}{2E} \frac{1}{r^3} \right)$.

Ex. 2. An elastic string, whose elements repel each other with a force proportional to the product of their masses into the square of their distance, rests in equilibrium on a smooth horizontal plane. If T be the tension at a point whose distance from one extremity is y , show that $\frac{d^4}{dy^4} (T + E)^2 + \frac{c^2}{T + E} = 0$, where c is a constant depending on the nature of the string. Explain also how the constants of integration are to be determined.

Ex. 3. An elastic string, whose elements repel each other with a force which varies as the distance, rests on a smooth horizontal plane. If $2l_1$ and $2l$ be the unstretched and stretched lengths of the string, show that $cl = \tan cl_1$, where $E c^2 dx$ is the force due to the whole string on an element whose unstretched length is dx when placed at a unit of distance from the middle point of the string.

Ex. 4. A uniform elastic string lying on a rough horizontal plane is fixed to two points, and forms a curve every part of which is on the point of motion. Show that the tension is given by the equation $\left(1 + \frac{t}{\lambda} \right)^2 \left\{ \left(\frac{dt}{d\psi} \right)^2 + t^2 \right\} = \mu^2 w^2 \rho^2$, where w is the weight per unit of length of the unstretched string, μ the coefficient of friction and ρ the radius of curvature. [Math. Tripos, 1881]

Ex. 5. An elastic string has its two ends fastened to points on the surface of a smooth circular cylinder of which the axis is vertical; show that in the position of equilibrium of the string on the surface the density of the string at any point varies as the tangent of the angle which the osculating plane at that point makes with a normal section of the cylinder through the direction of the string. [Math. T., 1886.]

500. A heavy elastic string is suspended from two fixed points and is in equilibrium in a vertical plane. To find its equation.

We may here use the same method as that employed in Art. 443 to determine the form of equilibrium of an inelastic string. Referring to the figure of that article, let the unstretched length of CP (i.e. the arc measured from the lowest point up to any point P) be s_1 , and let the rest of the notation be the same as before. Consider the equilibrium of the finite portion CP ;

$$T \cos \psi = T_0 \dots (1), \quad T \sin \psi = ws_1 \dots (2),$$

$$\therefore \frac{dy}{dx} = \tan \psi = \frac{ws_1}{T_0} = \frac{s_1}{c} \dots (3).$$

From these equations we may deduce expressions for x and y in terms of some subsidiary variable. Since $s_1 = c \tan \psi$ by (3), it will be convenient to choose either s_1 or ψ as this new variable.

Adding the squares of (1) and (2), we have

$$T^2 = w^2 (c^2 + s_1^2) \dots (4).$$

Since $dx/ds = \cos \psi$ and $dy/ds = \sin \psi$, we have by (1) and (2)

$$x = \int \frac{T_0}{T} ds = \int \frac{wc}{T} \left(1 + \frac{T}{E}\right) ds_1 = \frac{wc}{E} s_1 + c \log \frac{s_1 + \sqrt{(c^2 + s_1^2)}}{c},$$

$$y = \int \frac{ws_1}{T} ds = w \int \frac{s_1}{T} \left(1 + \frac{T}{E}\right) ds_1 = \frac{w}{2E} (c^2 + s_1^2) + \sqrt{(c^2 + s_1^2)},$$

where the constants of integration have been chosen to make $x=0$ and $y=c + c^2w/2E$ at the lowest point of the elastic catenary. The axis of x is then the statical directrix, Art. 494, Ex. 2.

501. Ex. 1. Prove the following geometrical properties of the elastic catenary

$$(1) \quad wy = T + \frac{T^2}{2E}, \quad (2) \quad \rho = \frac{c^2 + s_1^2}{c} \left\{1 + \frac{w}{E} \sqrt{(c^2 + s_1^2)}\right\},$$

$$(3) \quad s = s_1 + \frac{w}{2E} \left\{s_1 \sqrt{(c^2 + s_1^2)} + c^2 \log \frac{s_1 + \sqrt{(c^2 + s_1^2)}}{c}\right\},$$

all of which reduce to known properties of the common catenary when E is made infinite.

Ex. 2. Let M, M' be two points taken on the ordinate PN so that MM' is bisected in N by the statical directrix and let each half be equal to $T^2/2Ew$. If M be above the directrix draw ML perpendicular to the tangent at P . Show that $T = w \cdot PM$, $s_1 = PL$, $c = ML$, $w \cdot MN = T^2/2E$ and that M' is the projection of the anti-centre on the ordinate.

Ex. 3. An elastic string, uniform when unstretched, is hung up by two points. Prove that the intrinsic equation of the catenary in which it will hang under

$$\text{gravity is} \quad s = c \tan \psi + \frac{c^2}{2\lambda} \left\{ \tan \psi \sec \psi + \log \tan \left(\frac{\pi}{4} + \frac{\psi}{2} \right) \right\},$$

where c is the natural length of the string whose weight is equal to the tension at the lowest point, from which s is measured, and λ is the natural length of the string whose weight is equal to the modulus of elasticity. [Coll. Exam., 1880.]

CHAPTER XI

THE MACHINES

502. It is usual to regard the complex machines as constructed of certain simple combinations of cords, rods and planes. These combinations are called the *mechanical powers*. Though given variously by different authors, they are generally said to be six in number, viz. the lever, the pulley, the wheel and axle, the inclined plane, the wedge and the screw*.

Mechanical advantage. In the simplest cases they are usually considered as acted on by two forces. One of these, viz. the force applied to work the machine, is usually called *the power*. The other, viz. the force to be overcome, or the weight to be raised, is called *the weight*. The ratio of the weight to the power is called the *mechanical advantage* of the machine.

503. As a first approximation, we suppose that the several parts of the machine are smooth, the cords used perfectly flexible, the solid parts of the machine rigid, and so on. In some of the machines these suppositions are nearly true, but in others they are far from correct. It is therefore necessary, as a second approximation, to modify these suppositions. We take such account as we can of the roughness of the surfaces in contact, the rigidity of the cords and the flexibility of the materials. After these corrections have been made, our result is still only an approximation to the truth, for the corrections cannot be accurately made. For example, in making allowance for friction we assume that the bodies in contact are equally rough throughout, and that the coefficient of friction is properly known. The results however thus obtained are much nearer the real state of things than our first approximation.

504. Efficiency. Suppose a machine to be constructed of a combination of levers, pulleys, &c., each acting on the next in order.

* In the descriptions of the machines given in this chapter, the author has derived much assistance from Capt. Kater's *Treatise on Mechanics* in Lardner's *Cyclopædia*, 1830, Pratt's *Mechanical Philosophy*, 1842, Willis' *Principles of Mechanism*, 1870, and other books.

Let a force P acting at one extremity of the combination produce a force at the other extremity such that it could be balanced by a force Q acting at the same point. Then, for this machine, P may be regarded as the power and Q as the weight.

Let the machine be made to work, so that its several parts receive small displacements consistent with their geometrical relations. Such a displacement is called an *actual displacement* of the machine. Taking this as a virtual displacement, the work of the force P is equal to that of the force Q together with the work of the resistances of the machine. These resistances are friction &c., in overcoming which some of the work done by the power is said to be wasted or lost. The work done by the force Q is called the *useful work* of the machine. *The efficiency of a machine is the ratio of the useful work to that done by the power when the machine receives any small actual displacement.* It appears that the efficiency of a machine would be unity if all its parts were perfectly smooth, the solid parts perfectly rigid, and so on. In all existing machines however the efficiency is necessarily less than unity.

505. Ex. In any machine for raising a weight show that, if the weight remains suspended by friction when the machine is left free, the efficiency is less than one half. If however a force P be required to raise the weight, and a force P' be required to prevent it from descending, show that the efficiency will be $(P + P')/2P$, supposing the machine to be itself accurately balanced. [St John's Coll., 1884.]

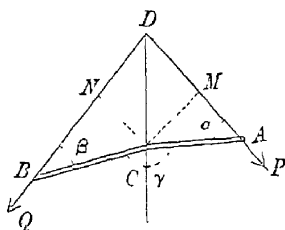
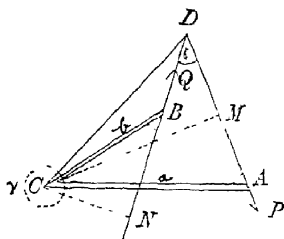
When the force P just raises a weight Q , the friction acts in opposition to the power P ; on the contrary it assists P' in supporting Q . The frictions in the two cases are evidently the same in magnitude, being the extreme amounts which can be called into play. Let x, y be the virtual displacements of the points of application of P, Q when the machine is worked, and let the same small displacement be given in each case. Let U be the work of the frictions. Then $Px = Qy + U$, and $P'x = Qy - U$. The efficiency of the machine is measured by the ratio Qy/Px . Eliminating U , we easily obtain the result given. If any of the resistances, other than friction, have no superior limit, but continually increase with the increase of the power, it is easy to see by the same reasoning that the efficiency will be less than the value found above.

506. The lever. A lever is a rigid rod, straight or bent, moveable about a fixed axis. The fixed axis is usually called the *fulcrum*. The portions of the lever between the fulcrum and the points of application of the power and the weight are called the *arms of the lever*. The forces which act on the lever are usually supposed to act in a plane which is perpendicular to the fixed axis.

When the forces act in any directions at any points of the body, the problem is one in three dimensions, the solution of which is given in Art. 268. In what follows we shall also neglect the friction at the axis, as that case has already been considered in Art. 179.

507. *To find the conditions of equilibrium of two forces acting on a lever in a plane perpendicular to its axis.*

The axis of the lever is regarded in the first approximation as a straight line; let C be its intersection with the plane of the forces.



Let the forces be P and Q . Let them act at A and B on the arms CA , CB in the directions DA , DB . When the lever is in its position of equilibrium, the forces P , Q and the reaction at the fulcrum must form a system of forces in equilibrium. Hence the resultant of P and Q must act along DC , and be balanced by the pressure on the fulcrum.

The conditions of equilibrium follow at once from the principles stated in Art. 111. Let CM , CN be perpendiculars drawn from C on the lines of action of the forces. Taking moments about C , we have $P \cdot CM - Q \cdot CN = 0$. It follows that in a lever, *the power and the weight are to each other inversely as the perpendiculars drawn from the fulcrum on their lines of action.*

508. *To find the pressure on the fulcrum,* we find the resultant of the two forces P , Q by any one of the various methods usually employed to compound forces. For example, if the position of D be known, let ϕ be the angle ADB , we then have $R^2 = P^2 + Q^2 + 2PQ \cos \phi$, where R is the required pressure.

Let $CA = a$, $CB = b$, and let α , β be the angles the directions of the forces P , Q make with the arms CA , CB . Let γ be the angle ACB . If these quantities are known, we may find the pressure by another method. Let θ be the angle the line of action of R makes with the arm CA , so that the angle DCA is $\pi - \theta$. Then, resolving the forces along and perpendicular to CA , we have

$$\begin{aligned} R \cos \theta &= P \cos \alpha + Q \cos (\gamma - \beta) \\ R \sin \theta &= P \sin \alpha + Q \sin (\gamma - \beta) \end{aligned}$$

whence $\tan \theta$ and R can be easily found.

Other relations between P , Q and R may be found by taking moments about A , B or some other point suggested by the data of the question. In the same way

other resolutions will sometimes be more convenient than those given above as specimens.

509. When several forces act on the lever, we find *the condition of equilibrium by equating to zero the sum of their moments about the fulcrum*, each moment being taken with its proper sign. The moments are taken about the fulcrum to avoid introducing into the equation the reaction at the axis.

To find the pressure on the fulcrum we transfer each force parallel to itself, in the plane perpendicular to the axis, to act at the fulcrum. We thus obtain a system of forces acting at a single point, viz. the intersection of the axis with the plane of the forces. The resultant of these is the pressure on the axis.

510. In the investigation the weight of the lever itself has been supposed to be inconsiderable compared with the forces P and Q . If this cannot be neglected, let W be the weight of the lever. There are now three forces acting on the body instead of two. These are P , Q acting at A and B , and W acting at the centre of gravity G of the lever. Let the fulcrum be horizontal, and let CL be the perpendicular distance between the fulcrum and the vertical through G . Let us also suppose that in the standard figure the weight W and the force P tend to turn the lever round the fulcrum in the same direction. The equation of moments now becomes $P \cdot CM - Q \cdot CN + W \cdot CL = 0$. The pressure on the fulcrum is found by compounding the forces P , Q , W .

511. Levers are usually divided into three kinds according to the relative positions of the power, the weight, and the fulcrum. In the first kind, the fulcrum is between the power and the weight. In the second kind the weight acts between the fulcrum and the power, and in the third kind the power acts between the fulcrum and the weight. The investigation in Art 507 applies to all three kinds, the only distinction being in the signs given to the forces and the arms, in resolving and taking moments.

512. The *mechanical advantage* of the lever is measured by the ratio $Q:P$. This ratio has been proved to be equal to $CN:CM$. By applying the power so that its perpendicular distance from the fulcrum is greater than that of the weight, a small power may be made to balance a large weight. Thus a crowbar when used to move a body is a lever of the second kind. The ground is the fulcrum, the weight acts near the fulcrum, and the power is applied at the extreme end of the bar.

513. If the lever be slightly displaced by turning it round its fulcrum through a small angle, the points of application A , B of the forces P , Q are moved through small arcs AA' , BB' , whose centres are on the fulcrum. Thus the actual displacements of the points of application of the power and the weight are proportional to their distances from the fulcrum. It is however the resolved part of the displacement AA' in the direction of the force P which measures the speed of working. For example, if the force P were applied by pulling a rope attached to the point A , the amount of rope to be pulled in would be measured by the resolved part of AA' in the direction of the length of the rope. The resolved parts of AA' , BB' in the direction of the forces P , Q are evidently $AA' \cdot \sin \alpha$, $BB' \cdot \sin \beta$. These are proportional to $CA \sin \alpha$,

$CB \sin \beta$, i.e. to CM , CN . (See fig. of Art. 516.) These resolved displacements are clearly the same as the virtual displacements of the points of application; Art. 64.

If then mechanical advantage is gained by arranging the lever so that the weight is greater than the power, the displacement of the weight is less, in the same ratio, than that of the power, each displacement being resolved in the direction of its own force. It follows that *what is gained in power is lost in speed*.

514. The reader may easily call to mind numerous instances in which levers are used. As examples of levers of the first kind we may mention the common balance, poker, &c.

Wheelbarrows, nutcrackers, &c. are examples of levers of the second kind. In these the weight is greater than the power. They are used when we wish to multiply the force at our disposal.

In levers of the third kind the weight is less than the power, but the virtual displacement of the weight is greater than that of the power. Such levers therefore are used when economy of force is a consideration subordinate to the speed of working.

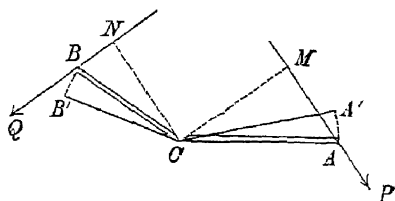
515. The most striking example of levers of the third kind is found in the animal economy. The limbs of animals are generally levers of this description. The socket of the bone is the fulcrum; a strong muscle attached to the bone near the socket is the power; and the weight of the limb, together with whatever resistance is opposed to its motion, is the weight. A slight contraction of the muscle in this case gives a considerable motion to the limb: this effect is particularly conspicuous in the motion of the arms and legs in the human body; a very inconsiderable contraction of the muscles at the shoulders and hips giving the sweep to the limbs from which the body derives so much activity.

The treddle of the turning lathe is a lever of the third kind. The hinge which attaches it to the floor is the fulcrum, the foot applied to it near the hinge is the power, and the crank upon the axis of the fly-wheel, with which its extremity is connected, is the weight.

Tongs are levers of this kind, as also the shears used in shearing sheep. In these cases the power is the hand placed immediately below the fulcrum or point where the two levers are connected. *Capt Kater's Mechanics*.

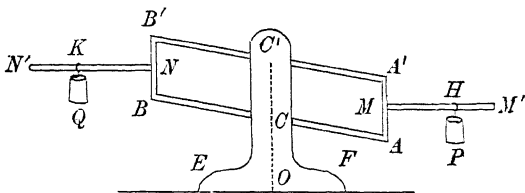
516. The principle of virtual work may be conveniently used to investigate the conditions of equilibrium in the lever. Let P , Q be two forces acting at A and B , and let C be the fulcrum. If the lever be displaced round C through a small angle $\delta\theta$, so that A , B come into the positions A' , B' , we have

$$P \cdot AA' \sin \alpha - Q \cdot BB' \sin \beta = 0,$$



where α, β have the same meanings as in Art. 507. This immediately leads to the result $P \cdot CM = Q \cdot CN$.

517. Roberval's Balance. This machine supplies an excellent example of the principle of virtual work. In this balance the four rods $AA', A'B', B'B, BA$ are hinged at their extremities and form a parallelogram. The sides $AB, A'B'$ are also hinged at the points C, C' to a fixed vertical rod $OC C'$. The line CC' must be parallel to AA' and BB' , but need not necessarily be equidistant from them. Two more rods MM', NN' are rigidly attached to AA', BB' so as to be at right angles to them. These support the weights P and Q suspended in scale-pans from any two points H and K . As the combination turns smoothly round the supports C, C' , the rods AA', BB' remain always vertical, and MM', NN' are always horizontal.



The peculiarity of the machine is that, if the weights P, Q balance in any one position, the equilibrium is not disturbed by moving either of the weights along the supporting rods MM', NN' . It may also be remarked that, if the machine be turned round its two supports C, C' so that one of the rods MM', NN' descends and the other ascends, the two weights continue to balance each other.

To show this, let the equal lengths $CA, C'A'$ be denoted by a , and the equal lengths $CB, C'B'$ by b . Let the inclination to the horizon of the parallel rods $AB, A'B'$ be θ . If the machine is displaced so that the angle θ is increased by $d\theta$, the rod AA' descends a vertical space $a \cos \theta d\theta$, and the rod BB' ascends a space $b \cos \theta d\theta$. When the weights of all the parts of the machine are neglected in comparison with P and Q , we have by the principle of virtual work $Pa \cos \theta d\theta = Qb \cos \theta d\theta$. This gives $Pa = Qb$; thus the condition of equilibrium is independent of the positions H, K at which P and Q act on the supporting rods, and is also independent of the inclination θ of the rods $AB, A'B'$ to the horizon.

If the balance is so constructed that the weights P, Q are equal, when in equilibrium, we can detect whether any difference in weight exists between two given bodies by simply attaching them to any points of the supporting rods. The advantage of the balance is that no special care is necessary to place them at equal distances from the fulcrum.

Ex. 1. If the weights of the rods $AB, A'B'$ are w, w' and the weights of the bodies $AA'M', BB'N'$ are W, W' , prove that the condition of equilibrium is

$$(P + W) a - (Q + W') b + \frac{1}{2} (w + w') (a - b) = 0.$$

Thence show that, if the weights P, Q balance in one position, they will as before balance in all positions. Find also the point of application of the resultant pressure of the stand EF on the supporting table.

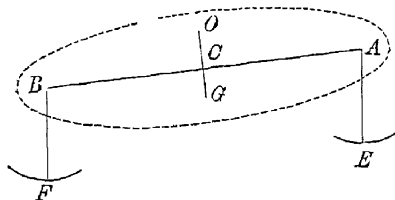
Ex. 2. If the balance be at rest and horizontal, prove that the horizontal pressure on either support bears to either weight the ratio of the difference of the horizontal distances of the centres of gravity of the weights from the central plane of the balance to the distance between the supports. [Math. Tripos, 1874.]

Let $X, Y; X', Y'$, be the horizontal and vertical components of the reactions at

A, A' . By taking moments about A' for the system $AM'A'$ we have $Xa = Ph$, where $AA' = a, MH = h$. We have also $X + X' = 0, Y + Y' = P$. Thus X, X' are known while the separate values of Y and Y' are indeterminate, Arts. 268, 148. Similarly if $X_1, Y_1; X'_1, Y'_1$ are the corresponding components at the points B, B' , we have $X_1a = Ph$ where $NK = h$. Since the rod AB is acted on by $X, Y; X_1, Y_1$ (reversed) at the extremities, the horizontal component of pressure at the pin C is $X - X_1$, which at once leads to the given result.

518. The Common Balance. In the common balance two equal scale-pans E, F are suspended by equal fine strings from the extremities A, B of a straight rod or beam. The rod AB can turn freely about a fulcrum O , with which it is connected by a short rod OC which bisects AB at right angles. The centre of gravity G of the beam AOB lies in the rod OC , and therefore, when the beam and the empty scales are in equilibrium, the straight line AB is horizontal.

The bodies to be weighed are placed in the scale-pans, and if their weights are unequal, the horizontality of the beam AB is disturbed. The centre of gravity G of the beam is now no longer under the point of support, and in the new position of equilibrium the inclination θ of the rod AB to the horizon is such that the moment of the weight of the beam about the fulcrum O is equal to that of the weight of the bodies and the scale-pans. It is therefore evident that the fulcrum should not coincide with the centre of gravity of the beam.



Let P, Q be the weights in the scales E and F , w the weight of either scale, let W be the weight of the beam AOB . Let $OG = h, OC = c, AB = 2a$. Let θ be the inclination of AB to the horizon when the system is in equilibrium. Taking moments about O , we have

$$(P + w)(a \cos \theta + c \sin \theta) - (Q + w)(a \cos \theta - c \sin \theta) + Wh \sin \theta = 0.$$

The coefficient of $P + w$ in this equation is the length of the perpendicular from O on the vertical AE , and is easily found by projecting the broken line OC, CA on the horizontal. The other coefficients are found in the same way. We therefore have $\tan \theta = \frac{(Q - P)a}{(P + Q + 2w)c + Wh}$.

For a minute account of a balance with illustrative diagrams the reader is referred to the tract, "The theory and use of a physical balance," by J. Walker, 1887.

519 A good balance has three requisites. The first is that when loaded with equal weights in the pans the rod AB should be horizontal. This is secured by making the arms AC, CB equal. To determine when the beam is horizontal, a small rod called the *tongue* is attached to it at right angles at its middle point. The beam is usually suspended from a point above O , and when the beam is horizontal the direction of the tongue should pass through the point of suspension.

The second requisite is *sensibility*. When the weights P, Q differ by a small quantity, the angle θ should be so large that it can be easily observed. For a given difference $Q - P$ the sensibility increases as $\tan \theta$ increases. We may therefore measure the sensibility by the ratio $\frac{\tan \theta}{Q - P} = \frac{a}{(P + Q + 2w)c + Wh}$. The

sensibility is therefore secured by so constructing the balance that the expression on the right-hand side of this equation is as large as possible.

The sensibility is therefore increased (1) by increasing the length of the rod AB , (2) by diminishing the length of the rod OC , (3) by diminishing the weight of the beam. If the balance is so constructed that h and c have opposite signs, the sensibility can be greatly increased. This requires that the fulcrum O should lie between G and C .

The third requisite of a balance is usually called *stability*. When the balance is disturbed, it should return readily to its horizontal position. The beam oscillates about its position of equilibrium, and the quicker the oscillation the sooner can it be determined by the eye whether the mean position of the beam is or is not horizontal. The balance should be so constructed that the times of oscillation are as short as possible. The discovery of the nature of the oscillations is a problem in dynamics, and cannot properly be discussed from a statical point of view.

520. Ex. 1. If one arm of a common balance, whose weight can be neglected, is longer than the other, prove that the true weight of a body is the geometrical mean of the apparent weights when weighed first in one scale and then in the other. [Coll. Exam.]

Ex. 2. A balance has its arms unequal in length and weight. A certain article appears to weigh Q_1 or Q_2 according as it is put in the one scale or the other. Similarly another article appears to weigh R_1 or R_2 . Find the true weights of these articles; and show that if an article appears to weigh the same in whichever scale it is put, its weight is $\frac{Q_1 R_2 - Q_2 R_1}{Q_1 - Q_2 - R_1 + R_2}$. [Coll. Exam., 1886.]

Ex. 3. In a false balance a weight P appears to weigh Q , and a weight P' to weigh Q' : prove that the real weight X of what appears to weigh Y is given by $X(Q - Q') = Y(P - P') + P'Q - PQ'$. [Math. Tripos, 1870.]

Ex. 4. A true balance is in equilibrium with unequal weights P, Q in its scales. If a small weight be added to P , the consequent vertical displacement of Q is equal to that which would be the vertical displacement of P were the same small weight to be added to Q instead of to P . [Math. Tripos, 1878.]

Looking at the expression for $\tan \theta$ in Art. 518, we notice that the changes produced in θ by altering either P or Q by the same small quantity are equal with opposite signs. The effect of increasing P or Q is therefore to turn the balance the one way or the other through the same small angle. The vertical displacements of the weights are therefore equal in the two cases.

Ex. 5. If the tongue of the balance be very slightly out of adjustment, prove that the true weight of a body is nearly the arithmetic mean of its apparent weights, when weighed in the opposite scales. [Coll. Exam.]

Ex. 6. A delicate balance, whose beam was originally suspended by a knife-edged portion of itself (higher than its centre of gravity) resting upon a horizontal agate plate, has its knife-edge worn down a distance ϵ so that it becomes curved (curvature = $1/r$), and has a corresponding hollow made in the agate plate (curvature = $1/\rho$). If slightly different weights P and Q be placed in the scales (whose weights may be neglected), show that the reciprocal of the sensibility is increased by $(P + Q + W) \left(\epsilon + \frac{r\rho}{\rho - r} \right) \frac{1}{a}$. [Coll. Exam., 1890.]

521. The Steelyards. The common steelyard is a lever ACB with unequal arms AC , CB , the fulcrum being situated at a point a little above C . The body Q to be weighed is suspended from the extremity B of the shorter arm, and a given weight P is moved along the longer arm CA to some point H such that the system balances. Let G be the centre of gravity of the beam, w its weight. The three weights, P acting at H , w at G , and Q at B are in equilibrium. Taking moments about C , we have

$$P \cdot HC + w \cdot GC = Q \cdot CB \quad \dots \quad (1).$$

Let D be a point on the shorter arm CB , such that $w \cdot GC = P \cdot CD$, the equation (1) then becomes $P \cdot HD = Q \cdot CB \quad \dots \quad (2).$

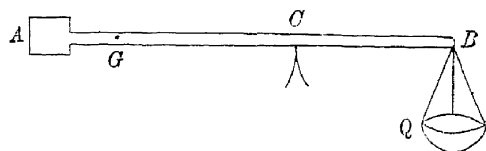
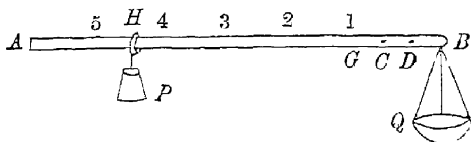
Thus the weight of Q is determined by measuring the distance HD . To effect this easily, we measure from D towards A a series of lengths DE_1 , E_1E_2 , E_2E_3 , &c. each equal to CD . The weight of the body Q is therefore equal to P , $2P$, $3P$, &c. according as the weight P is placed at the points E_1 , E_2 , E_3 , &c. when the system is in equilibrium. The intervals E_1E_2 , E_2E_3 , &c. are usually graduated into smaller divisions, so that the length HD can be easily read. The points E_1 , E_2 , &c. are marked 1, 2, &c. in the figure.

An instrument of this form was used by the Romans and is therefore often called the Roman steelyard.

522 In the Danish steelyard the weights P and Q act at fixed points of the lever, but the fulcrum or point of support C is made to slide along the rod AB until the system balances. The weight P , being fixed, can be conveniently joined to that of the lever. Let, then, P' be the weight of the instrument, so that $P' = P + w$, and let G be the centre of gravity. Taking moments about C , we evidently have $P' \cdot GC = Q \cdot CB$, and $\therefore BC = \frac{P' \cdot BG}{P' + Q}$. This expression enables us to calculate the values of BC when $Q = P'$, $2P'$, $3P'$, &c. Marking these points of the rod AB with the figures 1, 2, 3, &c., the weight of any body placed at B can be read off when the place of the fulcrum C has been found by trial.

If C , C' be two successive marks of graduation when the weights suspended at B are Q and $Q + S$, we easily find that $\frac{1}{BC'} - \frac{1}{BC} = \frac{S}{P' \cdot BG}$; since the right-hand side is constant when S is given, we infer that the marks of graduation on the bar are such that their distances from B form a harmonical progression when the weights form an arithmetical progression. Thus in the common steelyard the distances of the graduations from a certain point are in arithmetical progression, and in the Danish steelyard in harmonical progression.

523. The advantages of a steelyard over the balance are, (1) the exact adjustment of the instrument is made by moving a single weight P along the rod, (2) when



the body to be weighed is heavier than the fixed weight the pressure on the point of support is less than in the balance. The steelyard is therefore better adapted to measure large weights. There is on the other hand this advantage in the balance, that by using numerous small weights the reading can be effected with greater precision than by subdividing the arm of the steelyard.

524. Ex. 1. The weight of a common steelyard is w , and the distance of its fulcrum from the point from which the weight hangs is a when the instrument is in perfect adjustment; the fulcrum is displaced to a distance $a + a$ from this end; show that the correction to be applied to give the true weight of a body which in the imperfect instrument appears to weigh W is $(W + P + w)a/(a + a)$, P being the moveable weight. [Math. Tripos, 1881]

Ex. 2. In a weighing machine constructed on the principle of the common steelyard the pounds are read off by graduations reaching from 0 to 14, and the stones by weights hung at the end of the arm; if the weight corresponding to one stone be 7 oz., the moveable weight $\frac{1}{2}$ lb., and the length of the arm one foot, prove that the distances between the graduations are $\frac{3}{4}$ in. [Math. Tripos, 1884.]

Ex. 3. In graduating a steelyard to weigh pounds, marks are made with a file, a weight x being removed for each notch. With the moveable weight P at the end of the beam, n lbs. can be weighed after the graduation is completed, $(n + 1)$ before it is begun. Show that $n(n + 1)x = 2P$, and find the error made in weighing m pounds. The centre of gravity of the steelyard is originally under the point of suspension. [Coll. Exam., 1885.]

Ex. 4. Show that, if a steelyard be constructed with a given rod whose weight is inconsiderable compared with that of the sliding weight, the sensibility varies inversely as the sum of the sliding weight and the greatest weight which can be weighed. [Math. Tripos, 1854.]

Ex. 5. A common steelyard is graduated on the assumptions that its weight is Q , and that the moveable weight is W , both which assumptions are incorrect. If two masses whose real weights are P and R appear to weigh $P + X$ and $R + Y$, then the weight of the steelyard and the moveable weight are less than their assumed values by $\frac{W}{D}(X - Y)$ and $\frac{Q}{D}(X - Y) + \frac{a}{bD}(PY - RX)$, where b , a are the distances from the fulcrum to the centre of gravity of the bar and to the point of attachment of the substance to be weighed, and $D = P - R + X - Y$. [Math. Tripos, 1887.]

Ex. 6. The sum of the weight of a certain Roman steelyard and of its moveable weight is S , the fulcrum is at the point C and the body to be weighed is hung at the end B . The steelyard is graduated and after graduation the fulcrum is shifted towards B to another point C' . A body is then weighed, the old graduation being used, and the apparent weight is W . Prove that the true weight is greater than the apparent weight by $(S + W)CC'/BC'$. [Trin. Coll., 1889.]

Ex. 7. If, on a common steelyard, the moveable weight P , which forms the power, be increased in the ratio $1 + k : 1$, prove that the consequent error in Q , the weight to be found, is kY , where Y is the weight which must be removed from Q in order to preserve equilibrium when P is moved close to the fulcrum. [Coll. Exam., 1885.]

Ex. 8. In the Danish steelyard, if a_n be the distance of the fulcrum from that end of the steelyard at which the weight is suspended, the weight being n lbs., prove that $\frac{1}{a_{n+2}} - \frac{2}{a_{n+1}} + \frac{1}{a_n} = 0$. [Math. Tripos, 1859.]

Ex. 9. An old Danish steelyard, originally of weight W lbs., and accurately graduated, is found coated with rust. In consequence of the rust, the apparent weights of two known weights of X lbs. and Y lbs. are found when weighed by the steelyard to be $(X - x)$ lbs., $(Y - y)$ lbs. respectively. Prove that the centre of gravity of the rust divides the graduated arm in the ratio $W(x - y) : Yx - Xy$; and that its weight is, to a first approximation, $\frac{W+Y}{X-Y}x + \frac{W+X}{Y-X}y$. [Math. Tripos, 1885.]

Ex. 10. A brass figure $ABDC$, of uniform thickness, bounded by a circular arc BDC (greater than a semicircle) and two tangents AB , AC inclined at an angle 2α , is used as a letter-weigher as follows. The centre of the circle, O , is a fixed point about which the machine can turn freely, and a weight P is attached to the point A , the weight of the machine itself being w . The letter to be weighed is suspended from a clasp (whose weight may be neglected) at D on the rim of the circle, OD being perpendicular to OA . The circle is graduated, and is read by a pointer which hangs vertically from O : when there is no letter attached, the point A is vertically below O and the pointer indicates zero. Obtain a formula for the graduation of the circle, and show that, if $P = \frac{1}{2}w \sin^2 \alpha$, the reading of the machine will be $\frac{1}{2}w$ when OA makes with the vertical an angle equal to $\tan^{-1} \left\{ \frac{(\pi + 2\alpha) \sin^2 \alpha + 2 \sin \alpha \cos \alpha}{(\pi + 2\alpha) \sin^3 \alpha + 2 \cos \alpha} \right\}$. [Math. Tripos, 1878.]

525. The Pulley. The common pulley consists of a wheel which can turn freely on its axis. A rope or cord runs in a groove formed on the edge of the wheel, and is acted on by two forces P and P' one at each end. If the pulley is smooth and the weight of the string infinitesimal, the tension is necessarily the same throughout the arc of contact. It follows that the forces P , P' acting at the extremities of the string are equal to each other and to the tension. See fig. 1 of Art. 527. The same thing is true if the pulley is rough and circular, but can turn freely about a smooth axis, Art. 197.

526. When the axis of the pulley is fixed one of the forces P , Q is the power and the other is the weight. Thus a fixed pulley has no mechanical advantage in the technical sense. A machine, however, which enables us to give the most advantageous direction to the moving power is as useful as one which enables a small power to support a large weight.

527. A moveable pulley can however be used to obtain mechanical advantage. Suppose a perfectly flexible string to be fixed at A , pass under a pulley C of weight Q , and to be acted on at B by a force P , see fig. 2. In the position of equilibrium the strings on each side of the pulley meet in the line of action of the force Q (Art. 34), and must therefore make equal angles with

the vertical (Art. 27). Let α be the inclination of either string to the vertical, then

$$2P \cos \alpha = Q.$$

Fig. 1.

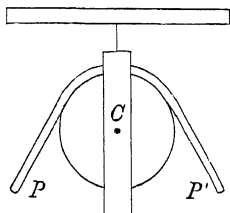
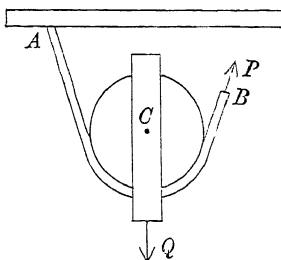


Fig. 2.



The mechanical advantage is therefore $2 \cos \alpha$. Unless α is less than 60° the mechanical advantage is less than unity. When the strings are parallel, we have $2P = Q$.

528. Ex. 1. In the single moveable pulley with parallel strings a weight W is supported by another weight P attached to the free end of the string and hanging over a fixed pulley. Show that, in whatever position the weights hang, the position of their centre of gravity is the same. [Math. Tripos, 1854.]

Ex. 2. A string is attached to the centre of a heavy circular pulley of radius r and is then passed over a fixed peg, then under the pulley, and afterwards passes over a second fixed peg vertically over the point where the string leaves the pulley and has a weight W attached to its extremity. The second peg is in the same horizontal line as the first peg and at a distance $\frac{3}{4}r$ from it. If there is equilibrium, prove that the weight of the pulley is $\frac{5}{8}W$, and find the distance between the first peg and the centre of the pulley. [Coll. Exam., 1886.]

Ex. 3. An endless string without weight hangs at rest over two pegs in the same horizontal plane, with a heavy pulley in each festoon of the string; if the weight of one pulley be double that of the other, show that the angle between the portions of the upper festoon must be greater than 120° . [Math. Tripos, 1857.]

529. Systems of pulleys may be divided into two classes, (1) those in which a single rope is used; and (2) those in which there are several distinct ropes. We begin with the first of these systems.

Two blocks are placed opposite each other, containing the same number of pulleys in each. Three are represented in each block in the figure. The string passes over the pulleys in the order $ADBECE$, and has one extremity attached to one of the blocks. The power P acts at the other extremity of the string, while the weight Q acts on a block.

Let n be the number of pulleys in either block, W the

weight of the lower block, we then have $Q + W$ supported by $2n$ tensions. Since the tension of the string is the same throughout, and equal to P , we have by resolving vertically $2nP = Q + W$.

If the pulleys were all of the same size, and exactly under each other, some difficulty might arise in their arrangement so that the cords should not interfere with each other. For this, and other reasons, the parts of the string not in contact with the pulleys cannot be strictly parallel. Except when the two blocks are very close to each other the error arising from treating the strings as parallel is very slight, and may evidently be neglected when we take no account of the other imperfections of the machine; Art. 503.

We may also deduce the relation between the power and the weight from the principle of virtual work. If the lower block, together with the weight Q , receive a virtual displacement upwards equal to q , it is clear that each string is slackened by the same space q . To tighten the string, P must descend a space q for each separate portion of string, i.e. P must descend a space $2nq$. We have therefore by the principle of work

$$P \cdot 2nq = (Q + W)q.$$

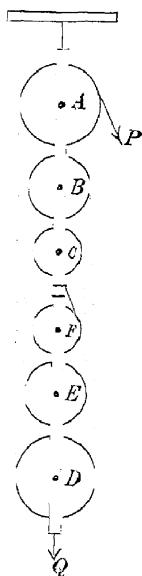
The result follows immediately

530. In some arrangements of this system the pulleys on each block have a common axis, but each pulley turns on the axis independently of the others. This change however does not affect the truth of the relation just established between the power and the weight

When the system works, it is clear that all the pulleys, if of equal size, do not move with equal angular velocities. To give greater steadiness to the several parts of the machine, it has been suggested that the pulleys in each block should not only have a common axis, but be of such radii that each turns with the same angular velocity. When this has been effected, the pulleys in each block may be welded into one and the string made to run in grooves cut out of the same wheel.

To understand how this may be done, we notice that if the lower block rises one foot, each string would be slackened one foot. To tighten the string between C and F on the right hand the pulley F must be turned round so that one foot of rope may pass over it. The string on the left hand between C and F is now slackened by two feet, hence the pulley C must be turned round so that two feet of rope may pass over it. In the same way the pulley E must be turned round so that three feet of rope may pass over it, and so on. If then the wheels in the upper block are constructed so that their radii are in the proportion $2 : 4 : 6 \cdot \&c.$, and those in the lower block so that the radii are in the proportion $1 : 3 \cdot 5 \cdot \&c.$, the wheels in each block will turn with the same angular velocity.

When very accurately constructed this arrangement works well. It is found



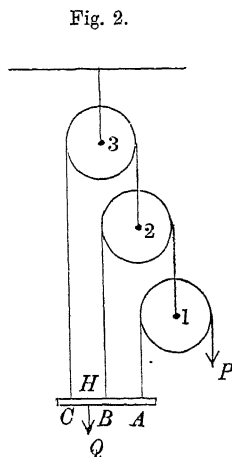
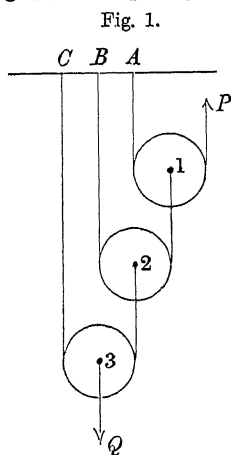
however that a very slight deviation from the true proportion of the radii will cause the rope to be unequally stretched, even the thickness of the rope must be allowed for. Some parts of the rope are therefore unduly tight, and others become nearly slack. This mode of arranging the pulleys is due to White. It is not now much used.

531. Ex. In that system of pulleys in which the same cord passes round all the pulleys it is found that on account of the rigidity of the cord and the friction of the axle a weight of P lbs. requires $aP + p$ lbs. to lift it by a cord passing over one pulley. Prove that when there are n parallel cords in the above system a power P can support a weight $Q = a \frac{a^n - 1}{a - 1} P + \frac{a(a^n - 1) - n(a - 1)}{(a - 1)^2} p$, and find the additional weight required to be added to P to raise Q . [Math. Tripos, 1884.]

The rigidity of cordage was made the subject of many experiments by Coulomb, Art. 170. The discussion of these would require too much space, but the general result may be shortly stated. Suppose a cord $ABCD$ to pass over a pulley of radius r , touching it at B and C , and moving in the direction $ABCD$. Then the rigidity of the portion AB of the cord which is about to be rolled on the pulley may be allowed for, by regarding the cord as perfectly flexible and applying a retarding couple to the pulley whose moment is $a + bT$, where a and b are constants which depend on the nature and size of the cord, but are sensibly independent of the velocity. If T' be the tension of the portion CD of the cord which is being unwound from the pulley, its rigidity may be represented in the same way by the application of a couple equal to $a' + b'T'$. The values of a' , b' are so much less than those of a , b , that this last correction is generally omitted. Taking moments about the centre this gives $T' - T = \frac{a + bT}{r}$, where r is the radius.

532. When several cords are used pulleys may be combined in various ways to produce mechanical advantage. Two systems are usually described in elementary books, both of which are represented in the figure.

In fig. (1) each pulley is supported by a separate string, one end



of which is attached to a fixed point of support, and the other to the pulley next in order. In fig. (2) the string resting on each pulley has one end attached to the weight and the other to the pulley next in order. The two systems resemble each other in the arrangement of the pulleys, but to a certain extent each is the inversion of the other.

Let $w_1, w_2, \&c.$ be the weights of the pulleys $M_1, M_2, \&c.$, $T_1, T_2, \&c.$ the tensions of the strings which pass over them. In the figures only the suffixes of $M_1, M_2, \&c.$ are marked on the pulleys to save space.

Considering fig. (1), the tension $T_1 = P$. The tensions of the parts of the string on each side of the pulley M_1 support the weight of that pulley and the tension T_2 , we have therefore

$$T_2 = 2T_1 - w_1 = 2P - w_1.$$

Considering the pulleys M_2, M_3 , we have in the same way

$$T_3 = 2T_2 - w_2 = 2^2P - 2w_1 - w_2,$$

$$T_4 = 2T_3 - w_3 = 2^3P - 2^2w_1 - 2w_2 - w_3,$$

and so on through all the pulleys. It is evident that the right-hand side of each equation is twice that of the one above with a w subtracted. We therefore have finally

$$Q = 2T_n - w_n = 2^nP - 2^{n-1}w_1 - 2^{n-2}w_2 - \&c. - 2w_{n-1} - w_n.$$

If all the pulleys are of equal weight this gives

$$Q = 2^nP - (2^n - 1)w.$$

The relation between the power and the weight follows easily from the principle of virtual work. If we suppose the lowest pulley to receive a virtual displacement upwards equal to q , each of the strings on its two sides is slackened by an equal space q . To tighten these we must raise the next lowest pulley through a space equal to $2q$. In the same way, the next in order must be raised a space twice this last, i.e. 2^2q , and so on. Hence the power P must be raised a space 2^nq . Multiplying each weight by the space through which it has been moved, we have, by the principle of work

$$(Q + w_n)q + w_{n-1}2q + w_{n-2}2^2q + \dots = P \cdot 2^nq.$$

Dividing by q we obtain the same relation as before.

533. Considering fig. (2), the tension $T_1 = P$. The tensions of the parts of the string on each side of the pulley M_1 , together with the weight of that pulley, are supported by the tension T_2 , we

therefore have $T_2 = 2T_1 + w_1 = 2P + w_1$. Taking the other pulleys in order, we see that we have the same results as before except that the w 's have opposite signs. We thus have

$$T_3 = 2T_2 + w_2 = 2^2P + 2w_1 + w_2,$$

$$T_4 = 2T_3 + w_3 = 2^3P + 2^2w_1 + 2w_2 + w_3,$$

and so on. Since the pulleys are all attached to the weight we have $T_1 + T_2 + \dots + T_n = Q + W$, where W is the weight of the bar.

Substituting the values of T_1, T_2 , &c. in this last equation, we find $Q + W = (2^n - 1)P + (2^{n-1} - 1)w_1 + (2^{n-2} - 1)w_2 + \dots + w_{n-1}$.

If all the pulleys are of equal weight this reduces to

$$Q + W = (2^n - 1)(P + w) - nw.$$

When the pulleys are arranged as in fig. (1), the mechanical advantage is decreased by increasing the weights of the pulleys. In fig. (2) the reverse is the case, for the weights of the pulleys assist the power in sustaining the weight.

To deduce the relation between the power and the weight from the principle of virtual work, let us first imagine the bar to be held at rest and the highest pulley to be moved downwards through a space q . Each of the strings on the two sides of that pulley is equally slackened by the space q . To tighten the string, the second highest pulley must be moved downwards through a space $2q$, and so on. The power must descend a space $2^n q$. To restore the upper pulley to its original position let us now suppose the whole system to be moved upwards through a space equal to q , Art. 65. On the whole, the weight Q , together with the bar ABC , has ascended a space q ; the downward displacements of the several pulleys in order, counting from the highest, are respectively $0, (2 - 1)q, (2^2 - 1)q, \dots$; while the downward displacement of the power P is $(2^n - 1)q$. The principle of work at once yields the equation

$$(Q + W)q = w_{n-1}(2 - 1)q + w_{n-2}(2^2 - 1)q + \dots + w_1(2^{n-1} - 1)q + P(2^n - 1)q.$$

Dividing by q we have the same relation as before.

534. We notice that the bar ABC will not remain horizontal unless the weight Q is fastened to it at the proper point. The bar is acted on at the points A, B , &c. by the tensions T_1, T_2 , &c., and these are to be in equilibrium with the weight Q acting at some point H and the weight W of the bar at its middle point G . The intervals AB, BC , &c. depend on the radii of the pulleys. If the radii be a_1, a_2 , &c.

we have $AB = 2a_2 - a_1$, $BC = 2a_3 - a_2$, and so on. Taking moments about A we have

$$T_2 \cdot AB + T_3 \cdot AC + \&c. = Q \cdot AH + W \cdot AG.$$

This equation determines the position of H .

If the weights of the strings or ropes cannot be neglected, we may suppose the weight of the portion of string between the pulleys M_1, M_2 included in the weight w_1 , that of the portion between the pulleys M_2, M_3 included in w_2 , and so on. The portions of string which join the points A, B, C , &c. to the pulleys are supported by the fixed beam ABC , &c. in fig. (1), and may be included in the weight of the bar in fig. (2). The weight of the string wound on any pulley may be included in the weight of that pulley.

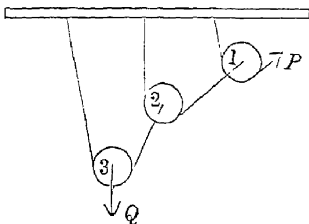
The system of pulleys represented in fig. (1) of Art. 532 is sometimes called the *first system*. That represented in Art. 529 is the *second system*; while the one drawn in fig. (2) of Art. 532 is the *third system*.

535. When the weights of the pulleys are neglected and each hangs by a separate string, we can easily find the relation between the power and the weight when the strings are not parallel.

Let $2\alpha_1, 2\alpha_2, 2\alpha_3$, &c. be the angles between free parts of the strings which pass over the pulleys M_1, M_2, M_3 , &c. respectively. Let also T_1, T_2, T_3 , &c. be the tensions. Then by the same reasoning as before

$$T_1 = P, \quad T_2 = 2T_1 \cos \alpha_1, \quad T_3 = 2T_2 \cos \alpha_2, \quad \&c.$$

If there are n pulleys we easily obtain $Q = 2^n P \cdot \cos \alpha_1 \cdot \cos \alpha_2 \cdot \&c. \cos \alpha_n$.



536. Ex. 1. In that system of pulleys in which all the strings are attached to the weight, if the weight of the lowest pulley be equal to the power P , of the second $3P$, and so on, that of the highest moveable pulley being $3^{n-2}P$, the ratio of $P : W$ will be $2 : 3^n - 1$. [Math. Tripos, 1856.]

Ex. 2. In that system of pulleys in which each hangs by a separate string from a horizontal beam the weights of the pulleys, beginning with the highest, are in arithmetical progression, and a power P supports a weight Q ; the pulleys are then reversed, the highest being placed lowest, and the second highest placed lowest but one, and so on, and now Q and P when interchanged are in equilibrium, show that $n(Q + P) = 2W$, where W is the total weight of the pulleys, and n the number of pulleys. [Coll Exam., 1882.]

Ex. 3. In a system of n pulleys where a separate string goes round each pulley and is attached to the weight, if the string which goes over the lowest have the end, at which the power is usually hung, passed under another moveable pulley and then over a fixed pulley, and attached to the weight Q ; and if the weight of each pulley be w and no other power be used, prove that $Q = (3 \cdot 2^{n-1} - n - 1)w$, and find the point of the beam at which Q must be hung. [Math. Tripos, 1876.]

Ex. 4. In that system of pulleys in which each of the strings, supposed parallel, is attached to the weight, if the power be equal to the weight of the lowest pulley, and if each pulley weigh three times as much as the one immediately below it, prove that the weight of each pulley is equal to the tension of the string passing over it. [Coll. Exam.]

Ex. 5. In the system of pulleys in which each hangs by a separate string, all

the strings being vertical, if W be the weight supported, and w_1, w_2, \dots, w_n the weights of the moveable pulleys, there will be no mechanical advantage unless

$$W - w_n + 2(W - w_{n-1}) + 2^2(W - w_{n-2}) + \dots + 2^{n-1}(W - w_1)$$

be positive.

[Math. Tripos, 1869.]

Ex. 6. In the system of n heavy pulleys in which each hangs by a separate string, P is the power (acting upwards), Q the weight, and R the stress on the beam from which the pulleys hang: show that R is greater than $Q(1 - 2^{-n})$ and less than $(2^n - 1)P$.

[Math. Tripos, 1880.]

Ex. 7. If there be two pulleys, without weight, which hang by separate strings, the fixed ends only of the string being parallel, and the power horizontal, prove that the mechanical advantage is $\sqrt{3}$.

[St John's Coll., 1883.]

Ex. 8. In that system of pulleys, in which all the strings are attached to the weight, if the power be made to descend through one inch, through what distance will the weight rise? Illustrate by reference to this system of pulleys the principle which is expressed by the words, "In machines, what is gained in power is lost in time."

[Math. Tripos, 1859.]

Ex. 9. In the system of pulleys in which all the strings are attached to the weight Q , prove that, if the pulleys be small compared with the lengths of the strings, the necessary correction for the weight of the strings is the addition to $Q, w_1, w_2, \dots, w_{n-1}$ respectively, of the weights of lengths

$$h_1 + h_2 + \dots + h_{n-1} + h, \quad 2(h_1 - h), \quad 2(h_2 - h_1), \dots, 2(h_{n-1} - h_{n-2})$$

of string; where $h_1, h_2, h_3, \dots, h_n$ are the heights of the n pulleys (whose weights are w_1, w_2, \dots, w_n respectively) above the line of attachment, supposed horizontal, of the strings to the weight Q , and h the height of the point of attachment of the power above the same line.

[Math. Tripos, 1877.]

Ex. 10. In that system of pulleys in which the strings are all parallel, and the weights of the pulleys assist the power, show that, if there are n pulleys, each of diameter $2a$ and weight w , the distance of the point of suspension of the weight from the line of action of the power is equal to

$$n \frac{2^{n+1}Q + [(n-3)2^n + n+3]w}{2(2^n - 1)Q} a,$$

where Q is the weight.

[Math. Tripos, 1883.]

Ex. 11. In a system of four pulleys, arranged so that each string is attached to a bar carrying the weight, the string which usually carries the power is attached to one end of the same bar, and the fourth string to the other end. The weight and diameter of each pulley are respectively double of those of the pulley below it, and the strings are all parallel. The weight being 33 times that of the lowest pulley, find at what point of the bar it is hung.

[Trin. Coll., 1885.]

Ex. 12. In the system of pulleys, in which each pulley hangs by a separate string with one end attached to a fixed beam, there are n moveable pulleys of equal weight w . The r th string, counting from the string round the highest pulley, cannot bear a greater tension than T . Prove that the greatest weight which can be sustained by the system is $2^{n-r+1}T - (2^{n-r+1} - 1)w$.

[Trin. Coll., 1890.]

Ex. 13. It is found that any force P being applied to the extremity of a string passing over a pulley can just raise a weight $P(1 - \theta)$. In the system of pulleys in which each hangs by a separate string a weight Q is just supported, the weight of each pulley being αQ . If α and θ are small quantities, whose squares and products may be neglected, show that an additional power equal to $n\theta Q/2^n$ can be applied without affecting the equilibrium.

[Coll. Exam., 1888.]

537. The Inclined Plane. *To find the relation between the power and the weight in the inclined plane.*

Let AB be the inclined plane, C any particle situated on it. Let CN be a normal to the plane and CV vertical; let α be the inclination of the plane to the horizon, then the angle $NCV = \alpha$. Let Q be the weight of C , P a force acting on C in the direction CK , where the angle $NCK = \phi$. It is supposed that CK lies in the vertical plane VCN .

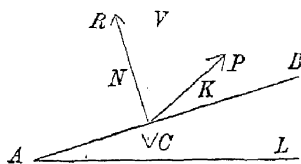


Fig. 1

If the plane is smooth the reaction R of the plane on the particle acts along the normal CN . We then have by Art. 35

$$\frac{P}{\sin \alpha} = \frac{Q}{\sin \phi} = \frac{R}{\sin (\phi - \alpha)} \dots \dots \dots (1).$$

It is necessary for equilibrium that R should be positive, for otherwise the particle would leave the plane. It follows from these equations that ϕ must be greater than α . This follows also from an examination of fig. (1), for Q acting along VC and R along CN cannot be balanced by a force P unless its direction lies within the angle formed by CV and NC produced. If P act up the plane, $\phi = \frac{1}{2}\pi$ and $P = Q \sin \alpha$, $R = Q \cos \alpha$. If P act horizontally, $\phi = \frac{1}{2}\pi + \alpha$, and $P = Q \tan \alpha$, $R = Q \sec \alpha$.

538. If the plane is rough, let $\mu = \tan \epsilon$ be the coefficient of friction. With the normal CN as axis describe a right cone whose semi-angle is ϵ ; this is the cone of friction, Art. 173. The resultant action R' of the plane on the particle lies within this cone; let CH be its line of action and let the angle $NCH = i$; then i lies between $\pm \epsilon$. Let the standard case be that in which α is greater than ϵ , and ϕ greater than either; this is represented in fig. (2). We therefore have

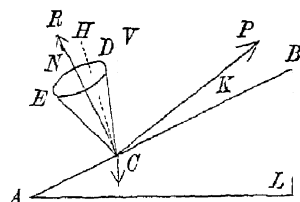


Fig. 2.

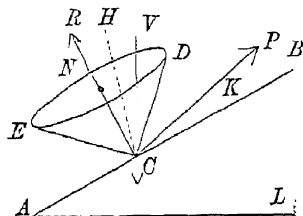


Fig. 3.

$$\frac{P}{\sin (\alpha - i)} = \frac{Q}{\sin (\phi - i)} = \frac{R'}{\sin (\phi - \alpha)} \dots \dots \dots (2).$$

When the force P is so great that the particle is on the point of ascending the plane, the reaction R' acts along CE , and $i = -\epsilon$. Let P_1 be this value of P , then

$$\frac{P_1}{\sin(\alpha + \epsilon)} = \frac{Q}{\sin(\phi + \epsilon)} = \frac{R'}{\sin(\phi - \alpha)} \dots\dots\dots(3).$$

When the force P is so small that the particle is only just sustained, the reaction R' acts along CD , and $i = \epsilon$. Let P_2 be the value of P , then

$$\frac{P_2}{\sin(\alpha - \epsilon)} = \frac{Q}{\sin(\phi - \epsilon)} = \frac{R'}{\sin(\phi - \alpha)} \dots\dots\dots(4).$$

If $\alpha > \epsilon$ as in fig. (2), it is clear that the particle will slide down the plane if not supported by some force P , Art. 166. When the particle is just supported the reaction R' acts along CD and Q along VC ; it is clear that these forces could not be balanced by any force P unless its direction lay within the angle made by CV and DC produced. Accordingly we see from (4) that R' is negative unless $\phi > \alpha$. In the same way it is impossible to pull the particle up the plane (without pulling it off) by any force whose direction does not lie between CV and EC produced. Assuming $\phi > \alpha$, the least force required to keep the particle at rest is given by (4), and the greatest by (3).

If $\epsilon > \alpha$ as in fig. (3), the particle will rest on the plane unless disturbed by some force P . To just pull the particle up the plane the force must act within the angle formed by CV and EC produced, and its magnitude is given by (3). In order that the particle may be just descending the plane the force must act within the angle formed by CV and DC produced, and its magnitude is given by (4).

539. Ex. 1. If a power P acting parallel to a smooth inclined plane and supporting a weight Q produce on the plane a pressure R , then the same power acting horizontally and supporting a weight R will produce a pressure Q . [Coll. Ex., 1881.]

Ex. 2. Find the direction and magnitude of the least force which will pull a particle up a rough inclined plane.

By (3) we see that P_1 is least when $\phi + \epsilon = \frac{1}{2}\pi$, i.e. when the force makes an angle with the inclined plane equal to the angle of friction.

Ex. 3. Find the direction and magnitude of the least force which will just support a particle on a rough inclined plane.

Ex. 4. A given particle C rests on a given smooth inclined plane and is supported by a force acting in a given direction. If the inclined plane is without weight and has its side AL moveable on a smooth horizontal table, find the force which when acting horizontally on the vertical face BL will prevent motion. Find also the point of application of the resultant pressure on the table.

Ex. 5. A heavy body is kept at rest on a given inclined plane by a force making a given angle with the plane; show that the reaction of the plane, when it is smooth, is a harmonic mean between the greatest and least reactions, when it is rough. [Math. Tripos, 1858.]

Ex. 6. A heavy particle is attached to a point in a rough inclined plane by a fine rigid wire without weight, and rests on the plane with the wire inclined at an angle θ to a horizontal line in the plane. Determine the limits of θ , the angle of inclination of the plane being $\tan^{-1}(\mu \sec \beta)$. [Coll. Exam.]

Ex. 7. Two equal particles on two inclined planes are connected by a string which lies wholly in a vertical plane perpendicular to the line of junction of the planes, and passes over a smooth peg vertically above this line of junction. If, when the particles are on the point of motion, the portions of the string make

equal angles with the vertical, show that the difference between the inclinations of the planes must be twice the angle of friction. [Math. Tripos, 1873.]

540. Wheel and Axle. *To find the relation between the power and the weight in the wheel and axle.*

Let a be the radius of the axle AB , c that of the wheel. The power P acts by means of a string which passes round the wheel several times and is attached to a point on the circumference. The weight Q acts by a string which passes similarly round the axle. Taking moments round the central line of the axle, we have $Pc = Qa$. The mechanical advantage is equal to c/a .

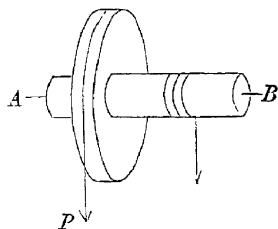


Fig 1.

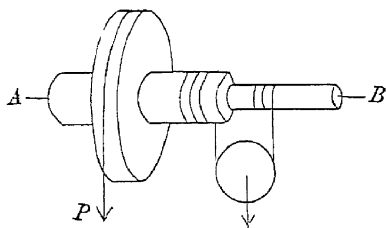


Fig 2.

If p, q be the spaces which the power and weight pass over while the wheel turns through any angle, we have

$$p/q = c/a = Q/P.$$

541. When a great mechanical advantage is required we must either make the radius of the wheel large or that of the axle small. If we adopt the former course the machine becomes unwieldy, if the latter the axle may become too weak to bear the strain put on it. In such a case we may adopt the plan represented in fig. (2). The two parts of the axle are made of different thicknesses, and the rope carried round both. As the power P descends, the rope which supports the weight is coiled on the thicker part of the axle and uncoiled from the thinner. Let a, b be the radii of these two portions of the axis. If Q be the weight attached to the pulley, the tension of the string is $\frac{1}{2}Q$. Taking moments about the central line of the axis, we have $Pc = \frac{1}{2}Q(a - b)$. The mechanical advantage is therefore equal to the radius of the wheel divided by half the difference of the radii of the axle. By making the radii of the two portions of the axis as nearly equal as we please, we can increase the mechanical advantage without decreasing the strength of the machine. This arrangement is called the *differential axle*.

542. Ex. 1. A rope passes round a pulley, and its ends are coiled opposite ways round two drums of different radii on the same horizontal axis. A person pulls vertically upon one part of the rope with a force P . What weight attached to the pulley can he raise, supposing the parts of the rope parallel? [Coll. Exam.]

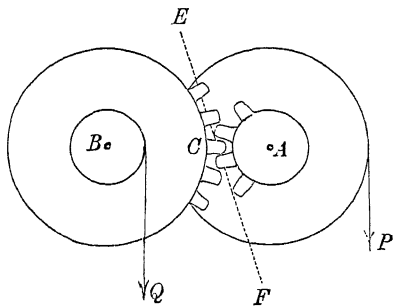
Ex. 2. In the differential axle if the ends of the chain, instead of being fastened to the axles, are joined together so as to form another loop in which another pulley and weight are suspended, find the least force which must be applied along the chain in order to raise the greater weight, the different parts of the chain being all vertical. [Math. Tripos.]

543. When both the power and the weight act on the circumference of wheels there are various methods of connecting the two wheels besides that of putting them on a common axis. Sometimes, when the wheels are at a distance from each other, they are connected by a strap passing over their circumferences. In some other cases one wheel works on the other by means of teeth placed on their rims.

544. Toothed Wheels. *To obtain the relation between the power and the weight in a pair of toothed wheels.*

Let A, B be the centres of two wheels which act on each other by means of teeth, the teeth on the axis of one wheel working into those on the circumference of the other at the point C . Let a_1, a_2 be the radii of the axes, b_1, b_2 those of the wheels.

Let p, q be the virtual velocities of the power P and weight Q , then $Pp = Qq$. If the teeth are small the average velocities of the points near C on the two wheels are equal, and the common direction is perpendicular to the straight line AB . If then θ_1, θ_2 are the angles turned through by the wheels when the power P receives a small displacement,



we have $a_1\theta_1 = b_2\theta_2$. But $p = b_1\theta_1, q = a_2\theta_2$. It follows that $\frac{Q}{P} = \frac{b_1b_2}{a_1a_2}$. We have here omitted the work lost in overcoming the friction at the teeth in contact and at the points of support.

545. Let a tooth on one wheel touch the corresponding tooth on the other in some point D , and let EDF be a common normal to the two surfaces in contact at D . The point D is not marked in the figure because the teeth are not fully drawn, but it is necessarily situated near C . The actual velocities of the points of the teeth in contact at D when resolved in the direction EDF are equal. If, then, h and k are the perpendiculars drawn from A, B on EDF , it is clear that $\theta_1h = \theta_2k$. As the wheels turn, the lengths h and k alter, and if the ratio h/k is not constant, there is more or less irregularity in the working of the machine. To correct this defect, the teeth are sometimes cut so that the normal at every point of the boundary of a tooth is a tangent to the circle to which the tooth is attached. When this is done, the line EDF is always a common tangent to the two circles. The ratio h/k is therefore constant throughout the motion and equal to the ratio of the radii of the circles. One cause of irregularity will thus be removed and the motion will be made more uniform. This method is commonly ascribed to Euler.

If the normal at every point of the surface of a tooth is a tangent to a circle, each of the two halves of that tooth is bounded by an arc of an involute of the

circle. The two involutes are unwrapped from the circle in opposite directions and portions of each form the sides of the tooth.

When the centres of the toothed wheels are given, and the ratio of the angular velocities at which they are to work, we may determine their radii in the following manner. Let A, B be the given centres, divide AB in C so that $AC \cdot \theta_1 = BC \cdot \theta_2$. Through C draw a straight line ECF , which should not deviate very much from a perpendicular to AB . With A and B as centres describe two circles touching the straight line ECF . The sides of the teeth are to be involutes of these circles. By this construction the common normal to two teeth pressing against each other at D is the straight line ECF . As the wheels turn round, and the teeth move with them, the point of contact D travels along the fixed straight line ECF . The perpendiculars h and k are equal to the radii of these circles and are constant during the motion. Their ratio also is evidently equal to the ratio of AC to BC , i.e. of θ_2 to θ_1 .

It has already been shown that $Pp = Qq$, and $p = b_1\theta_1$, $q = a_2\theta_2$. Since $\theta_1 h = \theta_2 k$, we find as before $\frac{Q}{P} = \frac{b_1 b_2}{a_1 a_2}$.

We may notice that, if the distance between the centres A and B is slightly altered, the pair of wheels will continue to work without irregularity and the ratio of the angular velocities will be the same as before. To prove this, we observe that the common normal to two teeth pressing against each other is still a common tangent to the two circles, though in their displaced positions. Thus, though the inclination to AB of the straight line ECF is altered, the lengths of the perpendiculars h and k are the same as before.

That the teeth should be made of the proper form is a matter of importance to the even working of the machine. Many other considerations enter into the theory besides that mentioned above. Thus defects may arise from the wearing of the teeth if the pressure be very great at the point of contact. There may also be jolts and jars when the teeth meet or separate. But the subject is too large to be treated of in a division of a chapter. The reader who is interested in this matter is referred to books on the principles of mechanism. In Willis' *Principles of Mechanism* (2nd edition, 1870) five different methods of constructing the teeth are described, in three of which epicycloids are used, the advantages and disadvantages of these constructions are also compared.

546 Ex 1. In a train of n wheels, the teeth on the axle of each wheel work on those on the circumference of the next in order. Show that the power and weight are connected by the relation $\frac{Q}{P} = \frac{b_1 b_2}{a_1 a_2} \frac{b_n}{a_n}$, where a_1, a_2 &c. are the radii of the axles and b_1, b_2 &c. those of the wheels.

Ex. 2. In a pair of toothed wheels show that, if the ratio of the power and weight is to be approximately constant, the height and breadth of the teeth must both be small relatively to the radius of each wheel.

Two equal and similar wheels, with straight narrow radial teeth, are started with a tooth of each in contact and in the same straight line; show that they will work together without locking, provided that the distance of their centres be greater than $2a \cos 2\pi/n$ and less than $2a \cos \pi/n$, where a is the radius of either wheel measured to the summit of a tooth, and n the number of teeth. [Math. T., 1872.]

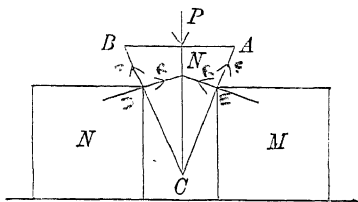
Ex. 3. Investigate the relation $Q/P = b_1 b_2 / a_1 a_2$ for a pair of toothed wheels without using the principle of virtual work.

The reaction R between two teeth acts along the straight line EDF . Taking moments in turn about A and B , we have $Pb_1 = Rh$, $Qa_2 = Rk$. As before, we have when the teeth are small $h/k = a_1/b_2$. The result follows at once.

547. The Wedge. *To find the relation between the power and the weight in the wedge.*

Let M , N be two obstacles which it is intended to separate by inserting a wedge ABC between them. For the sake of distinctness these obstacles are represented in the figure by two equal boxes placed on the floor, but it is obvious they may be of any kind.

We shall suppose that the wedge used is isosceles, and that it has its median line CN vertical.



Let the angle ACB be 2α . Let D , E be the points of contact with the obstacles (not marked in the figure), R , R the normal reactions at these points, F , F the frictions. When the wedge is on the point of motion we have $F = R \tan \epsilon$, where $\tan \epsilon$ is the coefficient of friction.

Let P be a force acting vertically at N urging the wedge downwards. Supposing P to prevail, the frictions on the wedge act along CA , CB ; we therefore find by resolving vertically

$$P = 2R (\sin \alpha + \tan \epsilon \cos \alpha) = 2R \sin (\alpha + \epsilon) \sec \epsilon.$$

The resultant reaction R' at D is then found by compounding R and μR .

If the obstacle M can only move horizontally, the whole of the reaction R' is not effective in producing motion. The horizontal component of R' tends to move M , but the vertical component presses the box on the floor and possibly tends to increase the limiting friction between the box and the floor. Let X be the horizontal component of R' ; we find

$$X = R \cos \alpha - R \tan \epsilon \cdot \sin \alpha = R \cos (\alpha + \epsilon) \sec \epsilon.$$

The mechanical advantage X/P is therefore equal to $\frac{1}{2} \cot (\alpha + \epsilon)$.

548. It may be noticed that the mechanical advantage of the wedge is increased by making the angle α more and more acute. There is of course a practical limit to the acuteness of this angle, for that degree of sharpness only can be given to the wedge which is consistent with the strength required for the purpose to which it is to be applied.

As examples of wedges we may mention knives, hatchets, chisels, nails, pins, &c. Generally speaking, wedges are used when a large power can be exerted through a small space. This force is usually applied in the form of an impulse.

It has not been considered necessary to consider separately the case in which the wedge is smooth, as the results obtained on so erroneous a supposition have no practical bearing.

549. If the force is applied in the form of a blow so that the wedge is driven forwards between the obstacles, the problem to determine its motion is properly one in dynamics. Our object here is merely to find the conditions of equilibrium of a triangular body inserted between two rough obstacles and acted on by a force P .

When a series of blows is applied to the wedge, we may however enquire what happens in the interval between two impulses. The wedge may either stick fast, held by the friction, or begin to return to its original position, being pressed back by the elasticity of the materials. Assuming that these forces of restitution may be represented by two equal pressures R, R , acting on the sides of the wedge, let P_1 be the force necessary to hold the wedge in position. The friction now acts to assist the power. To determine P_1 we write $-\epsilon$ for ϵ in the equations of equilibrium. We therefore have

$$P_1 = 2R \sin(\alpha - \epsilon) \sec \epsilon.$$

If α is greater than ϵ , P_1 is positive and therefore some force is necessary to hold the wedge in position. If α is less than ϵ , P_1 is negative, thus the friction is more than sufficient to hold the wedge fast. A force equal to this value of P_1 with the sign changed is necessary to pull the wedge out. The result is that the wedge will stick fast or come out according as the angle ACB is less or greater than twice the angle of friction.

Ex. 1. Referring to the figure of Art. 547, show that if either of the equal angles A or B of the wedge is less than the angle of friction, no force P however great could separate the obstacles M, N .

If the angle A is less than ϵ , we find that $\alpha + \epsilon$ is greater than a right angle, and therefore that X is negative. It is easy also to see that, if the angle A is equal to ϵ , the resultant reaction between one side of the wedge and an obstacle is vertical. The wedge therefore merely presses the obstacle against the floor.

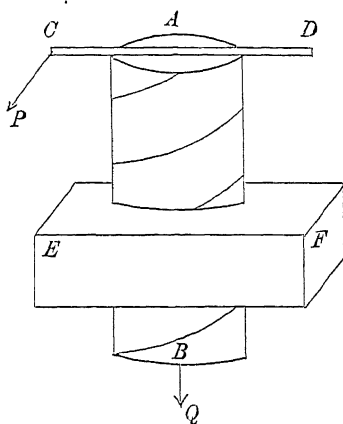
Ex. 2. If the obstacles M, N are not of the same altitude and are unequally rough, the position of the wedge when in equilibrium is such that the force P_1 and the resultant actions R_1', R_2' across the faces meet in a point. Supposing the force P_1 to act perpendicularly to the face AB of the wedge and to be just sufficient to hold the wedge at rest, show that $\frac{P_1}{\sin(2\alpha - \epsilon_1 - \epsilon_2)} = \frac{R_2'}{\cos(\alpha - \epsilon_1)} = \frac{R_1'}{\cos(\alpha - \epsilon_2)}$, assuming the obstacles to be of such form that the wedge must slip at both simultaneously. Show also that, if the wedge be such that the angle C is less than the sum of the

angles $\epsilon_1 + \epsilon_2$, the wedge can be held fast by the frictions without the application of any force.

Ex. 3. Deduce from the principle of virtual work the relation between the force X and the power P in a smooth isosceles wedge as represented in the figure of Art. 547. Discuss the two cases in which (1) one obstacle is immovable and (2) both move equally when the wedge makes an actual displacement.

550. The Screw. *To find the relation between the power and the weight in the screw.*

Let AB be a circular cylinder with a uniform projecting ridge running round its surface, the tangents to the directions of the ridges making a constant angle α with a plane perpendicular to the axis of the cylinder. The screw thus formed fits into a hollow cylinder with a corresponding groove on its internal surface, in which the ridge works. The grooves on the hollow cylinder have not been sketched, but are included in the beam EF .



The position of the ridge on the cylinder is easily understood by the following construction. Let a sheet of paper be cut into the form of a right-angled triangle LMN , such that the altitude MN is equal to the altitude of the cylinder AB and the angle the base LM makes with the hypotenuse LN is equal to α . Let this sheet of paper be wrapped round the cylinder AB ; if the base LM is long enough to go several times round the base of the cylinder, the hypotenuse will appear to wind gradually round the cylinder. The line thus traced by the hypotenuse is the curve along which the ridge lies.

Let P be the power applied perpendicularly at the end of a lever CD . Let $AC = a$, and let b be the radius of the cylinder. Supposing the body EF in which the screw works to be fixed in space, the end B of the cylinder will be gradually moved as C describes a circle round AB . Let Q be the force acting at B .

Let σ be any small length of the screw which is in contact with an equal length of the groove. Let $R\sigma$ be the normal reaction between these small arcs, $\mu R\sigma$ the friction.

In some screws the ridge is rectangular, so that it may be regarded as generated by the motion of a small rectangle moving round the cylinder with one side in contact with the surface and

its plane passing through the axis. When the ridge has this form, the line of action of R lies in the tangent plane to the cylinder and its direction makes with the axis of the cylinder an angle equal to α . In other screws the section of the ridge has some other form, such, for example, as a triangle. In such cases the line of action of R makes some angle θ with the tangent plane to the cylinder. We therefore resolve R into two components, one intersecting at right angles the axis of the cylinder and the other lying in the tangent plane. The magnitude of the latter is $R \cos \theta$, and its direction makes with the axis of the cylinder an angle equal to α . Since the ridge is uniform the angle θ will be the same throughout the length of the screw.

Let us suppose that the power P is about to prevail, then the friction acts so as to oppose the power. Resolving parallel to the axis of the cylinder and taking moments about it, we have

$$Q = \Sigma R \sigma \cdot \cos \theta \cos \alpha - \Sigma R \sigma \cdot \mu \sin \alpha,$$

$$Pa = \Sigma R \sigma \cdot b \cos \theta \sin \alpha + \Sigma R \sigma \cdot \mu b \cos \alpha.$$

Dividing one of these equations by the other we have

$$\frac{Q}{P} = \frac{\cos \theta \cos \alpha - \mu \sin \alpha}{\cos \theta \cos \alpha + \mu \sin \alpha} \cdot \frac{a}{b}.$$

551. If it be possible to neglect the friction and treat the screw as smooth we put $\mu = 0$. We then find for the mechanical advantage the expression $(a \cot \alpha)/b$. If a point travelling along the ridge or thread of the screw make one complete revolution of the cylinder, it advances parallel to the axis a space equal to the distance h between the ridges. This distance is therefore $h = 2\pi b \tan \alpha$. Substituting for $\tan \alpha$, we find that the mechanical advantage of a smooth screw is c/h , where c is the circumference described by the power and h is the distance between two successive threads of the screw measured parallel to the axis

552. We may easily deduce the relation between the power and the weight in a smooth screw from the principle of virtual work. When the power has turned the handle AC through a complete circle, the screw and the attached weight have advanced a space h equal to the distance between two threads of the screw measured parallel to the axis. When therefore friction is neglected and no work is otherwise lost in the machine, we have $Pc = Qh$, where c is the circumference of the circle described by P .

When the friction between the ridge and the groove is taken account of we see by Art. 550 that the efficiency of the machine is

given by $\frac{Qh}{Pc} = \frac{\cos \theta - \mu \tan \alpha}{\cos \theta + \mu \cot \alpha}.$

When the thread of the screw is rectangular the angle θ is zero. In that case the expression for the efficiency takes the simple form $\frac{Qh}{Pc} = \frac{\tan \alpha}{\tan(\alpha + \epsilon)}$, where ϵ is the angle of friction.

If the weight Q is about to prevail over the power, we change the signs of μ and ϵ in these formulæ.

553. Ex. 1. What force applied at the end of an arm 18 inches long will produce a pressure of 1000 lbs. upon the head of a smooth screw when 11 turns cause the head to advance two-thirds of an inch? [Trin. Coll., 1884.]

Ex. 2. A screw with a rectangular thread passes into a fixed nut: show that no force applied to the end of the screw in the direction of its length will cause it to turn in the nut, if the pitch of the screw is not greater than ϵ , where ϵ is the angle of friction. [Coll. Exam., 1878.]

Ex. 3. A rough screw has a rectangular thread: prove that the least amount of work will be lost through friction when the pitch of the screw is $\frac{1}{2}(\pi - 2\epsilon)$, where ϵ is the angle of friction. [St John's Coll., 1889.]

Ex. 4. The vertical distance between two successive threads of a screw is h , its radius is b , and the power acts perpendicularly to an arm a . If the thread be square and of small section, and the friction of the thread only be taken into account, show that if a and h are given, the efficiency of the machine is a maximum when $2\pi b = h \tan(\frac{1}{2}\pi + \frac{1}{2}\epsilon)$, ϵ being the limiting angle of friction. [Math. Tripos, 1867.]

Ex. 5. The axis AB of a screw is fixed in space and the beam EF through which the cylinder passes is moveable. The power P , acting at the end of a lever CD , tends to turn the cylinder, while a force Q , acting on EF parallel to the axis AB , tends to prevent motion. Show that the relation between P and Q is the same as that given in Art. 550.

Ex. 6. A weight is supported on a rough vertical screw with a rectangular thread without the application of any power. If l be the length and b the radius of the cylinder on which the thread lies, show that the screw has at least $\frac{l \cot \epsilon}{2\pi b}$ turns.

NOTE ON SOME THEOREMS IN CONICS REQUIRED IN ARTS. 126, 127.

THE following analytical proof of the two theorems in conics which are assumed in these articles requires a knowledge only of such elementary equations as those of the normal or of the chord joining two points

Let ϕ, ϕ' be the eccentric angles of two points P, Q on the conic. Taking the principal axes of the curve as the axes of coordinates, the equations of the normals at these points are

$$\frac{a\xi}{\cos \phi} - \frac{b\eta}{\sin \phi} = a^2 - b^2, \quad \frac{a\xi}{\cos \phi'} - \frac{b\eta}{\sin \phi'} = a^2 - b^2.$$

The ordinate η of their intersection is therefore given by

$$\frac{b\eta}{a^2 - b^2} = -\frac{\sin \frac{1}{2}(\phi + \phi')}{\cos \frac{1}{2}(\phi - \phi')} \sin \phi \sin \phi' \dots \dots \dots (1).$$

The ordinate of the middle point of the chord PQ is

$$\bar{y} = \frac{1}{2}b(\sin \phi + \sin \phi') = b \sin \frac{1}{2}(\phi + \phi') \cos \frac{1}{2}(\phi - \phi'),$$

$$\frac{b^2}{a^2 - b^2} \frac{\eta}{\bar{y}} = \frac{-\sin \phi \sin \phi'}{\cos^2 \frac{1}{2}(\phi - \phi')} = \frac{\cos^2 \frac{1}{2}(\phi + \phi')}{\cos^2 \frac{1}{2}(\phi - \phi')} - 1 \dots \dots (2).$$

Again, the equation to the chord PQ is

$$\frac{x}{a} \cos \frac{1}{2}(\phi + \phi') + \frac{y}{b} \sin \frac{1}{2}(\phi + \phi') - \cos \frac{1}{2}(\phi - \phi') = 0 \dots \dots (3).$$

If p, p' and q are the perpendiculars on the chord from the foci and the centre, we have the usual formula for the length of a perpendicular

$$\frac{pp'}{q^2} = \frac{\{\cos \frac{1}{2}(\phi - \phi') - e \cos \frac{1}{2}(\phi + \phi')\} \{\cos \frac{1}{2}(\phi - \phi') + e \cos \frac{1}{2}(\phi + \phi')\}}{\cos^2 \frac{1}{2}(\phi - \phi')}.$$

It follows by an easy reduction that

$$\left(\frac{\eta}{\bar{y}} - 1\right) \frac{b^2}{a^2} = -\frac{pp'}{q^2} \dots \dots \dots (4).$$

It is explained in the text that the corresponding form for ξ is an inconvenient one because the foci on the minor axis are imaginary. If the chord cut the axes in L and M , we find, from the equation to the chord PQ given above, that

$$\frac{CL}{a} = \frac{\cos \frac{1}{2}(\phi - \phi')}{\cos \frac{1}{2}(\phi + \phi')}, \quad \frac{CM}{b} = \frac{\cos \frac{1}{2}(\phi - \phi')}{\sin \frac{1}{2}(\phi + \phi')}.$$

We have immediately from (2)

$$\frac{b^2}{a^2} \left(\frac{\eta}{\bar{y}} - 1\right) = -\frac{CL^2 - a^2 + b^2}{CL^2}, \quad \frac{a^2}{b^2} \left(\frac{\xi}{\bar{x}} - 1\right) = -\frac{CM^2 - b^2 + a^2}{CM^2} \dots (5).$$

The second follows from the first by changing the letters. These are the formulæ used in Art. 126, Ex. 3 By introducing CM into the right-hand side of (1) we find

$$-\frac{CM}{a^2 - b^2} \frac{\eta}{\bar{y}} = \sin \phi \sin \phi', \quad \frac{CL}{a^2 - b^2} \frac{\xi}{\bar{x}} = \cos \phi \cos \phi' \dots \dots \dots (6).$$

When the points P, Q coincide, ξ, η become the coordinates of the centre of curvature at P . We then deduce from (1) the well-known formulæ

$$-\frac{b\eta}{a^2-b^2} = \sin^3 \phi, \quad \frac{a\xi}{a^2-b^2} = \cos^3 \phi. \dots\dots\dots (7).$$

The coordinates \bar{x}, \bar{y} of the middle point G of the chord being given, the chord itself is determinate. The equation to the chord is

$$\frac{(\xi - \bar{x})\bar{x}}{a^2} + \frac{(\eta - \bar{y})\bar{y}}{b^2} = 0.$$

We then readily find the intercepts CL, CM . We deduce from (2) or (5)

$$\left\{ \frac{b^2}{a^2-b^2} \frac{\eta}{\bar{y}} + 1 \right\} \left\{ \frac{\bar{x}^2}{a^2} + \frac{\bar{y}^2}{b^2} \right\}^2 = \frac{\bar{x}^3}{a^3} \left\{ -\frac{a^2}{a^2-b^2} \frac{\xi}{\bar{x}} + 1 \right\} \left\{ \frac{\bar{x}^2}{a^2} + \frac{\bar{y}^2}{b^2} \right\}^2 = \frac{\bar{y}^3}{b^3} \dots\dots\dots (8).$$

Let X, Y be the coordinates of the intersection T of the tangents at P, Q , then

$$\frac{X}{\bar{x}} = \frac{Y}{\bar{y}}, \quad \frac{\bar{x}X}{a^2} + \frac{\bar{y}Y}{b^2} = 1,$$

because G is the intersection of the straight line joining the origin to T with the polar line of T . We easily find \bar{x}, \bar{y} in terms of X, Y , and the equations (7) then become

$$\frac{\eta}{\bar{Y}} = \frac{(a^2-b^2)(X^2-a^2)}{a^2Y^2+b^2X^2}, \quad \frac{\xi}{\bar{X}} = -\frac{(a^2-b^2)(Y^2-b^2)}{a^2Y^2+b^2X^2} \dots\dots\dots (9),$$

which are the equations used in Art. 127.

Ex. 1. A uniform rod, whose ends are constrained to remain on a smooth elliptic wire, is in equilibrium under the action of a centre of force situated in the centre C and varying as the distance, see Art. 51. Show that the centre of gravity G must be either in one of the axes or at a distance from the centre equal to $CR^2/(a^2+b^2)^{\frac{1}{2}}$, where CR is the semi-diameter drawn through G . Show that in the latter case half the length of the rod is equal to $CD^2/(a^2+b^2)^{\frac{1}{2}}$, where CD is conjugate to CR . Show also that the tangents at the extremities of the rod are at right angles. Find the lengths of the shortest and longest rods which could be in equilibrium.

Ex. 2. One extremity of a string is tied to the middle point of a rod whose extremities are constrained to lie on a smooth elliptic wire. If the string is pulled in a direction perpendicular to the rod, show that there cannot be equilibrium unless the rod is parallel to an axis of the curve.

Ex. 3. When the conic is a parabola, show that the equations (5), (8), (9) take the simpler forms,

$$\eta = 2\bar{y} \cdot \frac{AR}{m} = \frac{2\bar{y}}{m} \left(\bar{x} - \frac{\bar{y}^2}{m} \right) = -\frac{2}{m} XY, \\ \xi = 2\bar{x} - AR + m = \bar{x} + \frac{\bar{y}^2}{m} + m = -X + \frac{2Y^2}{m} + m,$$

where A is the vertex, R the intersection of the chord with the axis, $2m$ the latus rectum, and the rest of the notation is the same as before.

Ex. 4. Show that the length L of a chord, when expressed in terms of its focal distances p, p' , is given by

$$L = \frac{2R^2}{a} \sqrt{1 - \frac{pp'}{b^2}}, \quad \frac{a^2b^2}{R^2} = b^2 + \left(\frac{p-p'}{2} \right)^2,$$

where R is the length of the semi-diameter parallel to the chord.

Ex. 5. Two chords of a conic are drawn parallel to any two conjugate diameters and touch a given confocal. Show that the sum of their lengths is constant

Ex. 6. If the normals at four points P, Q, R, S meet in a point whose co-ordinates are (ξ, η) , prove that the middle points of the six chords which join the points P, Q, R, S two and two lie on the conic

$$(a^2 - b^2) (a^2 y^2 - b^2 x^2) + a^2 b^2 (\xi x + \eta y) = 0.$$

This follows at once from (8).

Ex. 7. A heavy uniform rod is in equilibrium with both ends pressing against the interior surface of a smooth ellipsoidal bowl. If one axis of the bowl is vertical, show that the rod must lie in one of the principal planes.

The ellipsoid being referred to its axes, the normals at the extremities of the

$$\text{rod are } \frac{a^2}{x} (\xi - x) = \frac{b^2}{y} (\eta - y) = \frac{c^2}{z} (\zeta - z), \quad \frac{a^2}{x'} (\xi - x') = \frac{b^2}{y'} (\eta - y') = \frac{c^2}{z'} (\zeta - z').$$

It is necessary for equilibrium that each of these should be satisfied by $\eta = \frac{1}{2} (y + y')$, $\zeta = \frac{1}{2} (z + z')$. Substituting, we find that $y'/y = z'/z$, unless either both the y 's or both the z 's are zero. Putting $y' = \rho y$, $z' = \rho z$, the equations become

$$\frac{2a^2}{x} (\xi - x) = b^2 (\rho - 1) = c^2 (\rho - 1), \quad \frac{2a^2}{x'} (\xi - x') = b^2 \frac{1 - \rho}{\rho} = c^2 \frac{1 - \rho}{\rho}.$$

Unless $b^2 = c^2$, these give $\rho = 1$. It easily follows that $y' = y$, $z' = z$, $x' = x$ so that the two ends of the rod coincide. As this is impossible, we must have either both the y 's or both the z 's equal to zero. The rod must therefore be in a principal plane

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